MANDELBROT REVISITED

A new look at the dimensional relationship between the M-set and its associated J-sets.

Consider the iteration of the function z(n+1)=z(n)^2+c

Julia sets are defined by fixing c and iterating for varying values of z0. (normally corresponding to the computer screen pixel co-ordinates, zReal=x, zImag=y). The Julia set is in fact the boundary between the region where the iterates eventually escape to infinity, and the one where they remain contained.

The Mandelbrot set, in contrast, is obtained by fixing z0=0 and varying the parameter c.

The Mandelbrot set is in parameter space (c-plane) while the Julia set is in dynamical or variable space(z-plane).

Julia sets fall into two distinct classifications, those which are continuous are contained within the Mandelbrot set boundary, and those which are not (Cantor sets) lie outwith. The actual Mandelbrot set consists of all values of c for which the Julia set boundary is connected. Julia sets which are just connected lie on the boundary of the Mandelbrot set.

Conventional wisdom dictates that each point on the 2 dimensional Mandelbrot set indexes a unique 2 dimensional Julia set. However, this view does not extend to allow for any shared dimensionality or other linkage - considering the remarkable convergence in form under increasing localised magnification, can such conclusions be regarded as logical?

Let us now examine the conditions under which such linkage could occur.

Fractal images can be generated on four different planes, the z & c planes referred to above, and also two hybridised planes, firstly the "zc" plane defined by iterating zReal while varying cImag, and secondly the "cz" plane, defined by iterating zImag while varying cReal.

Consider firstly the zc-plane. The x-axis corresponds to the initial real value of z and the y-axis to the imaginary value of c. For each discrete value of y on this plane a scan is produced corresponding to that particular z-plane (Julia) x-axis at y=0. Each scan therefore corresponds to the centre of a different Julia set, and the assembled scans illustrate the variation over the depth of this "stack" of Julia sets.
It may help to imagine a stack of complete Julia sets sliced in two along the x-axis at y=0 and viewed from the side.

The stacked Julia array has a fractal-type structure with an inner lake where the iterate remains bounded, and an outer region of varying escape potential.

Now, if we change the fixed value of cReal, and iterate again, a different fractal image will be produced. In effect what is being done is to move the location of the stacked Julia array along the c-plane (Mandelbrot) x-axis (the value of cReal corresponds to c-plane x). Outwith the x limits of the Mandelbrot set, the bounded region vanishes, abruptly below x=-2 and disperses gradually above x=+0.25. Within the x limits, the points on the zc-plane where the y-axis at x=0 crosses the lake boundary correspond exactly to the position of the c-plane (Mandelbrot) boundary (at x=cReal).

Take now the second case, the cz-plane. The y-axis now corresponds to the initial imaginary value of z and the x-axis to the real value of c. An initial iteration (at cImag=0) produces a rather bland and uninteresting picture, consisting of a paraboloid-shaped bounded region, centred on the x-axis, with a minimum value at x=-2. At values of x greater than +0.25 the bounded region degenerates into a shape reminiscent of a comet's tail. Around the edge of most of the bounded region can be observed an abrupt change from containment to escape, with very little fractal-type detail. The bounded region extends along the x-axis between -2 and +0.25.

If the fixed value of cImag is now increased, even by as little as say 0.0000000001, a dramatic change occurs. A region of extreme chaos jumps into view, extending from the Myrburg-Feigenbaum point (-1.401...) (situated at the extreme negative tip of the main Mandelbrot cardioid) as far as -2, and contained within the paraboloid. Further variations of cImag can produce quite wide fluctuations in fractal form.

An overall view of the above reveals that we have a 4-dimensional space, with mutually perpendicular planes. The Mandelbrot set has a third dimension, which also constitutes one dimension of the z (Julia) plane. In fact, taking it one step further, Julia sets are not separate entities, but merely 2-dimensions of an abstract 4-dimensional mathematical construction. (The Mandelbrot set itself is an abstract 2-dimensional mathematical construction). As we now have one 4-dimensional set, the localised similarity between Mandelbrot & Julia sets referred to above is no longer quite so remarkable! The 4 dimensinal set would appear to be connected, not in the form of infinitely thin strands linking discs but rather infinitely thin shrouds linking cylinders.

Obviously much more work is needed on these and related topics.

The fractal explorer need not concern himself overmuch with the mathematical validity of this thesis - the main thing is the discovery of a completely new fractal universe largely unexplored. The most interesting areas in which to begin exploration tend to coincide with locations of interest on and around the 2 dimensional Mandelbrot set, such as Sea Horse valley (x=-0.75). Here on the zc- plane exist some really exotic structures - one has been observed with a striking resemblance to the Lorenz butterfly. There is also a property of uni-dimensional image compression - infinitely nested fractal structures which can be explored by repeatedly expanding the y dimension, x remaining constant, or being used merely to set the most appropriate frame for the image being produced.

It is also possible to rotate the actual fractal plane to an angle lying between the c-plane and the hybridised planes. Mandelbrot lambda sets have been found in this inter-planar region.

The constructions detailed above appear to be valid for other analytic complex mappings, such as those of the transcendental functions and the tetration function.

FRACTINT formulae and par files have been uploaded along with this document.

Finally, acknowledgements for those who have (unwittingly) helped with the development of this thesis. Dr Clifford A Pickover (Computers, Pattern, Chaos & Beauty) inspired my initial code to produce the stacked Julia array - it has taken over 12 months to get from there to the above! Also Prof Adrien Douady (Beauty of Fractals) for explaining the structure of the Mandelbrot set in terms that even I could understand. Last, but by no means least, the authors of FRACTINT, the tool I have used to hopefully fashion some kind of order out of Chaos!

Gordon Lamb
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