

e-book

## **Statistical Distributions with Applications and StatCalc Software**

Etext.net Publisher, Venice, CA

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## Binomial Distribution

**$n$  = Number of Trials;  $k$  = Number of Successes**

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$$P(X \leq k) = \sum_{i=0}^k \binom{n}{i} p^i (1-p)^{n-i}, \quad k = 0, 1, 2, \dots, n.$$

To compute probabilities:

Enter the values of the number of trials  $n$ , success probability  $p$ , and the observed number of successes  $k$ ; click {button P( $X \leq k$ ),PI(','Probability')}.

**Example 1.** When  $n = 20$ ,  $p = 0.2$ , and  $k = 4$ ,

$$P(X \leq 4) = 0.629648, \quad P(X > 4) = 0.588551$$

$$P(X = 4) = 0.218199.$$

To compute confidence interval:

**Example 2.** Suppose that a binomial experiment of 40 trials resulted into 5 successes. To find a 95% confidence interval, enter 40 for  $n$ , 5 for the observed number of successes  $k$ , and 0.95 for the confidence level; click {button C. I.,PI(','Confidence')} to get (0.0419, 0.2680).

To compute moments:

Enter the values of  $n$  and  $p$ ; click {button Momt,PI(','Moments')}.

## Hypergeometric Distribution

$N =$  Lot Size,  $M =$  Number of Defectives,  $n =$  Sample Size

$$P(X \leq k | M, N, n) = \frac{\sum_{i=L}^k \frac{\binom{M}{i} \binom{N-M}{n-i}}{\binom{N}{n}}}{\sum_{i=L}^n \frac{\binom{M}{i} \binom{N-M}{n-i}}{\binom{N}{n}}},$$

where  $L = \max\{0, M - N + n\}$ .

To compute probabilities:

Enter the values of the lot size  $N$ , number of defectives  $M$  in the lot, sample size  $n$ , and the observed number of defectives  $k$  in the sample; click {button P( $X \leq k$ ),PI(','Probability')} .

**Example 1.** When  $N = 100$ ,  $M = 36$ ,  $n = 20$ , and  $k = 3$ ,

$P(X \leq 3) = 0.023231$ ,  $P(X > 3) = 0.995144$ , and

$$P(X = 3) = 0.018375 .$$

To compute confidence intervals:

**Example 2.** Suppose that inspection of a sample of 20 items from a lot of 100 items revealed 3 defectives. To find a 95% confidence interval for the total number of defectives in the lot, enter 100 for  $N$ , 20 for  $n$ , 3 for  $k$ , and 0.95 for the confidence level; click {button C.I.,PI(','Confidence')} to get (4, 36). Because of the discreteness of the distribution, the actual coverage probability is 95.2276%.

To compute a **one-sided upper limit** for the number of defective items in the lot, enter 0.90 for the confidence level and click {button C.I.,PI(','Confidence')} to get 33.

To compute moments:

Enter the values of the  $N$ ,  $M$ , and  $n$ ; click {button Momt,PI(','Moments')} .

## Poisson Distribution

$\lambda = \text{mean}$

$$P(X \leq k) = \sum_{i=0}^k \frac{e^{-\lambda} \lambda^i}{i!}, \quad k = 0, 1, 2, \dots$$

To compute probabilities:

Enter the values of the mean, and  $k$  at which the probability is to be computed; click {button P(X <= k),PI(','Probability')}

**Example 1.** When the mean = 6,  $k = 5$ ,

$$P(X \leq 5) = 0.44568, \quad P(X \geq 5) = 0.714944$$

and

$$P(X = 5) = 0.160623.$$

To compute confidence intervals:

**Example 2.** Suppose that a sample 20 observations yielded a total of 140 occurrences. To find a 95% confidence interval for the mean, enter 140 for  $k$ , 20 for the sample size, and 0.95 for the confidence level; click {button C.I.,PI(','Confidence')} to get (5.88853, 8.26027).

To compute moments:

Enter the value of the mean, and click {button Momt,PI(','Moments')}.

## Geometric Distribution

$k$ : Number of Failures Until the First Success

$p$ : Success Probability

---

$$P(X = k) = (1 - p)^k p, \quad k = 0, 1, 2, \dots, K$$

$$P(X \leq k) = 1 - (1 - p)^{k+1}, \quad k = 0, 1, 2, \dots, K$$

To compute probabilities:

Enter the number of failures  $k$  until the first success and the success probability  $p$ ; click on {button P(X <= k),PI(','Probability')}.

**Example 1** The probability of observing the first success at the 12th trial, when the success probability is 0.1, can be computed as follows:

Enter 11 for  $k$ , and 0.1 for  $p$ ; click on {button P(X <= k),PI(','Probability')} to get

$$P(X \leq 11) = 0.71757, \quad P(X > 11) = 0.313811$$

and

$$P(X = 11) = 0.0313811,$$

where  $X$  is the number of failures until the first success to occur.

To compute confidence intervals:

Enter the observed number of failures  $k$  until the first success and the confidence level; click on click {button C.I.,PI(','Probability')}.

**Example 2** Suppose a binomial experiment took 12 trials until the first success. To find a 95% confidence interval for  $p$ , enter 11 for  $k$ , 0.95 for confidence level; click {button C.I.,PI(','Probability')} to get (0.002, 0.285).

To compute moments:

Enter the values of  $k$  and  $p$ ; click on {button Momt,PI(','Moments')}.

## Negative Binomial

$r$  = Number of Successes;  $k$  = Number of Failures Until the  $r$ th Success

$$P(X = k) = \binom{r+k-1}{k} p^r (1-p)^k, \quad k = 0, 1, 2, \dots; 0 < p < 1.$$

To compute probabilities:

Enter the number of successes  $r$ , number of failures until the  $r$ th success, and the success probability; click {button P(X <= k),PI(','Probability')} .

Example 1. When  $r = 20$ ,  $k = 18$ , and  $p = 0.6$ ,

$$P(X \leq 18) = 0.862419, \quad P(X > 18) = 0.181983,$$

and

$$P(X = 18) = 0.0444024.$$

To compute confidence intervals:

Suppose that a binomial experiment took 35 failures until the 5th success. To find a 95% confidence interval for the success probability  $p$ , enter 5 for  $r$ , 35 for  $k$ , and 0.95 for the confidence level; click {button C.I.,PI(','Confidence')} to get (0.04186, 0.24221).

To compute moments:

Enter the values of  $r$  and the success probability  $p$ ; click {button Momt,PI(','Moments')}.

## Logarithmic Series

$$P(X \leq k) = \sum_{i=1}^k \frac{\alpha \theta^i}{i}, \quad 0 < \theta < 1; \quad k = 1, 2, K$$

where  $\alpha = -1/\ln(1 - \theta)$ .

To compute probabilities:

Enter the value of  $\theta$ , and the observed value  $k$ ; click {button P(X <= k), PI(' , 'Probability')} .

**Example 1** When  $\theta = 0.3$ ,  $k = 3$ ,

$$P(X \leq 3) = 0.9925, \quad P(X > 3) = 0.032733$$

and

$$P(X = 3) = 0.025233 .$$

To compute the MLE of  $\theta$ :

Enter the sample mean, and click on .

**Example 2** When the sample mean = 2, the MLE of  $\theta$  is 0.715332.



**Logistic Variance**

$$\frac{b^2 p^2}{3}$$

Click this button to compute probabilities

Click this button to compute percentiles

Click this button to compute confidence interval

[Click this button to go to the previous screen](#)

Click this button for the topic that follows the current topic

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$\mu$ : mean;  $\sigma$ : standard deviation

$$P(X \leq x) = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^x \exp\left\{-\frac{(y-\mu)^2}{2\sigma^2}\right\} dy.$$

To compute probabilities:

Enter the values of the mean, standard deviation, and the value of  $x$  [Perc ( $x$ )] at which the cdf is to be computed; click {button P( $X \leq x$ ),PI(','Probability')}

**Example 1.** When mean = 1.0, standard deviation = 2.0, and the observed value [Perc ( $x$ )] = 3.5,  $P(X \leq 3.5) = 0.89435$  and  $P(X > 3.5) = 0.10565$ .

To compute percentiles:

Enter the values of the mean, standard deviation, and the cumulative probability {button P( $X \leq x$ ),PI(','Probability')}; click {button Perc ( $x$ ),PI(','Percentile')}

**Example 2.** When mean = 1.0, standard deviation = 2.0, and the cumulative probability = 0.95, the 95th percentile is 4.28971. That is,  $P(X \leq 4.28971) = 0.95$ .

To compute moments:

Enter the values of the mean, and standard deviation and click {button Momt,PI(','Moments')}

## Chi-square Distribution

$n$ : degrees of freedom

$$P(X \leq x) = \frac{1}{2^{n/2} \Gamma(n/2)} \int_0^x \exp(-y/2) y^{n/2-1} dy, \\ x > 0, n > 0.$$

To compute probabilities:

Enter the value of the degrees of freedom (df), and the value of  $x$  [Perc ( $x$ )] at which the cdf is to be computed; click {button P( $X \leq x$ ),PI(','Probability')}

**Example 10.6.1** When  $df = 13.0$ , and the observed value [Perc ( $x$ )] = 12.3,  
 $P(X \leq 12.3) = 0.496789$  and  $P(X > 12.3) = 0.503211$ .

To compute percentiles:

Enter the values of the degrees of freedom, and the cumulative probability {button P( $X \leq x$ ),PI(','Probability')} ; click {button Perc ( $x$ ),PI(','Percentile')}

**Example 10.6.2** When  $df = 13.0$ , and the cumulative probability {button P( $X \leq x$ ),PI(','Probability')} = 0.95, the 95th percentile is 22.362. That is,  
 $P(X \leq 22.362) = 0.95$ .

To compute moments:

Enter the value of the df and click {button Momt,PI(','Moments')}

**Remark:** For the degrees of freedom greater than 100000, a normal approximation is used to compute the cdf as well as the percentiles.

## Student's $t$ Distribution

$n$ : degrees of freedom

$$P(X \in x) = \frac{\Gamma[(n+1)/2]}{\Gamma(n/2)\sqrt{n\pi}} \int_{-x}^x \frac{1}{(1+y^2/n)^{(n+1)/2}} dy, \quad -\infty < x < \infty, n > 1.$$

To compute probabilities:

Enter the values of the degrees of freedom (df), and [Perc (x)] at which the cdf is to be computed; click {button P(X <= x),PI(','Probability')}.

**Example 1** When df = 12.0, and the observed value [Perc (x)] = 1.3,  
 $P(X \leq 1.3) = 0.890991$  and  $P(X > 1.3) = 0.109009$ .

To compute percentiles:

Enter the value of the degrees of freedom, and the cumulative probability {button P(X <= x),PI(','Probability')}

; click {button Perc (x),PI(','Percentile')}

**Example 2** When df = 12.0, and the cumulative probability =  $P(X \leq x) = 0.95$ , the 95th percentile is 1.78229. That is,  
 $P(X \leq 1.78229) = 0.95$ .

To compute moments:

Enter the value of the df and click {button Momt,PI(','Moments')}.

**Remark:** For  $df > 50000$ , the following approximation is used to compute the table values.

$$P(t_n \in t) \approx P\left(Z \in \frac{t \sqrt{2n} - \frac{1}{2n}}{\sqrt{1 + \frac{t^2}{2n}}}\right),$$

where  $Z$  is the standard normal random variable.

## F Distribution

***m***: Numerator Degrees of Freedom

***n***: Denominator Degrees of Freedom

$$f(x; m, n) = \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} \frac{a^a x^{a-1}}{b^b [1 + (a/b)x]^{a+b}}, \quad x > 0,$$

To compute probabilities:

Enter the values of the numerator degrees of freedom, denominator df, and the value of  $x$  [Perc (x)] at which the cdf is to be computed; click on {button P(X <= x),PI(','Probability')}

**Example 1** When numerator df = 3.3, denominator df = 44.5 and the observed value [Perc (x)] = 2.3,

$$P(X \leq 2.3) = 0.915262 \text{ and } P(X > 2.3) = 0.084738 .$$

To compute percentiles:

Enter the values of the degrees of freedom, and the cumulative probability {button P(X <= x),PI(','Probability')} ; click {button Perc (x),PI(','Percentile')}

**Example 2** When numerator df = 3.3, denominator df = 44.5, and the cumulative probability {button P(X <= x),PI(','Probability')} = 0.95, the 95th percentile is 2.73281. That is,

$$P(X \leq 2.73281) = 0.95 .$$

To compute moments:

Enter the values of the numerator df, denominator df, and click {button Momt,PI(','Moments')}

## Beta Distribution

$a$ : shape parameter;  $b$ : shape parameter

$$P(X \leq x) = \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} \int_0^x y^{a-1}(1-y)^{b-1} dy,$$
$$0 < x < 1, a > 0, b > 0.$$

To compute probabilities:

Enter the values of the parameters  $a$ , and  $b$  and the value of  $x$  [Perc ( $x$ )] at which the cdf is to be computed; click {button P( $X \leq x$ ),PI(','Probability')} .

**Example 1** When  $a = 2$ ,  $b = 3$ , and the observed value

Perc ( $x$ ) = 0.4,

$$P(X \leq 0.4) = 0.5248 \text{ and } P(X > 0.4) = 0.4752.$$

To compute percentiles:

Enter the values  $a$ ,  $b$ , and the cumulative probability {button P( $X \leq x$ ),PI(','Probability')}; click on {button Perc ( $x$ ),PI(','Percentile')}.

**Example 2** When  $a = 2$ ,  $b = 3$ , and the cumulative probability  $P(X \leq x) = 0.40$ , the 40th percentile is 0.329167. That is,

$$P(X \leq 0.329167) = 0.40.$$

To compute moments:

Enter the values  $a$  and  $b$  and click on {button Momt,PI(','Moments')}

**Remark 1** When both the shape parameters  $a$  and  $b$  are greater than 200000, the usual standard normal approximation is used to compute the table values. That is,

$$\frac{X - \text{mean}}{\text{Std Dev}} \sim N(0, 1).$$

## Gamma Distribution

$a$ : shape parameter;  $b$ : scale parameter

$$P(X \leq x) = \frac{1}{\Gamma(a)b^a} \int_0^x e^{-y/b} y^{a-1} dy, \quad x > 0, a > 0, b > 0.$$

To compute probabilities:

Enter the values of the shape parameter  $a$ , scale parameter

$b$ , and the observed value [Perc (x)]; click on {button P(X <= x),PI(','Probability')}

**Example 1** When  $a = 2$ ,  $b = 3$ , and the observed value [Perc (x)] = 5.3,

$$P(X \leq 5.3) = 0.527172 \text{ and } P(X > 5.3) = 0.472828.$$

To compute percentiles:

Enter the values of  $a$ ,  $b$ , and the cumulative probability {button P(X <= x),PI(','Probability')} ;  
click {button Perc (x),PI(','Percentile')}

**Example 2** When  $a = 2$ ,  $b = 3$ , and the cumulative probability {button P(X <= x),PI(','Probability')} = 0.05, the 5th percentile is 1.06608. That is,

$$P(X \leq 1.06608) = 0.05.$$

To compute moments:

Enter the values of  $a$  and  $b$ ; click {button Momt,PI(','Moments')}

## Noncentral Chi-square

$n = \text{Degrees of Freedom} > 0; \delta = \text{Noncentrality Parameter} > 0$

$$P(\chi_n^2(\delta) \leq x) = \sum_{k=0}^{\infty} \frac{\exp(-\delta/2)(\delta/2)^k}{k!} P(\chi_{n+2k}^2 \leq x).$$

To compute probabilities:

Enter the values of the df, noncentrality parameter, and the observed value  $x$  [Perc (x)]; click {button P(X <= x),PI(','Probability')} .

**Example 1** When df = 13.0, noncentrality parameter = 2.2 and the observed value [Perc (x)] = 12.3,

$$P(X \leq 12.3) = 0.346217 \text{ and } P(X > 12.3) = 0.653783 .$$

To compute percentiles:

Enter the values of the df, noncentrality parameter, and the cumulative probability  $P(X \leq x)$ ; click {button Perc (x),PI(','Percentile')} .

**Example 2** When df = 13.0, noncentrality parameter = 2.2, and the cumulative probability  $P(X \leq x) = 0.95$ , the 95th percentile is 26.0113. That is,

$$P(X \leq 26.0113) = 0.95 .$$

To compute moments:

Enter the values of the df, and the noncentrality parameter; click {button Momt,PI(','Moments')} .

## Noncentral t Distribution

$n$  = Degrees of Freedom;  $\delta$  = Noncentrality Parameter

$$f(x; n, \delta) = \frac{n^{n/2} \exp(-\delta^2/2)}{\sqrt{\pi} \Gamma(n/2) (n+x^2)^{(n+1)/2}} \sum_{i=0}^{\infty} \frac{\Gamma[(n+i)/2] \delta^i \exp(-\delta^2/2)}{i! \Gamma(n+x^2/2)^{i/2}},$$

$-\infty < x < \infty, -\infty < \delta < \infty.$

To compute probabilities:

Enter the values of the degrees of freedom (df), noncentrality parameter, and the value of  $x$  [Perc (x)] at which the cumulative probability is to be computed; click {button P(X <= x),PI(','Probability')}

**Example 1** When df = 13.0, noncentrality parameter = 2.2 and the observed value [Perc (x)] = 2.2,

$$P(X \leq 2.2) = 0.483817 \text{ and } P(X > 2.2) = 0.516183.$$

To compute percentiles:

Enter the values of the df, noncentrality parameter, and the cumulative probability P(X <= x); click {button Perc (x),PI(','Percentile')}

**Example 2** When df = 13.0, noncentrality parameter = 2.2, and the cumulative probability P(X <= x) = 0.90, the 90th percentile is 3.87082. That is,

$$P(X \leq 3.87082) = 0.90.$$

To compute moments:

Enter the values of the DF, and the noncentrality parameter; click {button Momt,PI(','Moments')}

## Noncentral F Distribution

$m$  = Numerator df;  $n$  = Denominator df;  $\delta$  = Noncentrality Parameter

$$P(X \leq x | m, n, \delta) = \sum_{k=0}^{\infty} \frac{\exp(-\delta/2)(\delta/2)^k}{k!} \cdot P_{F_{m+2k, n}}^{\text{nc}} \left( \frac{mx}{m+2k} \right), x > 0, \delta > 0.$$

To compute probabilities:

Enter the values of the numerator df, denominator df, noncentrality parameter, and the observed value  $x$  [Perc ( $x$ )]; click {button P( $X \leq x$ ),PI(','Probability')} .

**Example 1.** When numerator df = 4.0, denominator df = 32.0, noncentrality parameter = 2.2, and the observed value [Perc ( $x$ )] = 2,

$$P(X \leq 2) = 0.702751 \text{ and } P(X > 2) = 0.297249 .$$

To compute percentiles:

Enter the values of the df, noncentrality parameter, and the cumulative probability  $P(X \leq x)$ ; click {button Perc ( $x$ ),PI(','Percentile')} .

**Example 2.** When numerator df = 4.0, denominator df = 32.0, noncentrality parameter = 2.2, and the cumulative probability  $P(X \leq x) = 0.90$ , the 90th percentile is 3.22243. That is,  $P(X \leq 3.22243) = 0.90$ .

To compute moments:

Enter the values of the numerator df, denominator df and the noncentrality parameter; click {button Momt,PI(','Moments')} .

## Laplace Distribution

$a$  = Location Parameter;  $b$  = Scale Parameter

$$P(X \in x) = \frac{1}{2b} \exp\left(-\frac{|y-a|}{b}\right) dy, \\ -\infty < x < \infty, -\infty < a < \infty, b > 0,$$

To compute probabilities:

Enter the values of the parameters  $a$ , and  $b$  and the observed value  $x$  [Perc ( $x$ )]; click {button P( $X \leq x$ ),PI(','Probability')}

**Example 1** When  $a = 3$ ,  $b = 4$ , and the observed value

Perc ( $x$ ) = 4.5,

$$P(X \leq 4.5) = 0.656355 \text{ and } P(X > 4.5) = 0.343645 .$$

To compute percentiles:

Enter the values of  $a$ ,  $b$ , and the cumulative probability; click {button Perc ( $x$ ),PI(','Percentile')}

**Example 2** When  $a = 3$ ,  $b = 4$ , and the cumulative probability  $P(X \leq x) = 0.95$ , the 95th percentile is 12.2103. That is,

$$P(X \leq 12.2103) = 0.95 .$$

To compute moments:

Enter the values of  $a$  and  $b$  and click {button Momt,PI(','Moments')}

## Logistic Distribution

$a$  = Location Parameter;  $b$  = Scale Parameter

$$P(X \leq x) = \frac{1}{1 + \exp\left\{-\frac{x-a}{b}\right\}},$$

$-\infty < x < \infty, -\infty < a < \infty, b > 0.$

To compute probabilities:

Enter the values of the parameters  $a$ , and  $b$  and the observed value  $x$  [Perc (x)]; click {button P(X <= x),PI(','Probability')}.}

**Example 1** When  $a = 2$ ,  $b = 3$ , and the observed value  $x = 1.3$ ,

$$P(X \leq 1.3) = 0.44193 \text{ and } P(X > 1.3) = 0.55807.$$

To compute percentiles:

Enter the values  $a$ ,  $b$ , and the cumulative probability; click {button Perc (x),PI(','Percentile')}.}

**Example 2** When  $a = 2$ ,  $b = 3$ , and the cumulative probability = 0.25, the 25th percentile is  $-1.29584$ . That is,

$$P(X \leq -1.29584) = 0.25.$$

To compute moments:

Enter the values of  $a$  and  $b$  and click {button Momt,PI(','Moments')}.}

## Weibull Distribution

$m$  = location parameter;  $c$  = shape parameter;  $b$  = scale parameter

$$P(X \leq x | b, c, m) = 1 - \exp\left[-\left(\frac{x - m}{b}\right)^c\right], \quad x \geq m.$$

To compute probabilities:

Enter the values of the parameters  $m$ ,  $c$ , and  $b$  and the observed value  $x$  [Perc (x)]; click {button P(X <= x),PI(','Probability')}

**Example 1** When  $m = 0$ ,  $c = 2.3$ ,  $b = 2$ , and the observed value [Perc(x)] = 3.4,

$$P(X \leq 3.4) = 0.966247 \text{ and } P(X > 3.4) = 0.033753.$$

To compute percentiles:

Enter the values of  $m$ ,  $c$ ,  $b$ , and the cumulative probability; click {button Perc (x),PI(','Percentile')}

**Example 2** When  $m = 0$ ,  $c = 2.3$ ,  $b = 2$ , and the cumulative probability  $P(X \leq x) = 0.95$ , the 95th percentile is 3.22259. That is,

$$P(X \leq 3.22259) = 0.95.$$

To compute moments:

Enter the values of  $c$  and  $b$  and click {button Momt,PI(','Moments')}. The moments are computed assuming that  $m = 0$ .

## Extreme Value

$a$  = location parameter;  $b$  = scale parameter

$$P(X \leq x) = \exp\{-\exp[-(x - a)/b]\}, \quad -\infty < x < \infty, \quad b > 0.$$

### To compute probabilities:

Enter the values of the parameters  $a$ , and  $b$  and the observed value [Perc (x)]; click {button P(X <= x),PI(','Probability')}

**Example 1** When  $a = 1$ ,  $b = 2$ , and the observed value Perc (x) = 1.2,

$$P(X \leq 1.2) = 0.404608 \quad \text{and} \quad P(X > 1.2) = 0.595392 .$$

### To compute percentiles:

Enter the values  $a$ ,  $b$ , and the cumulative probability  $P(X \leq x)$ ; click {button Perc,PI(','Percentile')}

**Example 2** When  $a = 1$ ,  $b = 2$ , and the cumulative probability = 0.15, the 15th percentile is -0.280674. That is,

$$P(X \leq -0.280674) = 0.15 .$$

### To compute moments:

Enter the values  $a$  and  $b$  and click {button Momt,PI(','Moments')}

## Pareto Distribution

$$P(X \leq x) = 1 - \frac{a^b}{x^b}, x \geq a, b > 0.$$

To compute probabilities:

Enter the values of the parameters  $a$ , and  $b$  and the observed value [Perc (x)]; click {button P(X <= x),PI('','Probability')} .

**Example 1** When  $a = 2$ ,  $b = 3$ , and the observed value Perc (x) = 3.4,

$$P(X \leq 3.4) = 0.796458 \text{ and } P(X > 3.4) = 0.203542 .$$

To compute percentiles:

Enter the values  $a$ ,  $b$ , and the cumulative probability  $P(X \leq x)$ ; click {button Perc(x),PI('','Percentile')} .

**Example 2** When  $a = 2$ ,  $b = 3$ , and the cumulative probability = 0.15, the 15th percentile is 2.11133. That is,

$$P(X \leq 2.11133) = 0.15 .$$

To compute moments:

Enter the values  $a$  and  $b$  and click {button Momt,PI('','Moments')} .

## Cauchy Distribution

$a$  = location parameter;  $b$  = scale parameter

$$f(x; a, b) = \frac{1}{\pi b [1 + ((x - a)/b)^2]}, \quad -\infty < a < \infty, b > 0.$$

To compute probabilities:

Enter the values of the parameters  $a$ , and  $b$  and the observed value  $x$  [Perc (x)]; click {button P(X <= x), PI('', 'Probability')}

**Example 1** When  $a = 1$ ,  $b = 2$ , and the observed value Perc (x) = 1.2,

$$P(X \leq 1.2) = 0.531726 \text{ and } P(X > 1.2) = 0.468274.$$

To compute percentiles:

Enter the values of  $a$ ,  $b$ , and the cumulative probability; click {button Perc (x), PI('', 'Percentile')}.

**Example 2** When  $a = 1$ ,  $b = 2$ , and the cumulative probability  $P(X \leq x) = 0.95$ , the 95th percentile is 13.6725. That is,

$$P(X \leq 13.6725) = 0.95.$$

## Distribution of Runs

**Example 1.** Consider the sequence:

a a a b b a b b b a a a a b b a a a a b

Here  $m = 12$ ,  $n = 8$ , and the observed number of runs  $r = 8$ . To get the probabilities, enter 12 for the number of first type symbols, 8 for the number second type symbols, and 8 for the observed number of runs; click on {button  $P(R \leq r)$ ,PI(','Probability')}

$$P(R \leq 8) = 0.159085 \text{ and } P(R \geq 9) = 0.932603 .$$

Since the probability of observing 8 or fewer runs is not less than  $0.05/2 = 0.025$ , the null hypothesis that the arrangement is random will be retained at the 5% level.

To find the **critical value** at the 5% level, enter 0.025 for the tail probability, and click on {button Critical,PI(','Critical')}

$$P(R \leq 6) = 0.024609 \text{ and } P(R \geq 16) = 0.00654918 .$$

This means that the null hypothesis of randomness will be rejected (at 5% level) when the observed number of runs is less than or equal to 6 or greater than or equal to 16. Because of the discreteness of the distribution, the attained size is  $0.024609 + 0.00655 = 0.031159$  (not 0.05).

**Example 2.** Suppose that  $m = 50$  and  $n = 40$ . To find the 10% critical points, enter 50 for the first type symbols, 40 for the second type symbols, and 0.05 for the tail probability; click on {button Critical,PI(','Critical')}

$$P(X \leq 37) = 0.0438777 \text{ and } P(X \geq 54) = 0.0413445 .$$

Thus, the null hypothesis of randomness will be rejected at 10% level, if the total number of runs is less than or equal to 37 or is greater than or equal to 54.

## Sign Test and CI for Median

Let  $X_1, \dots, X_n$  be a sample of independent observations from a continuous population. Let  $X(j)$  denote the  $j$ th smallest of the  $X_i$ 's. For given confidence level  $(1 - \alpha)$ , and the sample size  $n$ , *StatCalc* computes the integers  $p$  and  $q$  such that the interval  $(X(p), X(q))$  would contain the population median with  $100(1 - \alpha)\%$  confidence. **StatCalc** also computes the p-values for hypothesis testing about the median.

**Example 1** To compute a 95% confidence interval for the median of a continuous population based on a sample of 40 observations, select **CI Med** from **StatCalc**, enter 40 for  $n$ , 0.95 for confidence level, click **OK** to get 14 and 27. That is, the required confidence interval is formed by the 14th and 27th order statistics from the sample.

**Example 2** Suppose that a sample of 40 observations yielded  $k = 13$ . To obtain the p-value of the test when  $H_a: M < M_0$ , select **CI Med** from **StatCalc**, enter 40 for  $n$ , 13 for  $k$  and click **P(K<=k)** to get  
 $P(K \leq 13) = 0.0192387$ .

Since this p-value is less than 0.05, the null hypothesis that  $H_0: M = M_0$  will be rejected at the 5% level.

## Wilcoxon Signed-Rank Test

**Example 1** Suppose that a sample of 20 observations yielded  $T^+ = 130$ . Then, enter 20 for  $n$ , 130 for  $T^+$ , and click on {button  $P(T^+ \leq k)$ , PI(','Probability')} to get

$$P(T^+ \leq 130) = 0.825595 \text{ and } P(T^+ \geq 130) = 0.184138 .$$

This program also computes the critical point for given  $n$  and the level  $\alpha$ .

**Example 2** When  $n = 20$ , cumulative probability = 0.95, the upper tail critical point is 150; that is

$$P(T^+ \geq 150) = 0.048654 .$$

**Remark :** For  $n > 60$ , the standard normal approximation is used to compute the table values.

## Wilcoxon Rank-Sum Test

WRS statistic is useful to test whether two continuous distributions are equal. Let  $X_1, \dots, X_m$  be independent observations from a continuous distribution  $F_X$  and  $Y_1, \dots, Y_n$  be independent observations from a continuous distribution  $F_Y$ . Pool  $X$  and  $Y$  observations, and arrange them in increasing order. Let  $W$  denote the sum of the ranks of the  $X$  observations in the pooled sample.

For given  $m$ ,  $n$ , and  $W$  this program computes the probabilities.

**Example 1** When  $m = 13$ ,  $n = 12$ , and the observed value of  $W$  is 180,  
 $P(W \leq 180) = 0.730877$  and  $P(W > 180) = 0.287146$ .

This program also computes the critical values for given  $m$ ,  $n$ , and the level of significance.

**Example 2** When  $m = 13$ ,  $n = 12$ , and the cumulative probability is 0.05, the left tail critical value is 138. Because of the discreteness of the distribution, the attained level is 0.048821; that is  
 $P(W \leq 138) = 0.048821$ .

**Remark 1** An exact method is used to compute the table values when  $m \leq 60$  and  $n \leq 60$ . For all other values, the standard normal approximation is used.

## NP Tolerance Limits

Let  $X_1, \dots, X_n$  be a sample of independent observations from a continuous population. Let  $X(k)$  denote the  $k$ th smallest of the  $X_i$ 's. For given  $0 < p < 1$ ,  $0 < g < 1$ , this program computes the value of  $n$  so that the interval

$$(X(1), X(n))$$

would contain at least  $p$  proportion of the population with confidence  $g$ .

**Example 1** When  $p = 0.90$ , and  $g = 0.95$ , the value of  $n$  is 46; that is the interval  $(X(1), X(46))$  would contain at least 90% of the population with confidence 95%. Further, the sample size required for a one-sided tolerance limit is 29; that is 90% of the population data are less than or equal to  $X(29)$  - the largest observation in a sample of 29 observations, with confidence 95%. Similarly, 90% of the population data are greater than or equal to  $X(1)$  - the smallest observation in a sample of 29 observations, with confidence 95%.

## Tolerance Factors for Normal Population

Let  $\bar{x}$  and  $s$  denote respectively the mean and standard deviation of a random sample of  $n$  observations from a normal population. For given  $n$ ,  $df$ ,  $0 < p < 1$ , and  $0 < g < 1$ , this program computes the value of  $k$  such that the interval

$$\bar{x} \pm ks$$

contains at least  $100p\%$  of the population with confidence  $100g\%$ . We refer this tolerance interval as  $(p, g)$  tolerance interval. **StatCalc** uses an exact method of computing the tolerance factor  $k$ .

**Remark 1** Here, the  $df$  associated with  $s$  is  $n - 1$ . In some situations, the  $df$  associated with the  $s$  could be different from  $n - 1$ . For example, in one-way ANOVA, the  $df$  associated with the pooled sample variance is  $(n - m)$ , where  $m$  denotes the number of groups. If one is interested in computing  $(p, g)$  tolerance interval

$$\bar{x}_i \pm k_i s_p$$

for the first population, then for this case, sample size is  $n_i$  and the degrees of freedom associated with the pooled variance is

$$\sum_{i=1}^m n_i - m,$$

where  $n_i$  denotes the size of the sample from the  $i$ th group,  $i = 1, \dots, m$ .

This program also computes one-sided tolerance factor  $c$  such that at least  $100p\%$  of the population data are less than or equal to

$$\bar{x} + cs$$

with confidence  $100g\%$ . The lower tolerance limit is

$$\bar{x} - cs.$$

That is, at least  $100p\%$  of the population data are greater than or equal to  $\bar{x} - cs$ .

**Example 1** When  $n = 23$ ,  $df = 22$ ,  $p = 0.90$ , and  $g = 0.95$ , the one-sided tolerance factor is 1.86902, and the two-sided tolerance factor is 2.25125.

## Fisher's Exact Test for 2 x 2 Table

For a given 2 x 2 table, **StatCalc** computes the probability of observing  $k$  or more successes (as well as the probability of observing  $k$  or less number of successes) in the cell (1,1). If either of this probability is less than  $\alpha/2$ , then the null hypothesis of equal proportions will be rejected at the level of significance  $\alpha$ .

Further, for given level of significance, power and guess values on  $p_1$  and  $p_2$ , **StatCalc** also computes an approximate sample size needed for each group to satisfy the requirements.

To find the p-value, enter the cell values and click Prob  $\leq$  (1,1) cell .

To compute the sample size, enter the level of significance, power, guess values of  $p_1$  and  $p_2$ , and then click S Size .

## Multivariate Tolerance Factors

Let  $\bar{X}$  and  $S$  denote respectively the mean and variance-covariance matrix of a random sample of  $n$  observations from an  $m$ -variate normal population. For a given  $n$ ,  $m$ ,  $0 < p < 1$ , and  $0 < g < 1$ , this program computes an approximate tolerance factor  $k$  such that the region

$$R = \{Y : (Y - \bar{X})' S^{-1} (Y - \bar{X}) \leq k\}$$

would contain at least  $100p\%$  of the population with confidence  $100g\%$ .

**Example 1** When  $n = 35$ ,  $m = 3$ ,  $p = 0.90$ , and  $g = 0.95$ , the 90% content – 95% coverage tolerance factor  $k$  is 9.83.

## Multiple Correlation Coefficient

### To compute probabilities:

Enter the values of the sample size  $n$ , number of variates  $m$ , squared population multiple correlation coefficient  $\rho^2$ , and the observed value of the squared sample multiple correlation coefficient  $r^2$ ; click {button P( $X \leq x$ ),PI('','Probability')}

**Example 1** When  $n = 40$ ,  $m = 4$ ,  $\rho^2 = 0.8$ ,  $r^2 = 0.75$ ,

$P(X \leq 0.75) = 0.151447$  and  $P(X > 0.75) = 0.84855$ .

### To compute percentiles:

Enter the values of the sample size  $n$ , number of variates  $m$ , squared population multiple correlation coefficient  $\rho^2$ , and the cumulative probability; click {button Perc,PI('','Percentile')}

**Example 2** When  $n = 40$ ,  $m = 4$ ,  $\rho^2 = 0.8$ , the 90th percentile is 0.874521.

### To compute confidence intervals:

Enter the values of  $n$ ,  $m$ ,  $r^2$ , and the confidence level; Click {button C.I.,PI('','Confidence')}

**Example 3** When  $n = 40$ ,  $m = 4$ ,  $r^2 = 0.75$ , the 95% confidence interval for  $\rho^2$  is (0.544189, 0.848982).

## Correlation Coefficient

### To compute probabilities:

Enter the sample size  $n$ , population correlation  $\rho$ , and the observed sample correlation  $r$ ; click {button P( $X \leq r$ ),PI(','Probability')}

**Example 1** When  $n = 30$ ,  $\rho = 0.7$ ,  $r = 0.75$ ,

$P(X \leq 0.75) = 0.687844$  and  $P(X > 0.75) = 0.312156$ .

### To compute percentiles:

Enter the sample size  $n$ , population correlation  $\rho$ , and the cumulative probability; click {button Perc,PI(','Percentile')}

**Example 2** When  $n = 30$ ,  $\rho = 0.7$ , and the cumulative probability is 0.95, the 95th percentile is 0.832097. That is,

$P(X \leq 0.832097) = 0.95$ .

### To compute confidence intervals:

Enter the sample size, sample correlation, and the confidence level; click {button C.I.,PI(','Confidence')}

**Example 3** When  $n = 30$ ,  $r = 0.6$ , a 95% confidence interval for  $\rho$  is (0.30584, 0.78959).

**Remark 1** An exact method is used to compute the cdf and the percentiles when  $n$  in  $[4, 100]$  and  $\rho$  in  $[-0.83, 0.83]$ . For the other values, Fisher's z transformation is used to compute cdf and the percentiles. Further, confidence intervals are based on the Fisher's z transformation.

## Bivariate Normal Distribution

Let  $(X, Y)$  be a bivariate normal random vector with mean  $= (0, 0)$ , and the correlation coefficient  $\rho$ . For given  $x, y$ , and  $\rho$ , this program computes the following probabilities:

$$P(X \leq x, Y \leq y), \quad P(X \leq x, Y > y)$$

$$P(X > x, Y \leq y), \quad P(X > x, Y > y)$$

$$\text{and } P(|X| < x, |Y| < y).$$

**Example 1** When  $x = 1.1, y = 0.8$ , and  $\rho = 0.6$ ,

$$P(X \leq 1.1, Y \leq 0.8) = 0.730456$$

$$P(X \leq 1.1, Y > 0.8) = 0.133878$$

$$P(X > 1.1, Y \leq 0.8) = 0.057688$$

$$P(X > 1.1, Y > 0.8) = 0.077977$$

$$P(|X| < 1.1, |Y| < 0.8) = 0.465559 .$$

If  $(X, Y)$  is a normal random vector with mean  $= (\mu_1, \mu_2)$ , and covariance matrix

$$S = \begin{pmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{21} & \sigma_2^2 \end{pmatrix}$$

then to compute the above probabilities at  $(x, y)$ , enter the standardized values

$$\frac{x - \mu_1}{\sigma_1}, \quad \frac{y - \mu_2}{\sigma_2}, \quad \text{and } \rho = \frac{\sigma_{12}}{\sigma_1 \sigma_2}.$$

## Maximum of $|t|$ Variables

Let  $X_1, \dots, X_k$  be independent normal random variables with mean  $\mu$  and common standard deviation  $\sigma$ . Let

$ms^2 / \sigma^2$  follow a chi-square distribution with  $df = m$ . **StatCalc** computes the distribution function of

$$X = \max_i \frac{|X_i - \bar{y}|}{s} \sqrt{\frac{m}{k}}$$

The percentiles of  $X$  are useful for constructing simultaneous confidence intervals for the treatment effects and orthogonal estimates in the analysis of variance, and to test extreme values.

**To compute probabilities:**

Enter the values of the number of groups  $k$ ,  $df$ , and the observed  $x$ ; click {button P(X <= x),PI('','Probability')}

**Example 1.** When  $k = 4$ ,  $df = 45$ , and  $x = 2.3$ ,

$$P(X \leq 2.3) = 0.900976 \text{ and } P(X > 2.3) = 0.099024.$$

**To compute percentiles:**

Enter the values of  $k$ ,  $df$ , and the cumulative probability; click {button Perc,PI('','Percentile')}

**Example 2.** When  $k = 4$ ,  $df = 45$ , and the cumulative probability is 0.95, the 95th percentile is 2.5897. That is,

$$P(X \leq 2.5897) = 0.95.$$

## Lognormal Distribution

$$P(X \in x) = \frac{1}{(\sqrt{2\pi})y\sigma} \exp\left[-\frac{(\ln y - \mu)^2}{2\sigma^2}\right] dy,$$

$x > 0, \sigma > 0, -\infty < \mu < \infty.$

### To compute probabilities:

Enter the values of the parameters  $\mu$  and  $\sigma$ , and the observed value  $x$  [Perc (x)]; click {button P(X <= x),PI(','Probability')}.

**Example 1** When  $\mu = 1$ ,  $\sigma = 2$ , and the observed value

[Perc (x)] = 2.3,

$P(X \leq 2.3) = 0.466709$  and  $P(X > 2.3) = 0.533291$ .

### To compute percentiles:

Enter the values  $\mu$ ,  $\sigma$ , and the cumulative probability  $P(X \leq x)$ ; click on {button Perc (x),PI(','Percentile')}.

**Example 2** When  $\mu = 1$ ,  $\sigma = 2$ , and the cumulative probability  $P(X \leq x) = 0.95$ , the 95th percentile is 72.9451. That is,

$P(X \leq 72.9451) = 0.95$ .

### To compute moments:

Enter the values of  $\mu$  and  $\sigma$  and click {button Momt,PI(','Moments')}.

