Solver Step-by-Step Example

Month
Seasonality
Units Sold
Sales Revenue
Cost of Sales
Gross Margin

Salesforce
Advertising
Corp Overhead
Total Costs

Prod. Profit
Profit Margin

Product Price
Product Cost

The following steps first show how to use the Solver to maximize a single quarter's sales, or sales for the full year. Then, in a more realistic example, the Solver is used to allocate an advertising budget by quarter to maximize sales for the year, taking advantage of a seasonal sales pattern in the model.

Row
4

5

6

7

8

10

11

12

13

15

16

18

19

This is a typical marketing model that shows sales rising from a base figure (perhaps due to the sales force effort) along with increases in advertising, but with diminishing returns. For example, the first $5,000 of advertising in Q1 yields about 1,092 incremental units sold, but the next $5,000 yields only about 775 units more.

You can use the Solver to find out whether the advertising budget is too low, and whether advertising should be allocated differently over time to take advantage of the changing seasonality factor.

Solving for a Value to Maximize Another Value

One way you can use the Solver is to determine the maximum value of a cell by changing one other cell. The two cells must be related through the formulas on the worksheet. If they are not, the Solver will report that "the Set Cell values do not converge."

For example, in this model, you want to know how much you need to spend on advertising to generate the maximum profit for the first quarter.

You will see messages in the title bar as the problem is set up and Solver starts working. After a moment, you'll see a message that Solver has found a solution. Solver finds that Q1 advertising of $17,093 yields the maximum profit $15,093.

Resetting the Solver Parameters

If you want to return the options in the Solver Parameters dialog box to their original settings so that you can start a new problem, you can click Reset All.

Solving for a Value by Changing Several Values

You can also use the Solver to solve for several values at once to maximize or minimize another value. For example, you can find the advertising budget for each quarter that will result in the best profits for the entire year. Because the seasonality factor in row 3 enters into the calculation of unit sales in row 5 as a multiplier, it seems logical that you should spend more of your advertising budget in Q4 when the sales response is highest, and less in Q3 when the sales response is lowest. Use the Solver to determine the best quarterly allocation.

You've asked the Solver to solve a moderately complex nonlinear optimization problem; that is, to find values for the four unknowns in cells B11 through E11 that will maximize profits. (This is a nonlinear problem because of the exponentiation that occurs in the formulas in row 5). The results of this unconstrained optimization show that you can increase profits for the year to $79,706 if you spend $89,706 in advertising for the full year.

However, most realistic modeling problems have limits on available resources such as your advertising budget. The Solver allows you to define constraints which express these limits. These constraints may be applied to any of the Changing Cells (i.e. Variables), or to any other cell containing a formula which depends on the values of the Variables.

Adding a Constraint

So far, the budget recovers the advertising cost and generates additional profit, but you're reaching a point of diminishing returns. Because you can never be sure that your model of sales response to advertising will be valid next year (especially at greatly increased spending levels), it doesn't seem prudent to allow unrestricted spending on advertising.

Suppose you want to maintain your original advertising budget of $40,000. Add the constraint to the problem that limits the sum of advertising during the four quarters to $40,000.

The solution found by the Solver allocates amounts ranging from $5,117 in Q3 to $15,263 in Q4. Total Profit has increased from $69,662 in the original budget to $71,447, without any increase in the advertising budget.

Saving a Solver Model

When you select Save on the File menu, the last selections you made in the Solver Parameters dialog box are attached to the worksheet and retained when you save the workbook. However, you can define more than one problem for a worksheet and save them individually using Save Model in the Solver Options dialog box. Each Solver model consists of the cells and constraints that you entered in the Solver Parameters dialog, and the options you chose in the Solver Options dialog.

When you select Save Model, the Save Model dialog box appears with a default selection as the area for saving the Solver model. Make sure that this cell range is an empty range on the worksheet.

You can also enter a reference to a single cell in the Select Model Area box. The Solver will use this reference as the top cell of a column range into which it will copy the problem specifications.

To load these problem specifications later, click Load Model in the Solver Options dialog box, type H15..H18 in the Model area box or select cells H15..H18 on the sample worksheet, and then click OK. The Solver displays a message asking if you want to reset the current Solver option settings with the settings for the model you are loading. Click OK to proceed.

	Q1	Q2
	0.9	1.1
	3591.55258906228	4389.67538663168
	143662.103562491	175587.015465267
	89788.8147265571	109741.884665792
	53873.2888359343	65845.1307994752
	8000	8000
	10000	10000
	21549.3155343737	26338.0523197901
	39549.3155343737	44338.0523197901
	14323.9733015606	21507.0784796851
	0.099705997241853	0.122486725016061
	40	
	25	

Contains
Fixed values

35*B3*(B11+3000)^0.5

+B5*B18

+B5*B19

+B6-B7

Fixed values

Fixed initial values

0.15*B6

@SUM(B10..B12)

+B8-B13

+B15/B6

Fixed values

Fixed values

Select Range Analyze Solver. In the Set Cell box, type B15 or select cell B15 (first-quarter profits) on the worksheet. The Max option is selected by default. In the By Changing Cells box, type B11 or select cell B11 (first-quarter advertising) on the worksheet. Click Solve.

After you examine the results, select Restore Original Values and click OK to discard the results and return cell B11 to its former value.

Select Range Analyze Solver. In the Set Cell box, type F15 or select cell F15 (total profits for the year) on the worksheet. Make sure the Max option is selected. In the By Changing Cells box, type B11..E11 or select cells B11..E11 (the advertising budget for each of the four quarters) on the worksheet. Click Solve.

After you examine the results, click Restore Original Values and click OK to discard the results and return all cells to their former values.

Select Range Analyze Solver, and then click Add. The Add Constraint dialog box appears. In the Cell reference box, type F11 or select cell F11 (advertising total) on the worksheet. Cell F11 must be less than or equal to $40,000. The relationship in the Constraint box is <= (less than or equal to) by default, so you don't have to change it. In the box next to the relationship, type 40000. Click OK, and then click Solve.

After you examine the results, click Restore Original Values and then click OK to discard the results and return the cells to their former values.

Select Range Aanlyze Solver, and then click Options. Click Save Model.
In the Select Model Area box, type H15..H18 or select cells H15..H18 on the worksheet. Click OK.

Q3	Q4	Total
0.8	1.2	
3192.49119027759	4788.73678541638	15962.4559513879
127699.647611103	191549.471416655	638498.238055517
79812.2797569397	119718.419635409	399061.398784698
47887.3678541638	71831.0517812457	239436.839270819
9000	9000	34000
10000	10000	40000
19154.9471416655	28732.4207124983	95774.7357083276
38154.9471416655	47732.4207124983	169774.735708328
9732.42071249828	24098.6310687474	69662.1035624914
0.0762133717247004	0.125808914483134	0.109103047448714

Explanation
Seasonality factor: sales are higher in quarters 2 and 4, and lower in quarters 1 and 3.

Forecast for units sold each quarter: row 3 contains the seasonality factor; row 10 contains the cost of advertising.

Sales revenue: forecast for units sold * product price

Cost of sales: forecast for units sold * product cost

Gross margin: sales revenues - cost of sales

Sales force personnel expenses.

Advertising budget (Solver will adjust these values).

Corporate overhead expenses: sales revenues * 15%

Total costs: sales force expenses + advertising + overhead.

Product profit: gross margin - total costs.

Profit margin: profit / sales revenue

Product price.

Product cost.

			100	100	100
Part Name	*Inventory*	*No. Used*			
Chassis	450	200	1	1	0
Picture Tube	250	100	1	0	0
Speaker Cone	800	500	2	2	1
Power Supply	450	200	1	1	0
Electronics	600	400	2	1	1
		Profits:			
		By Product	4732.180084	3154.786722	2208.350706
		Total	**10095.31751**		

Entries in the Solver Parameters dialog

Objective	D18	Objective is to maximize profit.
Variables	D9..F9	Units of each product to build.
Constraints	C11..C15<=B11..B15	Number of parts used must be less than or equal to the number of parts in inventory.

The formulas for profit per product in cells D17..F17 include the factor ^H15 which makes profit per ui diminish with volume when H15 < 1.0. H15 contains 0.9, which makes the problem nonlinear. If you change H15 to 1.0 to indicate that profit per unit remains constant with volume, and then click Solve a the optimal solution will change. This also makes the problem linear, so you can check the Assume Linear Model box in the Solver Options dialog to use the Simplex method solver, and obtain additiona information in the Sensitivity Report.

Diminishing Returns Exponent:
0.9

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	310	50	10	80	100	70
Tennessee	260	0	0	100	60	100
Arizona	280	130	80	20	0	50
		------	------	------	------	------
Totals:		180	90	200	160	220
Demands by Whse -->		180	80	200	160	220
Plants:	*Supply*	*Shipping costs from plant x to warehouse y (at intersection):*				
S. Carolina	310	10	8	6	5	4
Tennessee	260	6	5	4	3	6
Arizona	280	3	4	5	5	9
Shipping:	**4280**	890	400	980	680	1330

Entries in the Solver Parameters dialog

Objective	B20	Goal is to minimize total shipping cost.
Variables	C8..G10	Amount to ship from each plant to each warehouse.
Constraints	B8..B10<=B16..B18	Total shipped must be less than or equal to supply at plant.
	C12..G12>=C14..G14	Totals shipped to warehouses must be grea than or equal to demand at warehouses.

This is a classic transportation model, applicable in any industry where goods are shipped from many sources to many destinations. Goods can be shipped from any plant to any warehouse, but it costs more to ship goods over long distances than over short distances. The Solver will find the amounts to ship from each plant to each warehouse to minimize shipping cost, while meeting the metropolitan area demand at each warehouse, and not exceeding the plant supplies.

Note that the Assume linear model check box in the Solver Options dialog is checked, since this is a linear programming problem. If all of the supply and demand amounts are integers, the amounts shipped at the optimal solution will also be integers, in any problem of this type.

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		4	0	0
B	*Monday, Tuesday*	4	1	0
C	*Tuesday, Wednesday*	4	1	1
D	*Wed., Thursday*	6	1	1
E	*Thursday, Friday*	6	1	1
F	*Friday, Saturday*	6	1	1
G	*Saturday, Sunday*	4	0	1
	Schedule Totals:	34	26	26
	Total Demand:		22	17
	Pay/Employee/Day:	40		
	Payroll/Week:	**6800**		

Entries in the Solver Parameters dialog

Objective	D20	Objective is to minimize payroll cost.
Variables	D7..D13	Employees on each schedule.
Constraints	D7..D13=Integer	Number of employees must be an integer.
	F15..L15>=F17..L17	Employees working each day must be grea equal to the demand.
Schedules	Rows 7-13	1 means employee on that schedule works

In this example, there is an integer constraint so that the solution does not result in fractional number of employees on each schedule. Note that the Assume linear model box is checked in the Solver Options dialog, since this is an integer linear programming problem.

1	1	1	1	1
0	1	1	1	1
0	0	1	1	1
1	0	0	1	1
1	1	0	0	1
1	1	1	0	0
1	1	1	1	0
26	24	22	22	24
13	14	15	18	24

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that day.

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	0.01	1		1, 2, 3, 4, 5 and 6		
3-mo CDs:	0.04	3		1 and 4		
6-mo CDs:	0.09	6		1		***Total***

Month:	***Month 1***	***Month 2***	***Month 3***	***Month 4***	***Month 5***	***Month 6***
Init Cash:	400000	205000	216000	237000	158400	109400
Matur CDs:		100000	100000	110000	100000	100000
Interest:		1000	1000	1400	1000	1000
1-mo CDs:	100000	100000	100000	100000	100000	100000
3-mo CDs:	10000			10000		
6-mo CDs:	10000					
Cash Uses:	75000	-10000	-20000	80000	50000	-15000
End Cash:	205000	216000	237000	158400	109400	125400

One of the tasks of a company's financial officer or manager is to manage cash and short-term invest
a way that maximizes interest income, while keeping funds available to meet expenditures. You mus
the higher interest rates available from longer-term investments against the flexibility provided by kee
in short-term investments.

This model calculates ending cash based on initial cash (from the previous month), inflows from matu
certificates of deposit (CDs), outflows for new CDs, and cash needed for company operations for eac

There are a total of nine decision variables: the amounts to invest in one-month CDs in months 1 thr
the amounts to invest in three-month CDs in months 1 and 4; and the amount to invest in six-month C
month 1.

Entries in the Solver Parameters dialog

Objective	H8	Objective is to maximize interest earned.
Variables	B14..G14 B15, E15, B16	Dollars invested in each type of CD.
Constraints	B18..H18>=100000	Ending cash must be greater than or equal $100,000.

The optimal solution determined by the Solver earns a total interest income of $16,531 by investing a
as possible in six-month and three-month CDs, and then turns to one-month CDs. This solution satis
the constraints.

Interest Earned:
7700

End
125400
120000
2300

60000
187700

:ments in
t trade off
ping funds

uring
h month.

ough 6;
CDs in

to

s much
fies all of

		0.06	*Market variance*		0.03
Market rate		0.15	*Maximum weight*		1
	Beta	*ResVar*	*Weight*	**Beta*	**Var.*
Stock A	0.8	0.04	0.2	0.16	0.0016
Stock B	1	0.2	0.2	0.2	0.008
Stock C	1.8	0.12	0.2	0.36	0.0048
Stock D	2.2	0.4	0.2	0.44	0.016
T-bills	0	0	0.2	0	0
Total			1	1.16	0.0304
			Return		***Variance***
		Portfolio Totals:	**0.1644**		0.070768

Maximize Return:	**Minimize Risk:**
0.1644	0.070768
5	5
TRUE	TRUE
TRUE	TRUE
TRUE	TRUE
1380.602243	1380.602243

One of the basic ideas of investment management is diversification. By holding a portfolio of several stocks, for example, you can earn a rate of return that is the average of the returns from the individua stocks, while reducing your exposure to the risk that any one stock will perform poorly.

This worksheet contains figures for beta (market-related risk) and alpha, or residual variance, for four stocks. Your portfolio can also include Treasury bills (T-bills), assumed to have the risk-free rate of return and a variance of zero. Initially equal amounts (20 percent of the portfolio) are invested in eac stock.

Use the Solver to find allocations of funds to stocks and T-bills to either maximize the portfolio rate of return for a specified level of risk, or minimize the risk for a given rate of return. With the initial allocat of 20 percent across the board, the portfolio return is 16.4 percent and the variance is 7.1 percent.

Entries in the Solver Parameters dialog

Objective	E18	Objective is to maximize port
Variables	E10..E14	Weight of each stock.
Constraints	E16=1	Sum of weights must equal 1

	G18<=0.071	Portfolio variance (risk) must than or equal to 0.071.
Beta for each stock	B10..B13	
Residual variance for each stock	C10..C13	

Cells D21..D26 contain the model definition to minimize risk for a required rate of return of 16.4 percent. To load these problem specifications into the Solver, select Range Analyze Solver, click Options, click Load Model, select cells D21..D26 on the worksheet, and then click OK until the Solver Parameters dialog box is displayed. Click Solve. As you can see, Solver finds portfolio allocations in both cases that improve on the initial allocation of 20 percent across the board.

You can earn a higher rate of return (17.1 percent) for the same risk, or you can reduce your risk without giving up any return. These two allocations both represent efficient portfolios.

Cells A21..A26 contain the original problem model. To reload this problem, select Range Analyze Solver, click Options, click Load Model, select cells A21..A26 on the worksheet, and then click OK.

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				9
			q[t] =	0.09
			t =	0.05
Battery	Capacitor (C)	Inductor (L)	L =	8
			C =	0.0001
	Resistor (R)		R =	400
	1/(L*C)	1250	q[t] =	0.813072324
	(R/(2*L))^2	625		
	SQRT(B15-B16)	25		
	COS(T*B17)	0.315322362		
	-R*T/(2*L)	-1.25		
	Q0*EXP(B19)	2.578543172		

This model depicts an electrical circuit containing a battery, switch, capacitor, resistor, and inductor. With the switch in the left position, the battery charges the capacitor. When the switch is thrown to the right, the capacitor discharges through the inductor and the resistor, both of which dissipate electrical energy.

Using Kirchhoff's second law, you can formulate and solve a differential equation to determine how the charge on the capacitor varies over time. The formula relates the charge q[t] at time t to the inductance L, resistance R, and capacitance C of the circuit elements.

The Solver will pick an appropriate value for the resistor R (given values for the inductor L and the capacitor C) that will dissipate the charge to one percent of its initial value within 1/20th of a second after the time the switch is thrown. This very simple problem can also be solved using the Range Analyze Backsolver menu command.

Entries in the Solver Parameters dialog

Objective	G15	Target value for this cell is 0.09.
Variable	G12	Resistor.

This problem and solution are appropriate for a narrow range of values; the function represented by the charge on the capacitor over time is actually a damped sine wave.

volts
volts
seconds
henrys
farads

ohms

5	
0	0
4280	
15	
TRUE	
TRUE	
1375.100854	