e-book

Statistical Distributions with Applications and StatCalc Software

Etext.net Publisher, Venice, CA

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Binomial Distribution

n = Number of Trials; k = Number of Successes

$$P(X \le k) = \sum_{i=0}^{k} \binom{n}{i} p^i (1-p)^{n-i}, \quad k = 0, 1, 2, \ldots, n.$$

To compute probabilities:

Enter the values of the number of trials n, success probability p, and the observed number of successes k; click {button P(X <= k),PI(`',`Probability')}.

Example 1. When $n = 20$, $p = 0.2$, and $k = 4$,

$P(X \le 4) = 0.629648$, $P(X \ge 4) = 0.588551$

$P(X = 4) = 0.218199$.

To compute confidence interval:

Example 2. Suppose that a binomial experiment of 40 trials resulted into 5 successes. To find a 95% confidence interval, enter 40 for n, 5 for the observed number of successes k, and 0.95 for the confidence level; click {button C. I.,PI(`',`Confidence')} to get (0.0419, 0.2680).

To compute moments:

Enter the values of n and p; click {button Momt,PI(`',`Moments')}.

Hypergeometric Distribution

N = Lot Size, M = Number of Defectives, n = Sample Size

$$P(X \le k \mid M, N, n) = \sum_{i=L}^{k} \frac{\binom{M}{i}\binom{N-M}{n-i}}{\binom{N}{n}},$$

where $L = \max\{0, M - N + n\}$.

To compute probabilities:

Enter the values of the lot size N, number of defectives M in the lot, sample size n, and the observed number of defectives k in the sample; click {button P(X <= k),PI(`',`Probability')} .

Example 1. When $N = 100$, $M = 36$, $n = 20$, and $k = 3$,

$P(X \le 3) = 0.023231$, $P(X \ge 3) = 0.995144$, and

$$P(X = 3) = 0.018375.$$

To compute confidence intervals:

Example 2. Suppose that inspection of a sample of 20 items from a lot of 100 items revealed 3 defectives. To find a 95% confidence interval for the total number of defectives in the lot, enter 100 for N, 20 for n, 3 for k, and 0.95 for the confidence level; click {button C.I.,PI(`',`Confidence')} to get (4, 36). Because of the discreteness of the distribution, the actual coverage probability is 95.2276%.

To compute a one-sided upper limit for the number of defective items in the lot, enter 0.90 for the confidence level and click {button C.I.,PI(`',`Confidence')} to get 33.

To compute moments:

Enter the values of the N, M, and n; click {button Momt,PI(`',`Moments')}.

Poisson Distribution

λ = mean

$$P(X \le k) = \sum_{i=0}^{k} \frac{e^{-\lambda}\lambda^{i}}{i!}, \quad k = 0,1,2,\ldots$$

To compute probabilities:

Enter the values of the mean, and *k* at which the probability is to be computed; click {button P(X <= k),PI(`',`Probability')} .

Example 1. When the mean = 6, *k* = 5,

$P(X \le 5) = 0.44568$, $P(X \ge 5) = 0.714944$

and

$P(X = 5) = 0.160623$.

To compute confidence intervals:

Example 2. Suppose that a sample 20 observations yielded a total of 140 occurrences. To find a 95% confidence interval for the mean, enter 140 for *k*, 20 for the sample size, and 0.95 for the confidence level; click {button C.I.,PI(`',`Confidence')} to get (5.88853, 8.26027).

To compute moments:

Enter the value of the mean, and click {button Momt,PI(`',`Moments')}.

Geometric Distribution

k: Number of Failures Until the First Success
p: Success Probability

$P(X = k) = (1 - p)^k p, \quad k = 0,1,2,\ldots$

$P(X \le k) = 1 - (1 - p)^{k+1}, \quad k = 0,1,2,\ldots$

To compute probabilities:
Enter the number of failures *k* until the first success and the success probability *p*; click on {button P(X <= k),PI(`',`Probability')}.

Example 1 The probability of observing the first success at the 12th trial, when the success probability is 0.1, can be computed as follows:
Enter 11 for *k*, and 0.1 for *p*; click on {button P(X <= k),PI(`',`Probability')} to get
$P(X \le 11) = 0.71757$, $P(X \ge 11) = 0.313811$
and
$P(X = 11) = 0.0313811$,
where *X* is the number of failures until the first success to occur.

To compute confidence intervals:
Enter the observed number of failures *k* until the first success and the confidence level; click on click {button C.I.,PI(`',`Probability')}.

Example 2 Suppose a binomial experiment took 12 trials until the first success. To find a 95% confidence interval for *p*, enter 11 for *k*, 0.95 for confidence level; click {button C.I.,PI(`',`Probability')} to get (0.002, 0.285).

To compute moments:
Enter the values of k and p; click on {button Momt,PI(`',`Moments')}.

Negative Binomial

r = Number of Successes; k = Number of Failures Until the rth Success

$$P(X = k) = \binom{r + k - 1}{r - 1} p^r (1 - p)^k, \quad k = 0,1,2,\ldots;\ 0 < p < 1.$$

To compute probabilities:

Enter the number of successes r, number of failures until the rth success, and the success probability; click {button P(X <= k),PI(`',`Probability')} .

Example 1. When $r = 20$, $k = 18$, and $p = 0.6$,

$P(X \le 18) = 0.862419$, $P(X \ge 18) = 0.181983$,

and

$P(X = 18) = 0.0444024$.

To compute confidence intervals:

Suppose that a binomial experiment took 35 failures until the 5th success. To find a 95% confidence interval for the success probability p, enter 5 for r, 35 for k, and 0.95 for the confidence level; click {button C.I.,PI(`',`Confidence')} to get (0.04186, 0.24221).

To compute moments:

Enter the values of r and the success probability p; click {button Momt,PI(`',`Moments')}.

Logarithmic Series

$$P(X \le k) = \sum_{i=1}^{k} \frac{\alpha\theta^{i}}{i}, \quad 0 < \theta < 1; \; k = 1,2,\ldots$$

where $\alpha = -1/[\ln(1 - \theta)]$.

To compute probabilities:

Enter the value of θ, and the observed value *k*;click {button P(X <= k),PI(`',`Probability')} .

Example 1 When $\theta = 0.3$, $k = 3$,

$P(X \le 3) = 0.9925$, $P(X \ge 3) = 0.032733$

and

$P(X = 3) = 0.025233$.

To compute the MLE of θ:

Enter the sample mean, and click on MLE.

Example 2 When the sample mean = 2, the MLE of θ?is 0.715332.

Logistic Variance

$$\frac{b^{2}p^{2}}{3}$$

Click this button to compute probabilities

Click this button to compute percentiles

Click this button to compute confidence interval

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Contents

Normal Distribution

μ: mean; σ: standard deviation

$$P(X \le x) = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{x} \exp\left[-\frac{(y-\mu)^2}{2\sigma^2}\right] dy.$$

To compute probabilities:

Enter the values of the mean, standard deviation, and the value of *x* [Perc (x)] at which the cdf is to be computed; click {button P(X <= x),PI(`',`Probability')}

Example 1. When mean = 1.0, standard deviation = 2.0, and the observed value [Perc (*x*)] = 3.5,
$P(X \le 3.5) = 0.89435$ and $P(X > 3.5) = 0.10565$.

To compute percentiles:

Enter the values of the mean, standard deviation, and the cumulative probability {button P(X <= x),PI(`',`Probability')}; click {button Perc (x),PI(`',`Percentile')}

Example 2. When mean = 1.0, standard deviation = 2.0, and the cumulative probability = 0.95, the 95th percentile is 4.28971. That is,
$P(X \le 4.28971) = 0.95$.

To compute moments:

Enter the values of the mean, and standard deviation and click {button Momt,PI(`',`Moments')}

Chi-square Distribution

n: degrees of freedom

$$P(X \le x) = \frac{1}{2^{n/2}\Gamma(n/2)} \int_0^x \exp(-y/2)\, y^{n/2-1}\, dy,$$

$$x > 0,\ n > 0.$$

To compute probabilities:

Enter the value of the degrees of freedom (df), and the value of *x* [Perc (x)] at which the cdf is to be computed; click {button P(X <= x),PI(`',`Probability')}

Example 10.6.1 When df = 13.0, and the observed value [Perc (x)] = 12.3,
$P(X \le 12.3) = 0.496789$ and $P(X > 12.3) = 0.503211$.

To compute percentiles:

Enter the values of the degrees of freedom, and the cumulative probability {button P(X <= x),PI(`',`Probability')} ; click {button Perc (x),PI(`',`Percentile')}

Example 10.6.2 When df = 13.0, and the cumulative probability {button P(X <= x),PI(`',`Probability')} = 0.95, the 95th percentile is 22.362. That is,
$P(X \le 22.362) = 0.95.$

To compute moments:

Enter the value of the df and click {button Momt,PI(`',`Moments')}

Remark: For the degrees of freedom greater than 100000, a normal approximation is used to compute the cdf as well as the percentiles.

Student's *t* Distribution

n: degrees of freedom

$$P(X \le x) = \frac{\Gamma[(n+1)/2]}{\Gamma(n/2)\sqrt{n\pi}} \int_{-\infty}^{x} \frac{1}{(1+y^2/n)^{(n+1)/2}}\, dy,$$
$$-\infty < x < \infty,\ n \ge 1.$$

To compute probabilities:

Enter the values of the degrees of freedom (df), and [Perc (x)] at which the cdf is to be computed; click {button P(X <= x),PI(`',`Probability')}.

Example 1 When df = 12.0, and the observed value [Perc (x)] = 1.3,

$P(X \le 1.3) = 0.890991$ and $P(X > 1.3) = 0.109009$.

To compute percentiles:

Enter the value of the degrees of freedom, and the cumulative probability {button P(X <= x),PI(`',`Probability')}

; click {button Perc (x),PI(`',`Percentile')}

Example 2 When df = 12.0, and the cumulative probability = P(X <= x) = 0.95, the 95th percentile is 1.78229. That is,

$P(X \le 1.78229) = 0.95$.

To compute moments:

Enter the value of the df and click {button Momt,PI(`',`Moments')}.

Remark: For df > 50000, the following approximation is used to compute the table values.

$$P(t_n \le t) \simeq P\left(Z \le \frac{t\left(1 - \frac{1}{4n}\right)}{\sqrt{1 + \frac{t^2}{2n}}}\right),$$

where Z is the standard normal random variable.

F Distribution

m: Numerator Degrees of Freedom

n: Denominator Degrees of Freedom

$$f(x;m,n)=\frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)}\frac{a^{a}x^{a-1}}{b^{a}[1+(a/b)x]^{a+b}},\quad x>0,$$

To compute probabilities:

Enter the values of the numerator degrees of freedom, denominator df, and the value of *x* [Perc (x)] at which the cdf is to be computed; click on {button P(X <= x),PI(`',`Probability')}

Example 1 When numerator df = 3.3, denominator df = 44.5 and the observed value [Perc (*x*)] = 2.3,

$P(X \le 2.3) = 0.915262$ and $P(X > 2.3) = 0.084738$.

To compute percentiles:

Enter the values of the degrees of freedom, and the cumulative probability {button P(X <= x),PI(`',`Probability')} ; click {button Perc (x),PI(`',`Percentile')}

Example 2 When numerator df = 3.3, denominator df = 44.5,and the cumulative probability {button P(X <= x),PI(`',`Probability')} = 0.95, the 95th percentile is 2.73281. That is,

$P(X \le 2.73281) = 0.95$.

To compute moments:

Enter the values of the numerator df, denominator df , and click {button Momt,PI(`',`Moments')}

Beta Distribution

a: shape parameter; *b*: shape parameter

$$P(X \le x) = \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} \int_0^x y^{a-1}(1-y)^{b-1}dy,$$

$$0 < x < 1, a > 0, b > 0.$$

To compute probabilities:

Enter the values of the parameters *a*, and *b* and the value of *x* [Perc (x)] at which the cdf is to be computed; click {button P(X <= x),PI(`',`Probability')} .

Example 1 When *a* = 2, *b* = 3, and the observed value

Perc (x) = 0.4,

$P(X \le 0.4) = 0.5248$ and $P(X > 0.4) = 0.4752$.

To compute percentiles:

Enter the values *a*, *b*, and the cumulative probability {button P(X <= x),PI(`',`Probability')}; click on {button Perc (x),PI(`',`Percentile')}.

Example 2 When *a* = 2, *b* = 3, and the cumulative probability P(X <= x) = 0.40, the 40th percentile is 0.329167. That is,

$P(X \le 0.329167) = 0.40$.

To compute moments:

Enter the values *a* and *b* and click on {button Momt,PI(`',`Moments')}

Remark 1 When both the shape parameters *a* and *b* are greater than 200000, the usual standard normal approximation is used to compute the table values. That is,

$$\frac{X - \text{mean}}{\text{Std Dev}} \sim N(0, 1).$$

Gamma Distribution

a: shape parameter; *b*: scale parameter

$$P(X \le x) = \frac{1}{\Gamma(a) b^a} \int_0^x e^{-y/b} y^{a-1} dy, \quad x > 0, a > 0, b > 0.$$

To compute probabilities:

Enter the values of the shape parameter *a*, scale parameter *b*, and the observed value [Perc (x)]; click on {button P(X <= x),PI(`',`Probability')}

Example 1 When *a* = 2, *b* = 3, and the observed value [Perc (x)] = 5.3,
$P(X \le 5.3) = 0.527172$ and $P(X > 5.3) = 0.472828$.

To compute percentiles:

Enter the values of *a*, *b*, and the cumulative probability {button P(X <= x),PI(`',`Probability')} ; click {button Perc (x),PI(`',`Percentile')}

Example 2 When *a* = 2, *b* = 3, and the cumulative probability {button P(X <= x),PI(`',`Probability')} = 0.05, the 5th percentile is 1.06608. That is,
$P(X \le 1.06608) = 0.05$.

To compute moments:

Enter the values of *a* and *b*; click {button Momt,PI(`',`Moments')}

Noncentral Chi-square

n = Degrees of Freedom > 0; δ = Noncentrality Parameter > 0

$$P(\chi^2_n(\delta) \le x) = \sum_{k=0}^{\infty} \frac{\exp(-\delta/2)(\delta/2)^k}{k!} P(\chi^2_{n+2k} \le x).$$

To compute probabilities:

Enter the values of the df, noncentrality parameter, and the observed value *x* [Perc (x)]; click {button P(X <= x),PI(`',`Probability')} .

Example 1 When df = 13.0, noncentrality parameter = 2.2 and the observed value [Perc (x)] = 12.3,

$P(X \le 12.3) = 0.346217$ and $P(X > 12.3) = 0.653783$.

To compute percentiles:

Enter the values of the df, noncentrality parameter, and the cumulative probability P(X <= x); click {button Perc (x),PI(`',`Percentile')}.

Example 2 When df = 13.0, noncentrality parameter = 2.2, and the cumulative probability P(X <= x) = 0.95, the 95th percentile is 26.0113. That is,

$P(X \le 26.0113) = 0.95$.

To compute moments:

Enter the values of the df, and the noncentrality parameter; click {button Momt,PI(`',`Moments')}.

Noncentral t Distribution

n = Degrees of Freedom; δ = Noncentrality Parameter

$$f(x;n,\delta) = \frac{n^{n/2}\exp(-\delta^2/2)}{\sqrt{\pi}\,\Gamma(n/2)(n+x^2)^{(n+1)/2}} \sum_{i=0}^{\infty} \frac{\Gamma[(n+i+1)/2]\delta^i}{i!} \left(\frac{2x^2}{n+x^2}\right)^{i/2},$$

$$-\infty < x < \infty,\ -\infty < \delta < \infty.$$

To compute probabilities:

Enter the values of the degrees of freedom (df), noncentrality parameter, and the value of x [Perc (x)] at which the cumulative probability is to be computed; click {button P(X <= x),PI(`',`Probability')} .

Example 1 When df = 13.0, noncentrality parameter = 2.2 and the observed value [Perc (x)] =2.2,

$P(X \le 2.2) = 0.483817$ and $P(X > 2.2) = 0.516183$.

To compute percentiles:

Enter the values of the df, noncentrality parameter, and the cumulative probability P(X <= x); click {button Perc (x),PI(`',`Percentile')}.

Example 2 When df = 13.0, noncentrality parameter = 2.2, and the cumulative probability P(X <= x) = 0.90, the 90th percentile is 3.87082. That is,

$P(X \le 3.87082) = 0.90.$

To compute moments:

Enter the values of the DF, and the noncentrality parameter; click {button Momt,PI(`',`Moments')}.

Noncentral *F* Distribution

m = Numerator df; *n* = Denominator df; δ = Noncentrality Parameter

$$P(X \le x \mid m, n, \delta) = \sum_{k=0}^{\infty} \frac{\exp(-\delta/2)(\delta/2)^k}{k!} \times P\left(F_{m+2k,n} \le \frac{mx}{m+2k}\right), x > 0, \delta > 0.$$

To compute probabilities:

Enter the values of the numerator df, denominator df, noncentrality parameter, and the observed value *x* [Perc (x)]; click {button P(X <= x),PI(`',`Probability')} .

Example 1. When numerator df = 4.0, denominator df = 32.0, noncentrality parameter = 2.2, and the observed value [Perc (x)] = 2,

$P(X \le 2) = 0.702751$ and $P(X > 2) = 0.297249$.

To compute percentiles:

Enter the values of the df, noncentrality parameter, and the cumulative probability P(X <= x); click {button Perc (x),PI(`',`Percentile')}.

Example 2. When numerator df = 4.0, denominator df = 32.0, noncentrality parameter = 2.2, and the cumulative probability P(X <= x) = 0.90, the 90th percentile is 3.22243. That is,

$P(X \le 3.22243) = 0.90$.

To compute moments:

Enter the values of the numerator df, denominator df and the noncentrality parameter; click {button Momt,PI(`',`Moments')}.

Laplace Distribution

a = Location Parameter; *b* = Scale Parameter

$$P(X \le x) = \frac{1}{2b}\int_{-\infty}^{x} \exp\left[-\frac{|y-a|}{b}\right]dy,$$

$$-\infty < x < \infty,\ -\infty < a < \infty,\ b > 0,$$

To compute probabilities:

Enter the values of the parameters *a*, and *b* and the observed value *x* [Perc (x)]; click {button P(X <= x),PI(`',`Probability')} .

Example 1 When *a* = 3, *b* = 4, and the observed value

Perc (x) = 4.5,

$P(X \le 4.5) = 0.656355$ and $P(X > 4.5) = 0.343645$.

To compute percentiles:

Enter the values of *a*, *b*, and the cumulative probability; click {button Perc (x),PI(`',`Percentile')}.

Example 2 When *a* = 3, *b* = 4, and the cumulative probability P(X <= x) = 0.95, the 95th percentile is 12.2103. That is,

$P(X \le 12.2103) = 0.95$.

To compute moments:

Enter the values of *a* and *b* and click {button Momt,PI(`',`Moments')}.

Logistic Distribution

a = Location Parameter; *b* = Scale Parameter

$$P(X \le x) = \left[1 + \exp\left\{-\left(\frac{x - a}{b}\right)\right\}\right]^{-1},$$

$$-\infty < x < \infty,\ -\infty < a < \infty,\ b > 0.$$

To compute probabilities:

Enter the values of the parameters *a*, and *b* and the observed value *x* [Perc (x)]; click {button P(X <= x),PI(`',`Probability')} .

Example 1 When *a* = 2, *b* = 3, and the observed value *x* = 1.3,

$P(X \le 1.3) = 0.44193$ and $P(X > 1.3) = 0.55807$.

To compute percentiles:

Enter the values *a*, *b*, and the cumulative probability; click {button Perc (x),PI(`',`Percentile')}.

Example 2 When *a* = 2, *b* = 3, and the cumulative probability = 0.25, the 25th percentile is –1.29584. That is,

$P(X \le -1.29584) = 0.25$.

To compute moments:

Enter the values of *a* and *b* and click {button Momt,PI(`',`Moments')}.

Weibull Distribution

m = location parameter;   *c* = shape parameter;   *b* = scale parameter

$$P(X \le x \mid b, c, m) = 1 - \exp\left\{ -\left[\frac{x - m}{b}\right]^{c} \right\}, \quad x \ge m.$$

To compute probabilities:

Enter the values of the parameters *m*, *c*, and *b* and the observed value *x* [Perc (x)]; click {button P(X <= x),PI(`',`Probability')} .

Example 1 When *m* = 0, *c* = 2.3, *b* = 2, and the observed value [Perc(x)] = 3.4,
$P(X \le 3.4) = 0.966247$ and $P(X > 3.4) = 0.033753$.

To compute percentiles:

Enter the values of *m*, *c*, *b*, and the cumulative probability; click {button Perc (x),PI(`',`Percentile')}.

Example 2 When *m* = 0, *c* = 2.3, *b* = 2, and the cumulative probability P(X <= x) = 0.95, the 95th percentile is 3.22259. That is,
$P(X \le 3.22259) = 0.95$.

To compute moments:

Enter the values of *c* and *b* and click {button Momt,PI(`',`Moments')}. The moments are computed assuming that *m* = 0.

Extreme Value

a = location parameter; *b* = scale parameter

$$P(X \leq x) = \exp\{-\exp[-(x - a)/b]\}, \quad -\infty < x < \infty, \quad b > 0.$$

To compute probabilities:

Enter the values of the parameters *a*, and *b* and the observed value [Perc (x)]; click {button P(X <= x),PI(`',`Probability')} .

Example 1 When *a* = 1, *b* = 2, and the observed value Perc (x) = 1.2,

P(*X* £ 1.2) = 0.404608 and *P*(*X* > 1.2) = 0.595392 .

To compute percentiles:

Enter the values a, b, and the cumulative probability P(X <= x); click {button Perc,PI(`',`Percentile')}.

Example 2 When *a* = 1, *b* = 2, and the cumulative probability = 0.15, the 15th percentile is -0.280674. That is,

P(*X* £ -0.280674) = 0.15.

To compute moments:

Enter the values *a* and *b* and click {button Momt,PI(`',`Moments')}.

Pareto Distribution

$P(X \le x) = 1 - \left(\frac{a}{x}\right)^{b}, \; x \ge a, \; b > 0.$

To compute probabilities:

Enter the values of the parameters *a*, and *b* and the observed value [Perc (x)]; click {button P(X <= x),PI(`',`Probability')} .

Example 1 When *a* = 2, *b* = 3, and the observed value Perc (x) = 3.4,

$P(X \le 3.4) = 0.796458$ and $P(X > 3.4) = 0.203542$.

To compute percentiles:

Enter the values a, b, and the cumulative probability P(X <= x); click {button Perc(x),PI(`',`Percentile')}.

Example 2 When *a* = 2, *b* = 3, and the cumulative probability = 0.15, the 15th percentile is 2.11133. That is,

$P(X \le 2.11133) = 0.15.$

To compute moments:

Enter the values *a* and *b* and click {button Momt,PI(`',`Moments')}.

Cauchy Distribution

a = location parameter;   *b* = scale parameter

$$f(x; a, b) = \frac{1}{\pi b[1 + ((x - a)/b)^2]}, \quad -\infty < a < \infty,\ b > 0.$$

To compute probabilities:

Enter the values of the parameters *a*, and *b* and the observed value *x* [Perc (x)]; click {button P(X <= x),PI(`',`Probability')} .

Example 1 When *a* = 1, *b* = 2, and the observed value Perc (x) = 1.2,

$P(X \le 1.2) = 0.531726$ and $P(X > 1.2) = 0.468274$.

To compute percentiles:

Enter the values of *a*, *b*, and the cumulative probability; click {button Perc (x),PI(`',`Percentile')}.

Example 2 When *a* = 1, *b* = 2, and the cumulative probability P(X <= x) = 0.95, the 95th percentile   is 13.6725. That is,

$P(X \le 13.6725) = 0.95$.

Distribution of Runs

Example 1. Consider the sequence:

a a a b b a b b b a a a a b b a a a a b

Here $m = 12$, $n = 8$, and the observed number of runs $r = 8$. To get the probabilities, enter 12 for the number of first type symbols, 8 for the number second type symbols, and 8 for the observed number of runs; click on {button P(R <= r),PI(`',`Probability')} to get

$P(R \le 8) = 0.159085$ and $P(R \ge 9) = 0.932603$.

Since the probability of observing 8 or fewer runs is not less than 0.05/2 = 0.025, the null hypothesis that the arrangement is random will be retained at the 5% level.

To find the critical value at the 5% level, enter 0.025 for the tail probability, and click on {button Critical,PI(`',`Critical')} to get

$P(R \le 6) = 0.024609$ and $P(R \ge 16) = 0.00654918$.

This means that the null hypothesis of randomness will be rejected (at 5% level) when the observed number of runs is less than or equal to 6 or greater than or equal to 16. Because of the discreteness of the distribution, the attained size is 0.024609 + 0.00655 =0.031159 (not 0.05).

Example 2. Suppose that $m = 50$ and $n = 40$. To find the 10% critical points, enter 50 for the first type symbols, 40 for the second type symbols, and 0.05 for the tail probability; click on {button Critical,PI(`',`Critical')} to get 37 and 54. This means that

$P(X \le 37) = 0.0438777$ and $P(X \ge 54) = 0.0413445$.

Thus, the null hypothesis of randomness will be rejected at 10% level, if the total number of runs is less than or equal to 37 or is greater than or equal to 54.

Sign Test and CI for Median

Let $X_1, \ldots, X_n$ be a sample of independent observations from a continuous population. Let $X(j)$ denote the jth smallest of the X_i's. For given confidence level $(1 - \alpha)$, and the sample size n, *StatCalc* computes the integers p and q such that the interval $(X(p), X(q))$ would contain the population median with $100(1 - \alpha)\%$ confidence. **StatCalc** also computes the p-values for hypothesis testing about the median.

Example 1   To compute a 95% confidence interval for the median of a continuous population based on a sample of 40 observations, select C I. Med from **StatCalc**, enter 40 for n, 0.95 for confidence level, click C.I. to get 14 and 27. That is, the required confidence interval is formed by the 14th and 27th order statistics from the sample.

Example 2   Suppose that a sample of 40 observations yielded $k = 13$. To obtain the p-value of the test when H_a: $M < M_0$, select C I. Med from **StatCalc**, enter 40 for n, 13 for k and click P(K<=k) to get

$P(K \le 13) = 0.0192387.$

Since this p-value is less than 0.05, the null hypothesis that H_0: $M = M_0$ will be rejected at the 5% level.

Wilcoxon Signed-Rank Test

Example 1 Suppose that a sample of 20 observations yielded $T+ = 130$. Then, enter 20 for n, 130 for T+, and click on {button P(T+ <= k),PI(`',`Probability')} to get

$P(T+ \leq 130) = 0.825595$ and $P(T+ \geq 130) = 0.184138$.

This program also computes the critical point for given n and the level α.

Example 2 When $n = 20$, cumulative probability = 0.95, the upper tail critical point is 150; that is
$P(T+ \geq 150) = 0.048654$.

Remark : For $n > 60$, the standard normal approximation is used to compute the table values.

Wilcoxon Rank-Sum Test

WRS statistic is useful to test whether two continuous distributions are equal. Let $X_1, \ldots, X_m$ be independent observations from a continuous distribution F_X and $Y_1, \ldots, Y_n$ be independent observations from a continuous distribution F_Y. Pool X and Y observations, and arrange them in increasing order. Let W denote the sum of the ranks of the X observations in the pooled sample.

For given m, n, and W this program computes the probabilities.

Example 1 When $m = 13$, $n = 12$, and the observed value of W is 180,
$P(W \le 180) = 0.730877$ and $P(W \ge 180) = 0.287146$.

This program also computes the critical values for given m, n, and the level of significance.

Example 2 When $m = 13$, $n = 12$, and the cumulative probability is 0.05, the left tail critical value is 138. Because of the discreteness of the distribution, the attained level is 0.048821; that is
$P(W \le 138) = 0.048821$.

Remark 1 An exact method is used to compute the table values when $m \le 60$ and $n \le 60$. For all other values, the standard normal approximation is used.

NP Tolerance Limits

Let $X_1, \ldots, X_n$ be a sample of independent observations from a continuous population. Let $X(k)$ denote the kth smallest of the X_i's. For given $0 < p < 1$, $0 < g < 1$, this program computes the value of n so that the interval

$$(X(1), X(n))$$

would contain at least p proportion of the population with confidence g.

Example 1 When $p = 0.90$, and $g = 0.95$, the value of n is 46; that is the interval $(X(1), X(46))$ would contain at least 90% of the population with confidence 95%. Further, the sample size required for a one-sided tolerance limit is 29; that is 90% of the population data are less than or equal to $X(29)$ - the largest observation in a sample of 29 observations, with confidence 95%. Similarly, 90% of the population data are greater than or equal to $X(1)$ - the smallest observation in a sample of 29 observations, with confidence 95%.

Tolerance Factors for Normal Population

Let $\bar{x}$ and *s* denote respectively the mean and standard deviation of a random sample of *n* observations from a normal population. For given *n*, df, 0 < *p* < 1, and 0 < *g* < 1, this program computes the value of *k* such that the interval

$$\bar{x} \pm ks$$

contains at least 100*p*% of the population with confidence 100*g*%. We refer this tolerance interval as (*p*, *g*) tolerance interval. **StatCalc** uses an exact method of computing the tolerance factor *k*.

Remark 1 Here, the df associated with *s* is *n* - 1**.** In some situations, the df associated with the *s* could be different from *n* - 1. For example, in one-way ANOVA, the df associated with the pooled sample variance is (*n* -*m*), where *m* denotes the number of groups. If one is interested in computing (*p*, *g*) tolerance interval

$$\bar{x}_1 \pm k_1 s_p$$

for the first population, then for this case, sample size is $n1$ and the degrees of freedom associated with the pooled variance is

$$\sum_{i=1}^{m} n_i - m\,,$$

where ni denotes the size of the sample from the *i*th group, *i* = 1,…, *m*.

This program also computes one-sided tolerance factor *c* such that at least 100*p*% of the population data are less than or equal to

$$\bar{x} + cs$$

with confidence 100*g*%. The lower tolerance limit is

$$\bar{x} - cs\,.$$

That is, at least 100*p*% of the population data are greater than or equal to $\bar{x} - cs$.

Example 1 When *n* = 23, df = 22, *p* = 0.90, and *g* = 0.95, the one-sided tolerance factor is 1.86902, and the two-sided tolerance factor is 2.25125.

Fisher's Exact Test for 2 x 2 Table

For a given 2 x 2 table, **StatCalc** computes the probability of observing *k* or more successes (as well as the probability of observing k or less number of successes) in the cell (1,1). If either of this probability is less than $\alpha/2$, then the null hypothesis of equal proportions will be rejected at the level of significance α.

Further, for given level of significance, power and guess values on $p1$ and $p2$, **StatCalc** also computes an approximate sample size needed for each group to satisfy the requirements.

To find the p-value, enter the cell values and click Prob <= (1,1) cell .

To compute the sample size, enter the level of significance, power, guess values of $p1$ and $p2$, and then click S Size .

Multivariate Tolerance Factors

Let $\bar{X}$ and *S* denote respectively the mean and variance-covariance matrix of a random sample of *n* observations from an *m*-variate normal population. For a given *n*, *m*, $0 < p < 1$, and $0 < g < 1$, this program computes an approximate tolerance factor *k* such that the region

$$R = \{Y : (Y - \bar{X})' S^{-1} (Y - \bar{X}) \le k\}$$

would contain at least 100*p*% of the population with confidence 100*g*%.

Example 1 When $n = 35$, $m = 3$, $p = 0.90$, and $g = 0.95$, the 90% content – 95% coverage tolerance factor *k* is 9.83.

Multiple Correlation Coefficient

To compute probabilities:

Enter the values of the sample size *n*, number of variates *m*, squared population multiple correlation coefficient ρ, and the observed value of the squared sample multiple correlation coefficient r^2; click {button P(X <= x),PI(`',`Probability')}

Example 1 When $n = 40$, $m = 4$, $\rho^2 = 0.8$, $r^2 = 0.75$,

$P(X$ £ $0.75) = 0.151447$ and $P(X > 0.75) = 0.84855$.

To compute percentiles:

Enter the values of the sample size *n*, number of variates *m*, squared population multiple correlation coefficient ρ^2, and the cumulative probability; click {button Perc,PI(`',`Percentile')}

Example 2 When $n = 40$, $m = 4$, $\rho^2 = 0.8$, the 90th percentile is 0.874521.

To compute confidence intervals:

Enter the values of *n*, *m*, r^2, and the confidence level; Click {button C.I.,PI(`',`Confidence')}

Example 3 When $n = 40$, $m = 4$, $r^2 = 0.75$, the 95% confidence interval for ρ^2 is (0.544189, 0.848982).

Correlation Coefficient

To compute probabilities:

Enter the sample size *n*, population correlation ρ, and the observed sample correlation *r*; click {button P(X <= r),PI(`',`Probability')}

Example 1 When $n = 30$, $\rho = 0.7$, $r = 0.75$,
$P(X \le 0.75) = 0.687844$ and $P(X > 0.75) = 0.312156$.

To compute percentiles:

Enter the sample size *n*, population correlation ρ, and the cumulative probability; click {button Perc,PI(`',`Percentile')}

Example 2 When $n = 30$, $\rho = 0.7$, and the cumulative probability is 0.95, the 95th percentile is 0.832097. That is,
$P(X \le 0.832097) = 0.95$.

To compute confidence intervals:

Enter the sample size, sample correlation, and the confidence level; click {button C.I.,PI(`',`Confidence')}

Example 3 When $n = 30$, $r = 0.6$, a 95% confidence interval for ρ is (0. 30584, 0. 78959).

Remark 1 An exact method is used to compute the cdf and the percentiles when *n* in [4, 100] and ρ in [-0.83, 0.83]. For the other values, Fisher's z transformation is used to compute cdf and the percentiles. Further, confidence intervals are based on the Fisher's z transformation.

Bivariate Normal Distribution

Let (X, Y) be a bivariate normal random vector with mean = (0, 0), and the correlation coefficient ρ. For given x, y, and ρ, this program computes the following probabilities:

$P(X \le x, Y \le y)$, $P(X \le x, Y > y)$

$P(X > x, Y \le y)$, $P(X > x, Y > y)$

and $P(|X| < x, |Y| < y)$.

Example 1 When $x = 1.1$, $y = 0.8$, and $\rho = 0.6$,

$P(X \le 1.1, Y \le 0.8) = 0.730456$

$P(X \le 1.1, Y > 0.8) = 0.133878$

$P(X > 1.1, Y \le 0.8) = 0.057688$

$P(X > 1.1, Y > 0.8) = 0.077977$

$P(|X| < 1.1, |Y| < 0.8) = 0.465559$.

If (X, Y) is a normal random vector with mean = (μ_1, μ_2), and covariance matrix

$$\Sigma = \begin{pmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{21} & \sigma_2^2 \end{pmatrix}$$

then to compute the above probabilities at (x, y), enter the standardized values

$\frac{x - \mu_1}{\sigma_1}$, $\frac{y - \mu_2}{\sigma_2}$, and $\rho = \frac{\sigma_{12}}{\sigma_1 \sigma_2}$.

Maximum of | *t* | Variables

Let $X_1, \ldots, X_k$ be independent normal random variables with mean μ and common standard deviation σ. Let ms^2/σ^2 follow a chi-square distribution with df = m. **StatCalc** computes the distribution function of

$$X = \max_{1 \le i \le k} \left\{ \frac{|X_i|}{s} \right\}.$$

The percentiles of X are useful for constructing simultaneous confidence intervals for the treatment effects and orthogonal estimates in the analysis of variance, and to test extreme values.

To compute probabilities:

Enter the values of the number of groups k, df, and the observed x; click {button P(X <= x),PI(`',`Probability')}

Example 1. When $k = 4$, df = 45, and $x = 2.3$, $P(X \le 2.3) = 0.900976$ and $P(X > 2.3) = 0.099024$.

To compute percentiles:

Enter the values of k, df, and the cumulative probability; click {button Perc,PI(`',`Percentile')}

Example 2. When $k = 4$, df = 45, and the cumulative probability is 0.95, the 95th percentile is 2.5897. That is, $P(X \le 2.5897) = 0.95$.

Lognormal Distribution

$$P(X \le x) = \int_0^x \frac{1}{(\sqrt{2\pi})y\sigma} \exp\left[-\frac{(\ln y - \mu)^2}{2\sigma^2}\right] dy,$$
$$x > 0, \sigma > 0, -\infty < \mu < \infty.$$

To compute probabilities:

Enter the values of the parameters μ and σ, and the observed value *x* [Perc (x)]; click {button P(X <= x),PI(`',`Probability')}.

Example 1 When μ = 1, σ = 2, and the observed value [Perc (x)] = 2.3,

$P(X \le 2.3) = 0.466709$ and $P(X > 2.3) = 0.533291$.

To compute percentiles:

Enter the values μ, σ, and the cumulative probability P(X <= x); click on {button Perc (x),PI(`',`Percentile')}.

Example 2 When μ = 1, σ = 2, and the cumulative probability P(X <= x) = 0.95, the 95th percentile is 72.9451. That is,

$P(X \le 72.9451) = 0.95.$

To compute moments:

Enter the values of μ and σ and click {button Momt,PI(`',`Moments')}.