A **Countdown** conundrum

Carol Vorderman replaced by a machine? The very idea. But here, Daniel Norris-Jones and Julian Sweeting dare to ponder, as Mike Mudge sorts the vowels from the consonants.

This month's project is proposed by Daniel Norris-Jones and Julian Sweeting of Wheldrake, Yorkshire (Dan@akqa.com).

Carol Vorderman or machine?

Those of you who have watched the TV show, Countdown, will know that Carol Vorderman is not infallible when it comes to the numbers game. Now and then she is unable to solve the problem and this raises the question: "Is it *always* possible?"

The Countdown numbers game requires the contestants to pick six cards. Each card has a number on the back. The cards are arranged into four rows. The top row contains the numbers 10, 25, 50, 75 and 100. The other three rows have cards from one to ten.

A random number generator then creates a target number (an integer between 0 and 999) and the contestants must then use the six numbers and the four operators (+, -, x, ÷) to create a number as close to the target as possible. The contestants have 30 seconds in which to achieve this target, using only pencil, paper and mental power. Carol, on the other hand, gets a little longer because the contestants prove their solutions first.

There are two ways to approach this problem: intelligence, or brute force and ignorance. The Artificial Intelligence solution may only be as good as Carol Vorderman, and until the ADI Dynamic Link Library is available it would prove difficult to implement. So this results in the solution at which computers are best. Try every possible combination of numbers and operators and then you will know whether it is possible to achieve the target.

As a schoolboy, Julian Sweeting attempted this on an Atari 8-bit home computer. Naturally he suffered from low computing power and limited knowledge. He did, however, identify the problem of parenthesis which complicated the number of permutations and combinations of operators and numbers. Initial estimates of the number of potential calculations were in the order of tens of millions, far beyond the home computing power of the eighties.

Five years on, while learning to program LISP, Sweeting came across Reverse Polish Notation and recognised that it removed the need to consider parenthesis. Obviously this was the way to tackle the problem. Some time later, during a road trip around America, he happened to discuss the problem with his fellow traveller, programmer Daniel Norris-Jones. Their appetite for solving this problem was whetted and the project sparked into life. Travelling to LA from Las Vegas, the two applied the limited processing power they had available (two Psion 3a organisers) to parts of the problem.

The essential aspects of the problem are as follows: there are six numbers and so there are 6! = 6 * 5 * 4 * 3 * 2 * 1 = 720 possible ways of ordering the numbers, i.e.

```
1-      1,2,3,10,25,50
2-      2,1,3,10,25,50
```

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```
3-      2,3,1,10,25,50
        ......
719-    ...
720-    50,25,10,3,2,1
```

Ignoring parenthesis, an operator may be placed between each number pair. Hence five operators, each of which can take four values, 4 to power 5 or 4 * 4 * 4 * 4 * 4 = 1,024. That is:

```
1           +++++
2           ++++-
3           ++++*
.           ....
1,023       *////
1,024       /////
```

The Psion Organiser was used to attempt simple problems not requiring parenthesis. Because limited battery life made it possible to calculate only a few tens of numbers per second, the 700,000 was out of the question.

The road trips continued and in the desert near Roswell, Sweeting and Norris-Jones had their first inclination of whether the problem could be solved within 30 seconds. They required a compiled language (100 times faster than the interpreted OPL from Psion) run on a fast computer, perhaps a few hundred times faster than a Psion. The parenthesis problem proved simple. Reverse Polish Notation showed there are only ten ways to arrange the operators and operand, which brings the total to approximately seven million numbers to calculate in 30 seconds.

The two programmers arrived in Albany, New York, where they had access to some "real" computers (IBM RISC 6000 workstations). Not all solutions to the numbers game require all six numbers to be used. It is therefore necessary to check intermediate results to determine whether you have the answer. Successive calculations differ only slightly, so only the change requires calculation. This allows intermediate results to throw a helping hand to the floundering processor. For example, given the numbers 1, 2, 3, 10, 25, 50, the calculations may be done as follows:

```
check 50 + 25 + 10 + 3 + 2 - 1
check 50 + 25 + 10 + 3 + 1 - 2
                  10 calculations
```

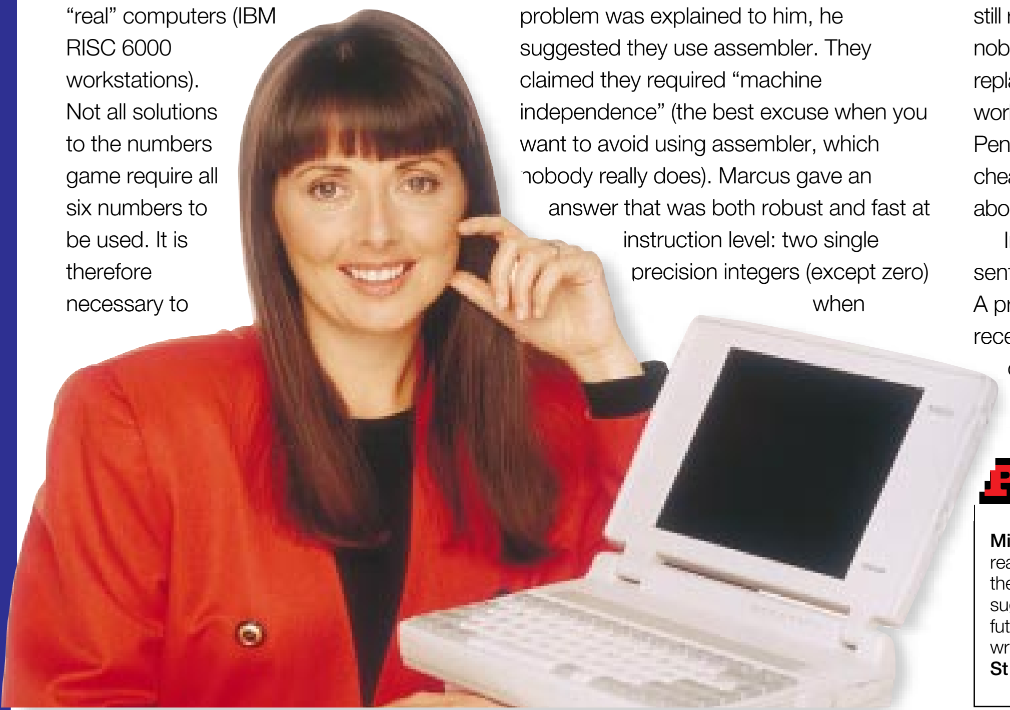
As can be seen, the calculations are reduced if the intermediate result is stored.

```
i = 50 + 25 + 10 + 3
check i
check i + 2 - 1
check i + 1 - 2
                  7 calculations
```

This may be applied simply in the nesting of the code and reduces the number of calculations per combination from five to between two and five.

At this stage, the solution was within reach. The code had been rewritten in C and was ready to go. The interpreter of the Psion had flagged overflows and these were dealt with easily. However, the Unix C compiler was not so accommodating. Overflows could go unnoticed and hence reproduce spurious results. Sweeting and Norris-Jones had come across a question which must have been asked by every serious programmer: "How do I detect integer overflow?"

Fortunately, at 3am that night they found Marcus, a diehard programmer, in an Albany bar. When the whole Countdown problem was explained to him, he suggested they use assembler. They claimed they required "machine independence" (the best excuse when you want to avoid using assembler, which nobody really does). Marcus gave an answer that was both robust and fast at instruction level: two single precision integers (except zero) when operated on with +, -, * or / cannot be larger than a double precision integer. So use single precision integers throughout and if the answer requires any of the bits of double precision, the operation has overflowed. The code was complete.



The program executed and found solutions within five seconds. When given a problem that was impossible to calculate, the solution required inspection of all the seven million combinations. In these cases the RISC 6000 completed the job in 25 seconds: a complete success.

The performance of the program was of interest and a small routine was created to simulate the picking of numbers. A reasonable sample would be required to analyse the performance properly, but at up to 25 seconds a game, such a sample would require a few hours of runtime. Unfortunately, as these machines were for university use, a different kind of access was required. Fortunately access was available, but not in the physical sense. The code was sent to a machine back in LA using ftp, and this machine was remotely set up to execute the program for 3,678 seconds every night from midnight. Generally, Unix machines are on day and night, so exclusive processor use could almost be guaranteed at that time. The NY workstations were set up for remote viewing of the LA machine's processor occupancy. The scrolling bar chart changed from a thin line to a solid black rectangle. The processor was running flat out.

That was October 1995. The program is still running with a log file of a few megabytes: nobody likes an idle computer. We can now replace Carol Vorderman if we wish. A Unix workstation does the trick, but nowadays a Pentium should be sufficient and a pretty cheap replacement. The next task? How about trying to replace Richard Whiteley?

Investigations of this problem should be sent to Mike Mudge at the address below. A prize will be awarded to the best entry received by 1st November. (SAE for return of entries, please.)

PCW Contact

Mike Mudge welcomes correspondence from readers on any subject within the areas of number theory and computational maths, together with suggested subject areas or specific problems for future articles. Email numbers@pcw.co.uk or write to Mike at **22 Gors Fach, Pwyll-Trap, St Clears, Carmarthenshire SA33 4AQ.**

Detector work

A prime number detector requiring only semi-colons? Pretty tight code, you may think. It lends Mike Mudge inspiration to set another poser for investigative readers.

Once upon a time, when Jonathan Cochrane started "messing about" with prime numbers, the first thing he did was to write a function to test whether a number is prime or not. It was easy enough, basically a function of the form:

```
int prime (int x)
{
algorithm;
return PRIME or NOT_PRIME
}
```

While getting the routine working and thinking of what to do next, he decided to try to optimise the prime number routine as much as possible (*haven't we all made this decision? — MM*) and he came up with a prime number detector that requires only semi-colons: pretty tight code, he thought! Can any readers implement a prime number detector satisfying the following specifications?

1. Use any amount of C code you want, but only two semi-colons are to be present.
2. Only allowed to pass one variable to the function, that is, the number to be tested.
3. No pointers are allowed.
4. The function must return a 1 or a 0 depending on prime or composite.
5. Semi-colons within the brackets of a for loop do not count, i.e. for

```
(x=2.3;x<99;x++)
```

^_ _ _ _ _ _ _ ^_ _ _ _ don't count.

6. The following style is also excluded, define semi_colon; Jonathan claims to have tried this on a number of colleagues without finding any solutions (other than his own!).

A different style of investigation, the responses from Numbers Count readers, will be examined with interest. Perhaps other code-based optimisation criteria might be applied?

An exercise in change of number base

Mr P Cowen of Middlesbrough has extended the recent result of JJ Clessa, *viz.* to find a number using the digits 1 to 9 once each only, such that the leading N digits of the number be divisible by N - to different number bases.

His first observation, that the number base must be even (why?) was followed by the use of a Pentium Pro 200 with 64Mb ROM "*which constipated with hard disk over-use at base 34,*" he tells us, but found results for bases 2, 4, 6, 8, 10 and 14. Can any readers extend this investigation, and, if

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possible, find an underlying theory which can be used to dramatically reduce the amount of computation needed to discover such numbers?

Any investigations of the above problems may be sent to me at the address below (see "PCW Contacts"), to arrive by 1st October 1997. All material received will be judged using suitable subjective criteria and a prize will be awarded by *PCW* to the best entry arriving by the closing date. (SAE for return of entry if required, please.) Each contribution should contain brief descriptions of the hardware and software used, together with coding, run times and a summary of the results obtained. General comments on the topics, with references to published or unpublished work in these general areas, would be appreciated.

JAMS: a result

Further to my column which dealt with the subject of JAMS (*Numbers Count, April*) the result X(34732165539) = 876 has been reported by both Mike Bennet (2hr, 11min, 3 sec on an Acorn Risc PC with a StrongARM processor) and by Nigel Backhouse ($4^1/_2$ days on a Pentium 133). So, do not become despondent at the lack of output from this investigation!

Appeals for reference material

Alexander Slack at 106431.2710@compuserve.com would like an *elementary* introduction to Mandelbrot Sets and wonders if there is any software available in QBASIC? Help for a 14-year-old embryo computer scientist would be appreciated.

Perhaps this is not quite in the spirit of Numbers Count*,* but the author would be interested to receive references to the problem of Tesselations in two dimensions. These need not involve any aspects of computing, although this is clearly a subject where computer graphics skills can be exploited both before and after the underlying maths has been understood.

Close relations

Going back to Numbers Count, December '96, John Sharp observes that the recurrence relation $T_n = 2T_{n-1} - T_{n-4}$ associated with $t^4 = 2t^3 + 1$ (number E) yields the sequence: 0, 1, 1, 1, 2, 3, 5, 9, 16, 29,… for which the ratio of successive terms converges, albeit very slowly, to the Tribonacci Number. Duncan Moore, Nigel Hodges and others found simple algebraic functions for A through I and partial results for the snubdodecahedron, e.g.

```
I = (t(t+2))**1/2, G = (2t+3)**1/2
```

NH also proved, in relation to Problem SL, that: T(2)=128, T(cubes)=12758, T(fourth powers)=5134240,T(fifth powers) =67898771, while T(6th powers) greater than 500 million and T (triangular numbers) = 33.

The worthy prizewinner, however, is Paul Richter of Tunbridge Wells, for a non-sophisticated approach to this investigation. Details from John Sharp at 20 The Glebe, Watford WD2 6LR (or from me).

Going back to your roots

This item in the November '96 column proved to be very popular. The Problem Function lead to a great deal of analysis. Ultimately, Nigel Hodges printed out the two roots to 700 places of decimals showing them to differ in the 647th place. Other analyses included using a program called "Mercury" on a 486DX by Martin Sewell. Duncan Gray refers to p3 of the Excel workbook, *Solutions*. James Lea cites *Numerical Recipes in C* (2nd edition), so this section is very well known.

RF Tindall has been aware of a very fast converging method of approximating to square roots, which is exactly equivalent to the algorithm given, for some time. But he observes that if N is at all large, there are difficulties finding the initial solution.

The worthy prizewinner is Matthew Davies of Luton, who offers an error estimate for the iteration scheme, a generalisation to rational rather than integers, a list of (m_0,n_0) seeds generated using a Turbo Pascal version 6.0 program in the range (1,100). And there's a concluding observation that "*If this technique were to be used as the basis of root calculations on something like an embedded system, I'd be inclined to compile a look-up table of* N....(m_0,n_0) *pairs rather than determine them on-the-fly.*"

■ *Correction: Dec'96 issue, p294, col.3 — for "Scientific American" read "American Scientist".*

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Mystery maths

Mike Mudge consults the book Unsolved Problems for his teasers this month, including distances and square numbers and circles. Plus, roll up for a number theory conference.

This is inspired by *Unsolved Problems in Geometry* by HT Croft, KJ Falconer and RK Guy, ISBN-0-387-97506-3, Springer Verlag 1991.

Question A. What is the maximum diameter of n equal circles that can be packed into a unit square? How should n points be arranged in a unit square so the minimum distance between them is greatest? These problems are equivalent: if a collection of points in a unit square are at a distance of at least d from each other, the points can serve as the centres of a collection of circles of diameter d that will pack into a square of side 1 + d.

Consider the second version of this problem and denote by d_n the greatest minimum distance between n points in a unit square. Exact results are known for n less than or equal to 9; also for n = 14, 16, 25 and 36. For n lying between 2 and 5 these are "easy" to obtain. Graham has established the result for n = 6, the results for n = 7, 8 & 9 are due to Schaer and Meir, those for 14, 16 & 25 & 36 are attributed to Wengerodt & Kirchner. Examples of both exact results and conjectual bounds are given in Fig 1 *(above)*.

Fig 1

n	2	3	4	7	10*	13*	17*	26*
d_n	$2^{1/2}$	$6^{1/2} - 2^{1/2}$	1	$2(2-3^{1/2})$	0.421	0.366	0.306	0.239

*Indicates a conjecture as far as the writer is aware.

Up to which square number is the square lattice packing the best? Certainly for up to 36. Wengerodt has found a packing of 64 points that is denser than the square lattice packing, but he conjectures that for 49, the square lattice packing is best.

Are there any values of n such that $d_n = d_{n+1}$? The problem can be asked for packing an equilateral triangle. Oler has shown that if n is a triangular number, of the form m(m+1)/2, the obvious configuration is the extremal one. The natural question is, can one do better if n is 1 less than a triangular number? Note that n spheres have been packed into a cube and certain other polyhedra, but even for a cube, exact results are only known for n less than eleven. A great deal of work is stilll to be done in this area!

Question B. Spreading points in a circle
The analog of the previous problem for the circle can be posed in a few equivalent ways:
1. What is the maximum radius of a disk, n copies of which can be packed into a circle of radius 1?
2. What is the radius of the smallest circle into which n unit disks can be packed?
3. What is the radius of the smallest circle containing n points, no pair of these points being a distance of less than 1 apart?
4. How large can the least distance between a pair chosen from n points in the circle be?

The last formulation yields the result that for n between two and six...inclusive...the least distance is 2*sin(PI/n) and for n = 7,8 & 9 the least distance is 2*sin(PI/(n-1)) with the obvious configurations. This is straightforward for n less than eight and was proved by Pirl for n = 8 & 9. He also solved the case of n = 10 and conjectured the values for n less than 20.

Suggested approaches to the problems include randomly generated points with analysis of large samples to estimate bounds, and the use of computer graphics to draw the optimum configurations, in the cases where these are known, and to examine and conjecture solutions for higher n.

Any investigations of the above problems may be sent to Mike Mudge at 22 Gors Fach, Pwll-Trap, St Clears, Carmarthenshire SA33 4AQ, tel 01994 231121, to arrive by 1st September 1997. All material received will be judged using suitable subjective criteria and a prize will be awarded by *PCW* to the best entry arriving by the closing date (SAE for return of entries, please). Each contribution should contain details of run times and a summary of the results obtained.

Comments on the topics would be appreciated. The topics included here range from tiling and dissection through packing and covering to include nets of polyhedra and lattice point problems.

■ *Details of the winner of November 1996 Numbers Count will appear next month.*

Conference on Smarandache-type Notions in Number Theory
21st-24th August 1997, Craiova, Romania. Bringing together those interested in Smarandache-type functions, sequences, algorithms, operations, criteria, theorums.

Contributed papers or plenary lectures are invited from all areas of Number Theory. For info: Dr C Dumitrescu, Mathematics Dept, University of Craiova, R-1100 Romania. Tel (40) 51-125302. Fax (40) 51-413728. kety prod@topedge.com , research37@aol.com.

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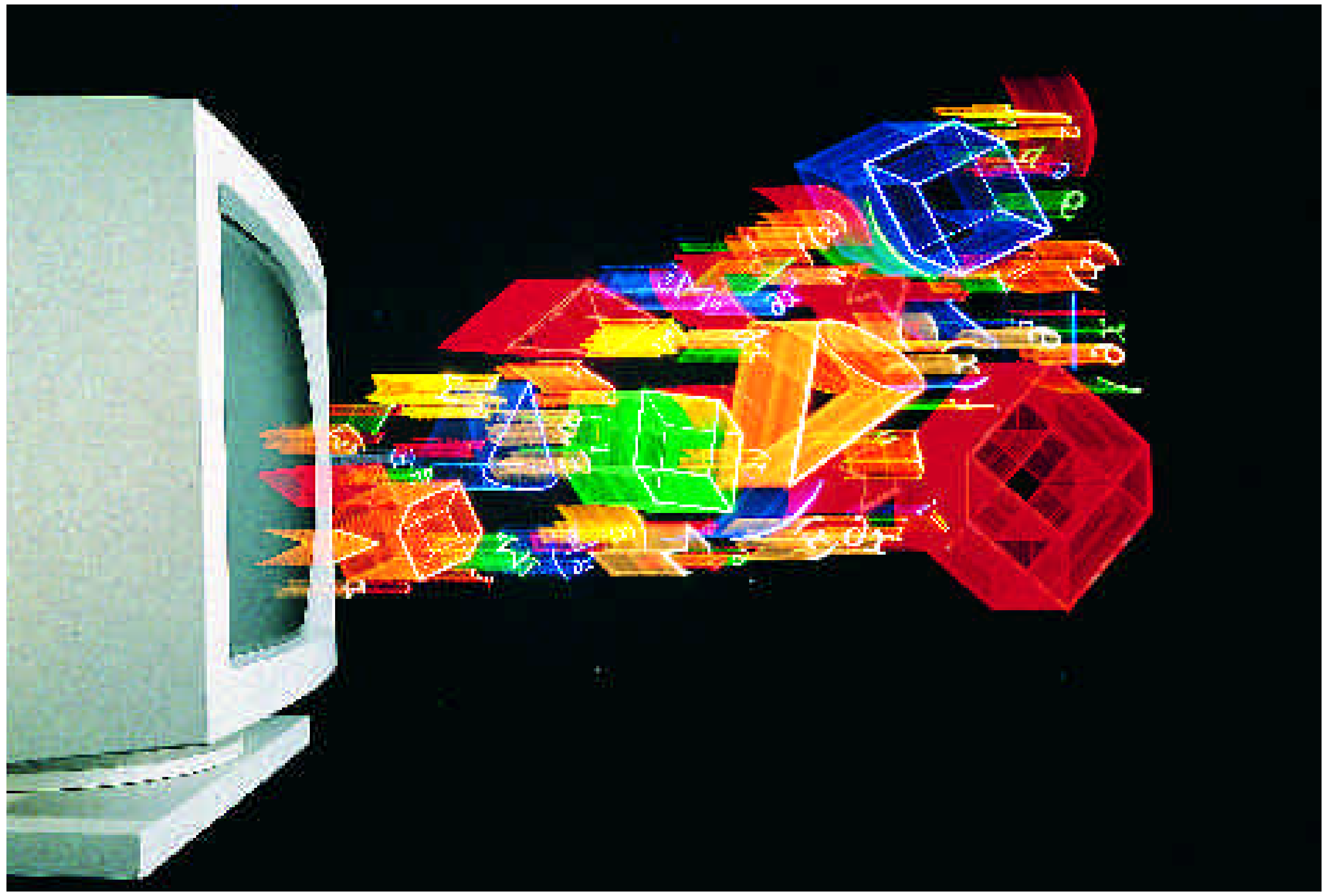
Morph code

Instead of dots and dashes, Mike Mudge checks his figures to find out whether numbers are nonamorphic or nonagonal. He also wonders why readers have been slow to respond.



Once upon a time... In the *Journal of Recreational Mathematics* Vol. 20(2), 1988, Charles W Trigg, of San Diego, addressed the problem of which primes had the sums of the squares of their digits also prime, e.g. if Prime (P) = 9431, then $9^2 + 4^2 + 3^2 + 1^2 = 107$ (Q) which is also prime. Among the 1229 prime numbers less than 10^4, Charles found 237 primes with this property… five two-digit, 47 three-digit and 185 four-digit primes. He observed that among the generating primes were the nine palindromes:

```
11, 101, 131, 191, 313, 353, 373,
797 & 919
```

The smallest of these is the sole prime repunit P = 11. For further study of repunits see *Repunits and Repetends* by Samuel Yates, Library of Congress Catalog Card Number 82-502451 (Star Publishing Co, Boiynton Beach, Florida 33435, in 1982).

There are also two near repunits, 223 and 8887. Among other structures present are members of the 25 reversal prime pairs such as 3169 and 9613. The smallest numbers of the pairs include

```
113, 179, 199… 3389, 3583, 7187,
7457, 7949 and 9479.
```

There are also some cases where the sums of the digits and the generating prime are equal, e.g. any prime permutation of 1136 giving 47 and 11, a prime permutation of 337, 1741 or 3037 giving 67 and 13, a prime permutation of 119 or 1019 giving 83 and 11. The most complex structure observed by Charles showed ten chains of primes wherein each Q is a P for the next link in the chain, e.g;

```
191, 83, 73, :443, 41, 17, :463,
61, 37, :1699, 199, 163 :6599, 223,
17, : 6883, 173, 59, : 467, 101, 2,
:883, 137, 59 : 449, 113, 11, 2,:
797, 179, 131, 11, 2,:
```

■ Problem CWT

Extend this analysis to both squares of digits of integers greater than 10^4, the cubes and higher powers of the digits of such prime numbers… and also address the problem to other "well-known" classes of integers like Fibonacci Numbers, Triangular Numbers, Tetrahedral Numbers etc. There may be underlying structures that deserve attention? Finally on this particular topic, the MM special: how do these results extend to other number bases? (Is there anything particular about base ten, from a number theoretic viewpoint? And if so, why?).

Nonamorphic numbers

Charles Trigg, the author cited above, introduced this terminology in the *Journal of Recreational Mathematics*, 13:1, pp 48-49 (1980-81). Definition: Nonagonal Numbers have the form N(n) = n(7n - 5)/2. A number is said to be nonamorphic if it teminates its nonagonal number.

Clearly, 1 is trivially nonamorphic in any number base. With this exception there are no nonamorphic numbers in bases two, three, four, five, eight and nine. In base ten there are five nonamorphic numbers less than 10^4, namely

```
N(1)=1, N(5)=75, N(25)=2125,
N(625)=1365625 and
N(9376)=307659376.
```

In base six there are five nonamorphic numbers less than 10^4, namely

```
N(1)=1, N(4)=114, N(13)=1113,
N(213)=253213 and
N(5344)=302505344.
```

Now, in base seven there are 42 such numbers!

■ Problem CWT nonamorph

Extend the above statistics to number bases greater than seven, and investigate any structure within these nonamorphs.

Finally, generate further "agonals" with associated "amorphs" and attempt to find an underlying general theory relating to their distributions within a given number base, and in particular the number bases in which non-trivial "amorphs" do not occur.

Can we consider "almost amorphs", where the termination differs from the input number in only one digit (by only one digit in that place)? Are we losing sight of number theory here and just playing with patterns? An underlying theory would say no.

Send any investigations of the above problems to Mike Mudge (see "PCW Contact", below) to arrive by 1st August, 1997. All material received will be judged using suitable subjective criteria and a prize will be awarded by *PCW* to the best entry arriving by the closing date (SAE for the return of entries, please). Each contribution should contain brief descriptions of the hardware and coding used, together with run times and a summary of the results obtained, and general comments on the topics. References to published or unpublished work in these areas would be appreciated.

Stop Press

In the March issue of *PCW* I requested a proof that $1^2 + 2^2 \ldots +n^2 = N^2$ had no solutions other than n = 1 and n = 24. The reference has been supplied by Robin John Chapman of the University of Exeter to WS Anglin, The Square Pyramid Puzzle, *American Mathematical Monthly* Vol. 97, pp 120-124 (February 1990). Thanks, Robin.

George Sassoon has investigated $x^2 = ny^2 = p$ and has so far (10/2/97) found that the value p = 316234801 leads to integer solutions for n = 1(1)30. He wonders what percentage of possible n values give solutions and suggests that there is no upper bound on values for p yielding such solution sets? Your comments, please.

Review of "Prime candidate", (Numbers Count 162, Oct '96)

For reasons totally beyond my comprehension, this did not prove to be a popular hunting ground for *PCW* readers. The worthy prizewinner is therefore the originator of the problem: Jonathon Ayres, 59 Watson Road, Leeds LS14 6AE.

Are there any readers with at least partial results to Jonathon's questions? If so, please contact him directly. There is also a fourth question to consider: What happens if you use different functions such as the highest Alliott Hailstone function, so that HAHF = highest alliott function (a*x + b)?

PCW Contact

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Mods and rockers

Mike Mudge JAMS with mod sequences. No, he hasn't joined a retro band; here he presents a stimulating exercise in occurrences to get your feet tapping and your calculators clicking.

JAMS, or Jonathon Ayres Mod Sequences, are believed to have their origins in Leeds in the autumn of 1996. I am indebted to Jonathon for the following presentation of the idea which both he, and I hope readers of this column, will find interesting and stimulating.

Mod sequences

The mod sequence is defined as
X(n) = (2*X(n-1)+1) mod n
where n starts at 1 and x(0) equals 0. The first few numbers in the mod sequence are 0, 1, 0, 1, 3, 1, 3, 7, 6 and 3.

1. Occurrence of X

When does a number occur in this sequence? The first occurrence of the numbers 0 to 19 in the mod sequence are shown in Fig 1.

All numbers less than 1,000 occur in this sequence, for n less than 10,000,000, with the exception of 204, 344, 614, 622, 876 and 964. These first occur at:

X(n)= 614, n= 10629529
X(n)= 204, n= 15245143
X(n)= 344, n= 26713415
X(n)= 622 n= 47286732
X(n)= 964 n= 67815823

I have not been able to find the first occurrence of X(n)=876, but if it does occur n is bigger than 75,000,000.

2. Special values of X(n)

- X(n)=0 for n = 1, 3, 79, 35, 431, 1503, 2943, 6059, 6619, 18911 and 54223.
- X(n)=n-1, for n=1, 2, 8, 32, 46, 392, 12230, 155942, 659488, 1025582, 10471228 and 3437088
- X(n)=n/2, for n=2, 78, 234, 430, 1502, 2942, 6058, 6618, 18910 and 54222
- X(n) and n end in the same last four digits for n=34875, 52363, 54975 and four others less than 100,000, and with the last five digits of both the same, the only values of n less than 1,000,000 are n=389103, 469599 and 742955.

3. Distribution of X(n)

- The most common occurring values of X(n) are of the form 2^p-1, so that for n less than 1,000,000, the number 63 occurs 47 times.
- The average value of x(n) is about n/4.
- There are no values of n greater than 1 so that X(n)=X(n+1), but for X(n)=X(n+2) this is true for n=6, 7, 12, 13, 24, 25, 174, 175, 2448, 2449, 3072, 3073, 6768 and 6769.
- X(n)+1=X(n+1) is true for the values of n, n=3, 5, 81, 237, 433, 1505, 2945…

Fig 2 shows the first values of n so that X(n)+a = X(n+1). All values of **a**, less than 500, occur for **n** less than 10,000,000 except for 205, 215, 345 and 391.

- For pairs of numbers x and y, y is at most 2x+1. The values of x where y has values other than 2x+1, are x=1,3,6,7,13,14,15, 16,17,18,20,23...

Fig 1

First occurrences of the numbers Y, Y=0 to 19, so that X(N)=Y

Y	N	Y	N
0	0	10	149
1	2	11	27
2	53	12	91
3	5	13	18
4	71	14	21
5	26	15	17
6	9	16	43
7	8	17	20
8	19	18	29
9	72	19	50

Fig 2

First values of n so that X(n)+a = X(n+1)

A	X	A	X
1	3	11	151
2	6	12	29
3	55	13	93
4	9	14	64
5	73	15	29823
6	28	16	33
7	63	17	45
8	18	18	42
9	21	19	71
10	74	20	52

Supernumerary

- On 6th December, Tony Forbes of Kingston-Upon-Thames announced his discovery of a triplet of 1,083-digit primes, believed to be the largest known prime triplet. *(Further details of these numbers and the underlying theory/computation on request — MM.)*
- Anyone who knows the means of obtaining a "zooming Mandlebrot plotter" please email Gogul@aol.com.
- Anyone wishing to get involved in the "Great Internet Mersenne Prime Search" mentioned in *PCW* (Jan) should contact Nigel Backhouse, Division of Applied Mathematics, University of Liverpool, M&O Building, Liverpool L69 3BX (*Kevin Edge, please note*).
- In response to frequent requests for reasonably priced (or free) software for long integer manipulation: I can provide UBASIC free of charge on receipt of a suitably stamped, addressed, padded bag.

Numbers Count (*PCW*, September '96) — 'Fraction Action'

Gareth Suggett obtained successive length records for the period of the continued fractions of the square roots of the non-square integers up to d=10,000, terminating with d=9,949 having cycle length 217. However, Gareth discovered a program called "CALC", written by KR Matthews of the University of Queensland. The MSDOS version is available from the Mathematics Archives ftp site: ftp://archives.math.utk.edu/software/msdos/number.theory/krm-calc. On a 25MHz 386 PC, each of the 10-digit results quoted in the original article can be obtained in about 20 minutes. The final 11-digit result was confirmed on a 133MHz Pentium in 15 minutes, producing a 6.8Mb output file!

John Borland observed that at some time, "continued fractions were a standard topic in higher mathematics". Readers' experiences of instruction in this topic would be most interesting, together with their personally recommended reference books both for numerical and function approximation theory applications.

This month's prizewinner, however, is Duncan Moore of Birkenhead for his major contribution to "Something Different", spread over August 1993 and January 1997. The total number of solutions now known is 30.

Also in relation to this problem, Henry Ibstedt reported *(November '96)* finding one with three of p, q, r, s, t sharing one factor and the other two sharing a different factor. This solution is p=286, q=154 sharing the factor 2, and r=s=t=11 sharing the factor 11 with (2, 11) = 1.

Henry points out that p and q also share the factor 11 but that this was not excluded from the question — there is still a great deal of work to be done before this problem is fully understood.

Questions

- Do all numbers occur in this sequence, and also, do they occur an infinite number of times?
- Is there always a value of n, for every a (positive or negative) so that X(n)+a = X(n+1)?
- Is there a way of predicting when a number will occur in the sequence?
- Is there a formula which gives the n'th value of the sequence, without calculating the rest of the series?
- What happens for other sequences, such as x(n)=ax(n-1)+b mod n or x(n)=(x(n-1) +x(n-2)) mod n?

Something different

This item was taken from *Computer Weekly* (19th January edition, 1989).

Following up on the observation that $15226_{10} = 62251_7$ and further that $99481_{10} = 18499_{16}$ (where the subscript denotes the base in which the number is represented), find the lowest five-digit number (in any base). Generalise this process to n-digit integers.

Answering back…

Please send any investigations of the above problems to Mike Mudge at 22 Gors Fach, Pwll-Trap, St Clears, Carmarthenshire, SA33 4AQ (tel 01994 231121), to arrive by 1st July, 1997. All material received will be judged according to suitable criteria and a prize will be awarded by *PCW* to the best entry arriving by the closing date (an SAE is required for the return of entries). Each contribution should contain brief descriptions of the hardware and coding used, together with run times and a summary of the results obtained.

General comments on the topic of JAMS would be welcome, together with any practical (or unusual) applications of integer arithmetic in number bases other than 2 and 10.

PCW Contact

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Power points

Mike Mudge faces a stiff challenge in proving a solution, and this leads him to considering a number of related problems concerned with the power sums of separate digits.

I was asked (by Cyprian Stockford) for a proof that the only solution to

$$1^2 + 2^2 + \ldots + n^2 = N^2$$

is n = 24 when N = 70, *viz.* positive integer solution of

$$n(n+1)(2n+1) = 6N$$

is unique as asserted in *The Penguin Book of Curious and Interesting Numbers* (David Wells, 1987) and elsewhere. Being unable to provide such a proof (can any readers help?) my attention was caught by a number of notionally related problems involving the power sums of the separate digits or the partitions of a given positive integer.

■ 1: 1201 seems to be the smallest prime number which can be represented by the expression x^2+ny^2 for all values of n from 1 to 10. Is this true? What other prime numbers can be so represented, and what happens if the range of values of n is increased to 1 to M for an arbitrary M?

■ 2: It is clear that $1233 = 12^2+33^2$ while $8833 = 88^2+33^2$. Under what circumstances is a given integer equal to the sums of the squares of its partitions into pairs? How does this result extend to the cases of higher powers (i.e. cubes) and also to the cases of partitions into ordered triples, 4-tuples, etc? Does this lead to a sensible problem in number bases other than 10?

■ 3: $3435 = 3^3+4^4+3^3+5^5$ while it is said that (Wells, p.190) 438579088 is the only other number exhibiting this behaviour when powers of a single digit are considered. Can this result be generalised to pairs, i.e. $abcdef\ldots = (ab)^{ab} + (cd)^{cd} + \ldots$ or even to triples, etc? What happens in other number bases?

■ 4: By inspection, $175 = 1^1 + 7^2 + 5^3$; when, in general, does

$$a^1{}_1 + a^2{}_2 + a^3{}_3 + \ldots + a^n{}_n = a_1a_2\ldots a_n$$

where the right-hand side is understood to mean the integer so written in any number base? It is more natural to reverse the powers and even to start at zero, thus requiring

$$b^0{}_0 + b^1{}_1 + b^2{}_2 + \ldots b^n{}_n = b_nb_{n-1}\ldots b_2b_1b_0$$

The Subfactorial Function is defined as

$$!N = N!\,(1 - 1/1! + 1/2! - 1/3! + 1/4! \ldots (-1)^N/N!)$$

where

$$N! = 1.2.3\ldots N \text{ e.g. } !5 = 5!\,(1 - 1/1! + 1/2! - 1/3! + 1/4! - 1/5!) = 44$$

while !7 = 1854. It is stated that 148349 is the only number equal to the sum of the subfactorials of its digits.

■ 5: Prove this result and attempt to generalise it to other number bases. Try replacing subfactorial by factorial and/or replacing sum by product. Comment on the function obtained from the subfactorial function by introducing only positive signs into the definition.

■ 6: Regarding the individual digits of an integer: is it possible to get a prime number from any given number by changing one of its digits? The answer is “No”. The smallest integer for which this is not possible is 200. Is it possible to get a prime number from any given integer by changing two of its digits? If not, what is the smallest number for which this is not possible?

Investigations of the above problems should be sent to Mike Mudge, 22 Gors Fach, Pwll-Trap, St Clears, SA33 4AQ, by 1st June 1997. All material will be judged using suitable subjective criteria and a prize will be awarded to the best entry arriving by the closing date (SAE for return of entries).

Golomb Rules, OK (*PCW*, Aug ’96)

This problem produced a large and varied response. In the problem PI seeking a solution greater than 7 to $n! + 1 = N^2$, Alan Cox extended Kraitchik’s lower bound from 1020 to 2500 using MAPLE V release 4 on a Dell 486D DX33 with 8Mb RAM and about 250Mb hard disk, in about six hours.

Problem P2 is solved completely. Dr John Cohen gave the reference to *Finkelstein & London* in *J. Number Th. 2* (1970), pp 310-321, together with references to work on $y^2 + k = x^3$ for a large range of k by Josef Gebel. Nigel Backhouse obtained a list of Golomb Rulers up to order 15, the final length being 151 with an example (0, 4, 20, 30, 57, 59, 62, 76, 100, 111, 123, 136, 144, 145, 151).

Gareth Suggett indicates that a group from Duke University have obtained optimum rulers up to 19 marks (*New Algorithms for Golomb Rulers Derivation and Proof of the 19 Mark Ruler,* Dollas, Rankin & McCracken, Nov ’95). Gareth speculated on the metric result for measuring all distances in centimetres from 1 to 100 on a metre rule. He refers to The *Dipole* column in *The IEE News* some years ago with the best known solution as 15 marks at 1, 2, 8, 14, 25, 36, 47, 58, 69, 80, 85, 90, 95, 98, 99. Is this minimal and/or unique?

Our prizewinner is RF Trindall, of Cambridge, for his extension to circular Golomb Rulers with n(n-1) + 1 points spaced round a circle uniformly and n of them marked to measure every distance from 1 to n(n-1). This was accompanied by analysis of P2 and P3 and some (accepted) criticism of their difficulty… sorry, readers!

PCW Contact

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Not numerology but numeralogy!

There's a world of difference between the o and the a, as Mike Mudge explains.

Numerology is variously defined as the study of numbers as supposed to show future events or the relationship between numbers and the occult. However, the term *numeralogy*, supplied by P Castini of Arizona, USA, is defined (by him) as "Properties of the Numbers": his proposal for a Numbers Count column includes some 37 sequences each with a rule of generation and some associated queries for investigation.

There follows a (random?) sample of these. Others may be included at a later date depending on the popularity of such research areas.

The **PROBLEM CAS. (n).** is the same in every case, *viz.* implement a computer algorithm to generate the defined sequence and hence, or otherwise, investigate the associated queries.

S(1). Non-arithmetic Progression. General definition: If m_1 & m_2 are the first two terms of the sequence, then m_k for k greater than 2 is the smaller number such that no 3-term arithmetic progression is in the sequence, i.e. we do not find

$m_p - m_q = m_q - m_r$

for distinct p,q & r.

e.g. if

`m1= 1 & m2 = 2`

we generate

`1,2,4,5,10,11,13,14,28,29,31,...`

Generalised S(1) Same initial conditions, but no t-term arithmetic progression in the sequence for t greater than 3.

Query How does the density of such a sequence, i.e. the fraction of the integers less than N which it contains, vary with N, (m_1,m_2) & t?

S(2). Prime-product sequence Here T_n is one greater than the product of the first n primes with the proviso that $T_1=2$. Sequence begins

`2,7,31,211,2311,30031...`

since 2*3*7*11*13+1 = 30031.

Query How many members of this sequence are prime numbers?

S(3). Square-product sequence As S(2) above with primes replaced by squares, viz.

`2,5,37,577,14401,`
`518401,....`

since

$1^2*2^2*3^2*4^2*5^2*6^2 + 1 = 518401$

Query How many members of this sequence are prime numbers?

Generalised S (3) Replace squares by cubes, fourth powers, etc. and investigate the same query. May also be generalised using the products of the factorial numbers

`1,2,6,24,120,720....`

Now let (T_n) be a sequence defined by a property P and screen this sequence, selecting only those terms whose individual digits hold the property P to obtain the S. P-digital subsequence. e.g. the S. square-digital subsequence

`0,1,4,9,49,100,144,..`

is obtained from

`0,1,4,9,16,25,36,49,..`

by selecting the terms whose digits are all perfect squares — only 0,1,4 & 9 allowed.

p280 ➢

Numbers Count, June 1996

"Sequence of events", Descriptive Number Sequences Part (1), *PCW* June 1996, proved very popular. It is intended to review at length the two parts of this topic in the next issue. Suffice it to announce the prizewinner as Jean Flower of The Mathematics Centre, Chichester IHE, Upper Bognor Road, Bognor Regis, West Sussex PO21 1HR, who used Mathematica on a Pentium 120 and (eventually) was able to find all cycles of length less than 17, with a greater than 1 and n greater than 13. All of this was accompished in about five minutes of processor time and was accompanied by a fascinating alphabetic version of the same problem. Consider the sequence of sentences. "This sentence contains three hundred and seventeen occurences of the letter 'e'", the next term being a sentence which decribes the previous one etc. What about carrying this analysis on a computer?

More to come on this topic.

Similarly for the S. cube-digital subsequence and higher powers.

S (4). Consider the S. prime-digital subsequence

```
2,3,5,7,37,53,73,..
```

Query Is this sequence infinite?

S (5). The S. odd sequence

```
1,13,135,1357,13579, 1357911,...
```

Query How many terms are prime?

S (6). The S. even sequence

```
2,24,246,2468,246810,..
```

Query How many terms are the nth powers of a positive integer?

S (7) The S. prime sequence

```
2,23,235,2357,235711,...
```

Query How many terms are prime?

For further study of S(4) through (7) see: Sylvester Smith, *Bulletin of Pure and Applied Sciences*, vol. 15. E (no. 1) 1996. pp101-107. A set of conjectures on Smarandache* Sequences.

**All the sequences discussed this month have appeared in print under Smarandache Notions.*

For further information on this area of work see Smarandache Notions Journal, vol. 7 no. 1-2-3, August 1996. ISSN 1084-2810. Department of Mathematics, University of Craiova, Romania.

Something totally different

Eric Adler has drawn my attention to the approximate sizes of elements in the Mathematica 3.0 Software Package where "Front end etc. 6.0Mb, Kernel etc. 18,5Mb, MathLink Libraries 0.5Mb and Fonts 4.5Mb total 27.5Mb whilst Standard Add-on Packages at 9.0Mb together with The Mathematica Book of 36Mb, Listing of Built-In Functions at 5.5Mb, Standard Add-on Packages occupying 11.0Mb and Additional Documentation of 15.0Mb (the latter four items totalling 66Mb) yield 74.5Mb. The total size of storage (again approximate) is quoted as 96Mb whilst strict addition yields 106.0Mb."

Eric asks: "How do they get that?" and offers ten IBM format 3.5in 1.44Mb floppy disks as first prize, with 40 IBM-format 3.5in 1.44Mb floppy disks with UBASIC as runners-up prizes. Facetious answers such as "They used a Microsoft Calculator" or "They are measuring using Microsoft Drive Space" will not be eligible for the first prize!

Stop press!

Would Duncan Moore please let me have his address as I have some information for him. Sorry, Duncan, for the inefficiency of my filing system!

Following on from the study of "Golomb rulers" in the August 1996 issue of *PCW,* at least one reader has expressed an interest in the "Circular Golomb Ruler". Here, the problem is essentially the same except that the points are spaced around the circumference of a circle and distances measured along the circumference also. Apparently solutions are known for some n (maximum distance to be measured); it is further known that for certain n, no solution is possible. What happens if the distance is measured in a straight line!?

Any investigations of this month's queries may be sent to Mike Mudge, 22 Gors Fach, Pwll-Trap, St Clears, Carmarthenshire SA33 4AQ, tel. 01994 231121, to arrive by 1st May 1997. All material received will be judged using suitable criteria and a prize will be awarded by *PCW* to the best entry (SAE for return of entries, please).

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Festive fractions

Mike Mudge gets stuck into a feast of fractions for Christmas, and appeals for help on behalf of readers.

This month, I have a number of appeals to make:

1. Nigel Backhouse of the Division of Applied Mathematics at the University of Liverpool wonders if any readers would be interested in joining the Great Internet Mersenne Prime Search? He tells me that George Woltman is asking for volunteers with Pentiums and 486s and access to the web, to join a team searching for new, large Mersenne Primes. He provides free software and full instructions on how to use it. This can be downloaded from our world.compuserve.com/home pages/just for fun/prime.htm.

2. Alan Cox has been studying the paper by Artur Ekert and Richard Jozsa in *Reviews of Modern Physics*, July '96, pp1-28, entitled "Quantum Computation and Shor's Facturing Algorithm". In common with your columnist, he finds it difficult to understand but realises the importance of the subject area. Is anyone willing to produce a simple guide to the concepts involved? *PCW* may consider such material for publication, as it would be to the benefit of many readers and relate to the very frontiers of computational theory.

3. Caryl Takvorian is anxious to access a paper on the subject of NP-complete and intractable problems. Is any reader able to supply a suitable reference or offer such a paper to *PCW* and/or Caryl directly?

FRACTRAN: a simple universal programming language for arithmetic

Fractran: Due to JH Conway, Open Problems Commun. Comput, pp4-26, published in 1986.

To play the Fraction Game corresponding to a given list: $f_1, f_2, \ldots, f_k$ of fractions and a starting integer N, we repeatedly multiply the integer which is defined at any stage (initially N) by the earliest f_i in the list for which the answer remains an integer. Whenever there is no such f_i the game stops.

Formally: the sequence (N_n) is defined by $N_O = N$ (given) while $N_{n+1} = f_i N_n$ where i between 1 & k inclusive is the least i for which $f_i N_n$ is integral, providing such an i exists.

Experiment 1 Consider the list of fractions $^{17}/_{91}$ $^{78}/_{85}$ $^{19}/_{51}$ $^{23}/_{38}$ $^{29}/_{33}$ $^{77}/_{29}$ $^{95}/_{23}$ $^{77}/_{19}$ $^{1}/_{17}$ $^{11}/_{13}$ $^{13}/_{11}$ $^{15}/_{2}$ $^{1}/_{7}$ $^{55}/_{1}$: these define PRIMEGAME (after Conway). Choosing N = 2, the other powers of 2 which are generated are those whose indices are the Prime Numbers in ascending order.

Experiment 2 Consider the list of fractions $^{365}/_{46}$ $^{29}/_{161}$ $^{79}/_{575}$ $^{679}/_{451}$ $^{3159}/_{413}$ $^{83}/_{407}$ $^{473}/_{371}$ $^{638}/_{355}$ $^{434}/_{335}$ $^{89}/_{235}$ $^{17}/_{209}$ $^{79}/_{122}$ $^{31}/_{183}$ $^{41}/_{115}$ $^{517}/_{89}$ $^{111}/_{83}$ $^{305}/_{79}$ $^{23}/_{73}$ $^{73}/_{71}$ $^{61}/_{67}$ $^{37}/_{61}$ $^{19}/_{59}$ $^{89}/_{57}$ $^{41}/_{53}$ $^{833}/_{47}$ $^{53}/_{43}$ $^{86}/_{41}$ $^{13}/_{38}$ $^{23}/_{37}$ $^{67}/_{31}$ $^{71}/_{29}$ $^{83}/_{19}$ $^{475}/_{17}$ $^{59}/_{13}$ $^{41}/_{291}$ $^{1}/_{7}$ $^{1}/_{11}$ $^{1}/_{1024}$ $^{1}/_{97}$ $^{89}/_{1}$: these define PIGAME (after Conway). Choosing N as 2^n the next power of 2 to appear is $2^{p(n)}$ where p(n) is the n^{th} digit after the point in the decimal expansion of PI.

Experiment 3 Consider the list of fractions $^{583}/_{559}$ $^{629}/_{551}$ $^{437}/_{527}$ $^{82}/_{517}$ $^{615}/_{329}$ $^{371}/_{129}$ $^{1}/_{115}$ $^{53}/_{86}$ $^{43}/_{53}$ $^{23}/_{47}$ $^{341}/_{46}$ $^{41}/_{43}$ $^{47}/_{41}$ $^{29}/_{37}$ $^{37}/_{31}$ $^{299}/_{29}$ $^{47}/_{23}$ $^{161}/_{15}$ $^{527}/_{19}$ $^{159}/_{7}$ $^{1}/_{17}$ $^{1}/_{13}$ $^{1}/_{3}$: these define POLYGAME (after Conway). Define $f_c(n)$ +m if, when Polygame is started at $c2^{2^n}$, then it stops at 2^{2^m}, otherwise leave $f_c(n)$ undefined. Then every computable function appears among f_0, f_1, f_2… The number c is called the Catalogue Number and is "easily computed for some quite interesting functions". Conway gives f_c for any c whose largest odd divisor is less than 2^{10}.

Problem

Understand and implement FRACTRAN in the form of the first two experiments. Follow this with an initial investigation of Experiment 3… and comment upon this approach to computable functions.

■ Send any implementation of the above algorithms to Mike Mudge, by 1st April, 1997. All material received will be judged using suitable subjective criteria and a prize will be awarded for the best entry (SAE for return of entries, please).

■ Responses to the three appeals should also be sent to Mike Mudge (for forwarding). George Woltman can be contacted directly as indicated above.

Report on Numbers Count May '96

Nigel Hodges examined "Problem MM" and used x = m/n, y = a/b (in their lowest terms) to distinguish two cases p does/does not divide m: obtaining solutions for p = 5 involving integers of 15 & 16 digits for m&n and 22, 23, 24 digits for a and b. Note that A. Bremner and J. Cassels, *Mathematics of Computation*, vol. 42, no.165, Jan 1984, pp 257-264, cite "*a most startling generator of all solutions for p = 877 where 42 & 40 digit integers arise as m & n whilst a & b have 63 & 60 digits respectively*". However, the prizewinner this month is Patrick Moss, of 26 Hillside, Grays RM17 5SX. His submission, "Rational Points on a Cubic Curve", includes an arithmetic/algebraic section followed by a section dealing with geometrical arguments, and finally, a set of special cases and generalisations. The computational aspects were programmed in C++ on a Gateway 2000 P5-120, prompting Patrick to ask if any reader has access to some decent code or knows of a not-too-expensive piece of software for handling large integer-length arithmetic? He used Microsoft Excel to draw the graphs but wonders whether other software could have done the job?

Details of this work on request to Patrick. A number of his results were subsequently confirmed in *The Arithmetic of Elliptic Curves* by JH Silvermann.

• PCW Contacts

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Close **relations**

Mike Mudge presents the relationship between Archimedean Polyhedra and the Tribonacci Series. Mersenne Non-Primes get some attention, too.

In a recent letter John Sharp, of Watford, wrote: "*It is well known that the dodecahedron and the icosahedron are intimately bound up with the Golden Section, which is in turn related to the Fibonacci Series.*" Readers whose knowledge of geometry is minimal may already feel discouraged. However, this is an arithmetic problem.

John has studied the Archimedean Polyhedra known as the snub-cube and the snub-dodecahedron and has found a similar relationship between the former and the Tribonacci Series, defined by the recurrence relationship:

$T_{k+1} = T_k + T_{k-1} + T_{k-2}$

with suitable initial values for

T_0, T_1 & T_2

The constant associated with this series is

t = 1.83928675521416..

this being the positive root of the quartic equation:

$t^4 - 2t^3 + 1 = 0$*

Now, relative to a snub-cube with unit sides, the diagonals have (approximate) length:

A= 1.68501832488972
B = 1.83928675521416
C = 2.16300104263277
D = 2.320124084592509
E = 2.382975767906236
F = 2.434474230834721
G = 2.584293619236854
H = 2.601144274317068
I = 2.657357374421356

$A = (t + 1)^{1/2}$, $B = t$, $E^* = t + 1/t = (2t + 2)^{1/2}$

Intuition tells John Sharp that the lengths of the other diagonals have "*some relatively simple relationships to t*" but how can these

be found computationally?

For the snub-dodecahedron with constant

m = 1.943151259243865

there are 28 lengths commencing with:

A = 1.715561499697342
B = m
C = 2.343373277136706
D = 2.467232466141474
E = 2.528610449446665
AA = 4.260575577706465
F = 2.775836816301074
G = 2.782298391314399
H = 3.059283956591891
I = 3.118888631147017
J = 3.144084782738732 down to
BB = 4.294380888587396

There is a database available on the internet called the Inverse Symbolic Calculator (ISC) by J. A. Sloane and S. Plouffe, having, on August 1996, 45 million entries which (reference: "A question of numbers", by Brian Hayes, Scientific American, vol. 84, Jan-Feb 1996) Plouffe foresees expanding to a billion entries. The internet address is www.cecm.sfu.ca/projects/IS/ISCmain.html. Here, the Tribonacci constant is easily found but there is no entry "close to" m. Help!

Mersenne NON-PRIMES

On 3rd September 1996, Cray Research announced that Slowinski and Gage had found the 34th Mersenne Prime, being $2^{1257787} - 1$ with 378632 decimal digits. (*Note: This may not be the 34th in order of magnitude as the search algorithm is not exhaustive*). However, Jonathan Ayres of Leeds, one of our regular readers, drew my attention to certain problems related to Mersenne NON-PRIMES.

Revision note: a Pseudo-Prime to base b is a number, n, such that bn-1-1 is divisible by n. For example, 15 is the smallest pseudo-prime to base 4, because 414 - 1 = 268435455 is divisible by 15. Similarly, 217 is the second smallest pseudo-prime to base 4. 91 is the smallest pseudo-prime to base 3, 341 and 641 are the first two pseudo-primes to base 2, while 161038 is the smallest *even* pseudo-prime to base 2.

A Carmichael Number (or Absolute Pseudo-Prime) is a pseudo-prime to any base. So, 561, 1729, 2821, 1105, 1729, 2464, 2821 are examples of such numbers, $a^{560} - 1$ being divisible by 561 whatever the value of a.

PROBLEMS MNP

1. Are all non-prime Mersenne numbers pseudo-prime to some base b, and more generally pseudo-prime to some base 2^p? Are there some Mersenne numbers that are Carmichael numbers?

2. Are all non-prime Mersenne numbers pseudo-primes to some base b, where b is not a power of 2, and how does this number relate to p? Furthermore, is there some base b, that is not a power of 2 but is a pseudo-prime basis for more than one Mersenne number?

3. Are all composite xy ± 1 pseudo-primes for some base b, and are there any Carmichael numbers of this form?

Some of the early numerical results relating to problems MNP can be obtained by sending a stamped addressed envelope to Mike Mudge.

Any investigations of Problems MNP and/or advice for John Sharp may be sent to me, Mike Mudge at the address shown in the *PCW* panel here, to arrive no later than 1st March 1997.

INTEGRAL BASES and Computer Experiments due to Shen Lin

I have a further item which follows on from last month's theme, based on an article by P. Shiu. Let $S=(s_1,s_2,...s_k...)$ be a sequence of positive integers and, consider the set P(S) consisting of all numbers which are representable as a sum of a finite number of distinct terms of S. We say that S is complete if all sufficiently large integers belong to P(S). For a complete sequence, we call the largest integer not in P(S) the threshold of completeness T(S). It is known that for the sequence of squares

S=(1,4,9,16,..) T(S) - 128

and for the sequence of cubes

S=(1,8,27,64..) T(S) = 12758

PROBLEM SL. Determine the value of T(S) for the sequence of fourth primes and triangular numbers. (Generated using n(n + 1)/2).

Report on "Chiefs and Indians" (*Numbers Count, PCW* April 1996)

"Stop Press": Rex Gooch analysed up to six consecutive prime pairs to 109 and also confirmed Nigel Backhouse's result, of 14 consecutive prime pairs starting at 678771479, 678771481, while John Sutton looked at the alternative problem of the span containing n prime pairs, relaxing the requirement of no intervening primes. A future research area?

Now to the "Chiefs and Indians". Alan Cox quotes from Rouse Ball where the "Josephus problem" is referred to with the reference Hegesippus's "De Bello Judaico". Nigel Hodges generates samples of the numbers of Indians needed for the Chief to be successful for "step-factors" up to 49. For example, 1169262 Indians will constitute good news for the Chief if the "step-factor" is 44.

However, the worthy prizewinner this month is Robert Newmark of Cleadon, Sunderland, who programmed in C on a Toshiba T2110-486DX for up to 5,000 Indians with jumps from two to 20: total analysis in one second. The program is available on request.

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Going back to your **roots**

Mike Mudge presents a square-root algorithm suitable for newcomers to this column, and rational approximations to square roots of integers should crank your brains into gear.

This month's theme is based upon an article by P. Shiu in *Mathematical Spectrum*, vol 4, no. 1, 1971/72, pp26.30.

To approximate to the square root of N, i.e. $N^{1/2}$, where N is a given square-free integer, first seek an integer solution m_0,n_0 of the equation $n(n+1) = Nm_2$. Then observe that this equation is also satisfied by the sequence:

$m_1 = 2m_0(2n_0 + 1)$, $n_1 = 4n_0(n_0 + 1)$
$m_2 = 2m_1(2n_1 + 1)$, $n_2 = 4n_1(n_1 + 1)$.....
$m_{k+1} = 2m_k(2n_k + 1)$; $n_{k+1} = 4n_k(n_k + 1)$

While $n^{1/2}$ is approximated to (from above) by:

$r_k = (2n_k + 1)/(2m_k)$

e.g. If N = 2 we may choose $m_0 = n_0 = 1$ when the above recurrence relations yield:

$m_1 = 6$, $n_1 = 8$; $m_2 = 204$, $n_2 = 288$; $m_3 = 235416$, $n_3 = 332928$; $m_4 = 313506783024$, $n_4 = 443365544448$;

These numbers yield an n_4 which differs from $2^{1/2}$ by less than 10^{-24}. We have an approximation to square root of two correct to 24 decimal places!

PROBLEM ROOTS. Implement the Shiu Algorithm to initially find an m_0,n_0 pair for a given N, followed by the sequence of fractions (r_k) which approximate to $N_{1/2}$.

PROBLEM ROOTS*. Attempt to generalise this process to cuberoots and beyond, comparing its computational efficiency with other, more commonly used algorithms.

An 'almost incomputable' function

The recently-published text by Arnold R. Krommer and Christoph W. Ueberhuber, "Numerical Integration on Advanced Computer Systems", Lecture Notes in Computer Science 848, Springer-Verlag 1994, has a 268-item bibliography and a commensurate body of text, an altogether outstanding publication. On page 186, readers are introduced to the function

$f(x) = 3x^2 + (PI)^{-4}\log((PI - x)^2) + 1$

which has a pole at x = PI, by which we mean that its value is unbounded below (infinitely large and negative) at x = PI.

Since clearly the function is positive over very large ranges of x, it must have two zeros (at least) one on either side of the pole. However, if it is sampled at ALL MACHINE NUMBERS differing by 2^{-54} (approximately 5.6×10^{-17}) and corresponding to Double Precision IEEE Arithmetic, the pole cannot be detected and indeed no negative values are generated.

PROBLEM FUNCTION. Devise a means of exhibiting either graphically or numerically the true behaviour of this function. Such revelations may come from a sophisticated programming technique, or by the use of some algebraic transformation?

PROBLEM FUNCTION*. Indicate some other functions which exhibit this type of behaviour. Do any of them have a practical application?

● Any investigations of the above problems may be sent to Mike Mudge, 22 Gors Fach, Pwll-Trap, St. Clears, Carmarthenshire SA33 4AQ, tel 01994 231121, to arrive by 1st February 1997. All material received will be judged using suitable subjective criteria and a prize will be awarded by Mike Mudge to the "best" entry arriving by the closing date. Each contribution should contain brief descriptions of the hardware and coding used, together with run times and a summary of the results obtained. (SAE for return entries, please.)

Report on Numbers Count -155- 'Pounding the beat', *PCW* March 1996

All aspects of this column generated interesting responses. The "Full Houses" or "Prime Decades" upto 100000 numbering 40 (less the two inadmissable 11,7,5,3 and 13,11,7,5) these consist of the 37 regular ones and the anomalous 2,3,5,7. Alan Cox obtained these with UBASIC and its NXTPRM(x) function (can any reader tell us how this function works?) in 48 seconds on a "slow 8086", while Hugh Spence used an AMD 585 running at 133MHz in Modula-2 ("the last Topspeed incarnation") to reproduce the results in 9.5 seconds.

Problem GS produced responses, including one from Tim Thorp who refers to Donald Knuth's *The Art of Programming* where the base three (being the integer nearest to e) is "in some sense" optimal for numerical operations.

This month's winner is David Price of 13 The Hall Close, Dunchurch, Rugby, Warwickshire CV22 6NP: his representation of numbers in various bases extended to complex bases and involved Fortran in double precision on a 486 PC. Altogether a commendable mixture of algebra/calculator arithmetic and programming.

•PCW Contributions Welcome

Mike Mudge welcomes correspondence from readers on any subject within the areas of number theory and computational mathematics, together with suggested subject areas and/or specific problems for future Numbers Count articles.

Prime candidate

Prime functions take centre stage and hailstones are a big hit, in this month's maths musings. With Mike Mudge.

The first area of investigation this month is due to Jonathon Ayres of Leeds, who writes as follows:

Highest prime function

I have recently been investigating what I have called the highest prime function — HPF(X), which is defined as the highest prime factor of x, so that HPF(7) = 7 and HPF(10) = 5.

1) Highest prime function sequence

HPFS(x_0,x_1) is defined so that

$Xn = HPF(X_{n-1}*X_{n-2}+1)$

So,

```
HPFS(1,2) = 3, 7, 11, 13, 3, 5,
2, 11, 23, 127, 487, 1237,
331,127,21019, 1811, 140983,
2239651, 10005473, ..
```

and

```
HPFS(3,2) = 7,. 5, 3, 2, 7, 5,
3, 2, and so on
```

(this has period 4).

Questions

1). Do all HPFSs eventually lead to recurring sequences? For example, HPFS(x,y) leads to a,b,c,d ... a,b,c,d, and so on. If not, do all the non-recurring HPFS go through all possible numbers? (The function

$HPFS(x_n) = HPF(x_{n-1}*x_{n-2}*... x_1 , x_0 +1)$, starting 2,3,7, 43, 139 ..

has been shown not to repeat, nor is it ever equal to

`5,11,13,17.)`

2). For recurring HPFS, what are the smallest numbers a,b so that HPFS(a,b) has period n, and are there any values of n so there are no HPFS(a,b) with period n?

3). How many different HPFS do numbers converge to? For instance, HPFS(2,3) and HPFS(2,11) converges to the same sequence?

4). What happens for related sequences such as:

a) Lowest prime factor sequence

`LPFS(3,7) = 2,3,1 .....`

b) Highest alliott divisor sequence (not including the number itself)

`HADS(3,5) =8, 1, 3, 2, 1,...`

c) HPFSm(a,b) = HPF(a*b+m)

d) HPFS(a,b,c) = HPF(a*b*c+1)

Highest Prime Hailstone Function

This is similar to the "Hailstone Function", which is defined as: if n is even, then n is divided by 2, else it is multiplied by 3 and one is added.

The highest prime hailstone function,

$HPHF_{(a,b)}\ (x_n) = HPF(a*x_{n-1}+b)$

```
HPHF(2,1) (1) = 3, 7, 5, 11, 23,
47, 19, 13, 3, 11, 23, 47, 19, 13,
...
```

has period 7, with the lowest value in the periodic sequence being 3 and the highest 47.

The table above shows the period, lowest and highest value for the recurring sequence which $HPHF_{(n\ ,\ 1)}$ (1) leads to.

Values for recurring sequence

N	Period	Lowest value	Highest value
2	7	3	47
3	5	2	17
4	7	5	71
5	3	2	11
6	18	13	13219
7	3	2	23
8	12	11	1097
9	5	13	97
10	6	43	15971
11	2	17	47

Questions

1). Do all $HPHF_{(a,1)}$ (1) lead to recurring sequences, and how does the period, lowest and highest value change for different a (for example, a being prime, then the period seems to be quite low)?

2). Do all $HPHF_{(a,b)}$ (x), for fixed a,b and variable x, lead to recurring sequences?

For fixed a and b, is there more than one recurring sequence? For example, $HPFH_{(9,1)}$(1) leads to a sequence with lowest value 13, highest 97 and length 5; and $HPF_{(9,1)}$(41) leads to a sequence with lowest value 37, highest 269 and length 7. If so, how many different recurring sequences?

For fixed a and b, what value of x takes the longest/shortest time to reach a repeating sequence, and what value of x reaches the highest values?

3). Do all HPHF lead to recurring sequences?

Any responses to these problems to be sent to: Mike Mudge, 22 Gors Fach, Pwll-Trap, Carmarthenshire SA33 4AQ (01994 231121), by 1st January 1997.

Spot the difference

Stephen Saxon, of Stockport, has suggested an interesting problem — it combines an area of mathematics predating computers "as we know them" by several centuries, with current programming techniques. The question is, how to fit a polynomial of the lowest possible degree to a set of equally spaced data points? An answer will be provided next month by The Calculus of Finite Differences or, as Stephen calls it, The Newtonian Difference Method.

•PCW Contributions Welcome

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Fraction **action**

Mike Mudge presents continued fractions — when are they periodic, and how long are the periods?

Definition: an expression of the form $a_0+1/(a_1+1/(a_2+1/(a_3\ldots$ is called a **regular**, or **simple**, continued fraction. Throughout this work a_r will denote positive integers. (a_0 may be zero.) The SIMPLE continued fraction numerically equal to any rational number (i.e. the quotient of two integers) must terminate. That is, have only a finite number of partial quotients a_r,; although such expressions have certain applications, including the design of gear trains, they have very limited appeal in computational or pure mathematics. For example, 105/38 = 2 + 1/(1 + 1/(3 + 1/(4 + 1/2))). To simplify this somewhat cumbersome notation, we write 105/38 = (2;1,3,4,2.).

Theorem A. Look at *Continued Fractions* by A. Ya Khinchin (Phoenix Science Series, The University of Chicago Press, 1964). The necessary and sufficient condition for a simple continued fraction to be finite is that it represents a rational number.

Theorem B. *loc.cit.* above. The necessary and sufficient condition for a simple continued fraction to be **periodic** is that it should represent a **quadratic irrational**. That is, a non-integer real root of a quadratic equation: $ax^2 + bx + c = o$ where a, b and c are integers, a not equal to zero.

● **Problem 1.** Write a simple computer program to generate the (**finite**) continued fraction corresponding to any given positive rational number, i.e. input p/q and output $(a_0;a_1;a_2;a_3,\ldots a_n.)$.

● **Problem 2.** Write a simple computer program to solve **exactly** any given quadratic equation with integer coefficients, i.e. input a, b & c as in $ax^2 + bx + c = 0$ and output the roots as P±SQRT(Q).

It is suggested that the reader now experiments with simple periodic continued fractions such as (0;1,1,1,...), also (2;3,4,3,4,3,4,...) to see the quadratic equation whose root they represent. Note in the first example, x = 0+1/(1+x), while in the second example,
x - 2 = (0;3,4,3,4,3,4,...) = y
say where
y = 1/(3+1/(4+y)).
Hence, the desired quadratic equations and exact values for x & y can be found.

The more complicated experiment is to start with a given quadratic equation and determine the continued fraction expansion of any positive real roots which it may have. Note: these **must** be periodic; the analysis may be beyond the mathematical experience of some readers, but its omission does not affect the continuity of the rest of this discussion. Now restrict the quadratic equation to the form, $x^2 - a = 0$, and focus on the root SQRT(a). In their paper *Some Periodic Continued Fractions with Long Periods* (*Mathematics of Computation* vol 44, number 170, April 1985 pp 523-532), CD Patterson and HC Williams used The University of Manitoba Sieve Unit (UMSU), "a machine similar to DH Lehmer's DLS-127", to investigate cases of long periodicity. Theoretically, they identified four classes of 'a' of interest: (1) a ≡ 3 (mod 8) 'a' prime; (2) a ≡ 7 (mod 8) 'a' prime; (3) a ≡ 6 (mod 8) 'a'/2 prime; and (4) a ≡ 1 (mod 8) 'a' prime. Denoting the period by p(a), typical results in each of these classes are:
a 2186009851 2763423391 2340752254 18901431649
p(a) 151838 170804 157036 433383

● **Problem 3.** Attempt to determine the period of the simple continued fraction expansion of SQRT(a) in such a manner that the investigation can be extended to the orders of integers indicated above. Verify that the period is bounded by:
$f(a) = a^{1/2}\log\log(a)$ if a ≡ 1(mod 8) and by
$f(a) = a^{1/2}\log\log(4a)$ otherwise.

● **Something different**
In March 1986, readers were invited to find integer solutions p,q,r,s,t for
5(p2+q2+r2+s2+t2)2 = 90pqrst + 7(p4+q4+r4+s4+t4).

An extensive investigation by *PCW* reader, Duncan Moore, generalised the 90 to 5n and led to the following questions:
(a) Are there any solutions with three of p,q,r,s,t sharing one factor and the other two sharing a different factor? If not, then the search for solutions with three only sharing a common factor could be significantly speeded up.
(b) Are there any solutions with n = 1 or with n = -1?

Any investigations of the above problems, together with answers (either complete or partial) to Duncan Moore's questions, should be sent direct to: Mike Mudge, 22 Gors Fach, Pwll-Trap, Carmarthenshire SA33 4AQ (tel 01994 231121), to arrive by 1st December. The author also welcomes comments on the subject areas chosen this month: namely, continued fraction theory and Diophantine equations. Details of recent results either published or unpublished in these areas would be particularly appreciated.

Interesting Powers of Ten

Hugo Steinhaus' problem (*PCW*, January) was of great interest. This produced a very interesting set of responses. Worthy of mention in the Interesting Powers of Ten, are Paul Leyland's conclusion that there are no less than 1063017, other than those quoted — the result of almost three hours' computing time on a DEC Alpha. Nigel Hodges used Microsoft C++ on his Packard Bell up to 2^{10000} in three seconds and then established some associated probabilities. Steinhaus, being simple to comprehend, yielded a great deal of results. However, the clear prize-winner is Richard M Tobin, 2 Flr, 53 Spottiswoode Street, Edinburgh, EH9 1DQ, who programmed a Sun Sparcstation 5/110 in C and summarised all of the Steinhaus cycles up to and including twenty fifth powers! This latter took eight days and revealed nine perfect digital invariants (including 1), the next one having 24 digits.

PCW Contributions Welcome

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Golomb rules, **OK**

Mike Mudge **deals with the concept of perfect and imperfect rulers.**

"Golomb rulers"
…with a pre-metric introduction
(Inspiration acknowledged from Malcolm E. Lines' Think of a Number *[1990. Adam Hilger. ISBN 0-85274-183-9] chapter 11, pp 101-105.)*

Consider a one-foot ruler having only inch markings: for the benefit of younger readers this represents a uniform scale of length, with 12 equal sub-divisions.

We see that there are 12 ways of measuring one unit of length, *viz.* 0-1, 1-2, 2-3,…11-12; also, seven ways of measuring six units of length, *viz.* 0-6, 1-7,…6-12; with four ways of measuring nine units of length 0-9, 1-10, 2-11, 3-12. Clearly,there is considerable redundancy in this instrument.

Starting with a "trivial prototype" ruler of unit length marked 0 & 1, this measures one possible distance in one possible way. However, a two-length ruler with three markings at 0, 1 & 2 has already introduced an inefficiency since it measures one unit in two different ways, *viz.* 0-1 & 1-2. However, if the marks are at 0, 1 & 3 we have a ruler measuring distances 1, 2 & 3 each one way only.

Pencil and paper study should convince the reader that it is not possible to construct a ruler which will achieve this for either 4 or 5. Marks at 0, 1, 4 & 6 generate such a ruler (called PERFECT) of length 6, since either of the distances 1, 2, 3, 4 & 5 can be measured in one way only. This idea is originally due to Professor Solomon Golomb of the university of Southern California; see later reference.

Now a ruler with five marks can measure ten distances, therefore if it were a PERFECT ruler it would be of length 10. Note that a ruler can be IMPERFECT in two distinct ways:
(a) there may be some distances which cannot be measured;
(b) there may be some distances which can be measured in two or more ways.

The "next best thing" to the non-existent perfect five-mark ruler might possibly be defined as one that contains each measurable distance only once, but which is unable to measure every possible distance up to the length of the ruler. Clearly not an adequate definition since a five-mark ruler, marked at 0, 4, 10, 27 & 101, measures distances of 4, 6, 10, 17, 23, 27, 74, 91, 97 & 101 each one way only. the challenge is to find the SHORTEST ruler which does not measure any one distance in more than one way.

The shortest five-mark ruler is of length 11; with mark positions at 0, 1, 4, 9, 11, i.e. one unit longer than the PERFECT TEN. The only length less than ten which it cannot measure is 6. In general, the shortest ruler with n-marks is called the "n-mark Golomb ruler" in honour of its inventor. Malcolm Lines lists all of the Golomb rulers known to him in *Fig 1.*

Fig 1 Golomb rulers

Number of marks	2	3	4	5	6	7…	13	14	15
Golomb length	1	3	6	11	17	25…	106	127	151

The fifteen-mark Golomb ruler has marks at 0, 6, 7, 15, 28, 40, 51, 75, 89, 92, 94, 121, 131, 147, 151.

Now to research! There exists a formula which yields the shortest length that a Golomb ruler with any particular length can possibly have. This yields the entries in row L (Lower bound) shown in *Fig 2.*

Fig 2 The entries in row L

Number of marks	16	17	18	19	20	21	22	23	24
Shortest known	179	199	216	246	283	333	358	372	425
L	154	177	201	227	254	283	314	346	380

Problem: Golomb
The following quotation is intended to inspire readers to investigate the problem of Golomb rulers: "In co-operation with a personal computer, it is quite likely that the enthusiast can improve on some of the 'shortest known' rulers in the above table, although a demonstration that the actual Golomb ruler has been located is probably beyond all but the most powerful of today's computers." (*M.E.L. 1990*).

Investigate the table at *Fig 2*, extending where possible and making a serious attempt to quantify the difference between any "shortest known" results, i.e. from a particular algorithm, and the actual length of the Golomb ruler. Further, how do these values differ from **L**?

Any investigation of the above problem, together with comments on the concept of Golomb rulers (which do have applications in both radio astronomy and satellite communications) may be sent to Mike Mudge, 22 Gors Fach, Pwll-Trap, Carmarthenshire SA33 4AQ (tel 01994 231121) to arrive by 1st November. All material received will be judged using suitable subjective criteria and a prize will be awarded, by Mike Mudge, to the "best " entry arriving by the closing date. Such contributions should contain brief descriptions of the hardware and coding used, together with run times and a summary of the results obtained. (SAE for entries to be returned, please.)

Some simply-posed problems for beginners

P1) Does there exist a positive integer n greater than 7 for which n! + 1 is the square of the integer? It is known that if n exists it must be greater than 1020. *M.Kraitchik (Paris, 1924)*.

P2) Obtain all solutions in integers of the equation $x3 - y^{2} = 18$. It has been proved that the number of solutions is finite but it is not known how many there are.

P3) Do there exist three rational numbers (i.e. fractions with integer numerators and integer denominators) whose sum and product are each equal to 1?

PCW Contributions Welcome

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Back in **sequence**

Descriptive Number Sequences, part two, presented by **Mike Mudge**.

Continuing the study of Numbers Count, June 1996: Recall the definition, due to Jonathan Ayres, of Leeds: $ds_n(m)$ where n is the index of the sequence and m the original number. Thus:

$ds_1(0) = 10$

because the original number consists of 1 zero; whilst

$ds_2(0) = 1011$

because $ds_1(0)$ consists of 1 zero and 1 one.

Problem A. Is there a way of deciding if a given initial number, x say, leads to a self-descriptive number (such as 1031223314) without calculating the whole descriptive sequence?

Empirical evidence suggests that as x increases, the likelihood of a sequence becoming self-descriptive decreases. Why is this?

Problem B. Is there any function which relates the chances of a number becoming self-descriptive with the magnitude of the number?

COMPLETELY DESCRIPTIVE SEQUENCES, Ds(n)

These are similar to descriptive sequences, but the next number in the sequence refers to all the digits zero to nine i.e. it does not omit the reference to non-occurring digits.

$Ds_1(0) = 10010203040506070809$,
$Ds_2(0) = 10021121314151617181 9$

This process converges to the amicable descriptive pair:

$Ds_6(0) = 10714213141516171819$,
$Ds_7(0) = 10812213241516271819$

Problem C. Do all numbers n lead to the above amicable descriptive pair?

WHAT HAPPENS IN DIFFERENT NUMBER BASES?

In binary, for example,

$ds_1(0) = 10$, $ds_2(0) = 1011$ whilst $ds_3(0) = 10111$

We have 11 ones since 3 is represented in binary as 11; subsequently

$ds_9(0) = ds_{10}(0) = \ldots = 1101001$

a self-descriptive number in binary, having three zeros and four ones.

Example: in base 6 there is an amicable descriptive pair consisting of

103142132415 & 104122232415

Problem D. Are there any number bases with period four or larger amicable descriptive sequences?

DESCRIPTIVE SEQUENCES OF ORDER GREATER THAN ONE

Here the digits are regarded in groups of order n which may be either CONSECUTIVE...TYPE I, or GROUPED..TYPE II?

In type 1 the number zero generates the following:

$dsc^2_1(0) = 0100$

which is then split into 01, 10 & 00

$dsc^2_2(0) = 010001010110$ and $dsc^2_3(0) = 0200040104100111 \ldots$

whilst in type II the number zero generates the following:

$dsg^2_1(0) = 0100$

(because order two uses two digits so 0 goes to 00 and 1 goes to 01)

$dsg^2_2(0) = 01000101$

i.e. one zero and one one.

$dsg^2_3(0) = 01000301$

i.e. one zero and three ones etc.

It is found that $dsg^2_{989}(0)$ and its amicable descriptive partner are each 395 digits long; whilst $dsc^2_{41}(0)$ having 395 digits also is part of an amicable descriptive pair.

Problem E. Analyse completely the behaviour of type I & type II descriptive sequences of order two, consider the extension to higher orders. (Remember the order is the size of the subsets of digits being counted.)

TWO-DIMENSIONAL DESCRIPTIVE SEQUENCES, $^2DS(n)$

Descriptive sequences can be generalised from one-dimensional "lines" of numbers to two-dimensional "planes" of numbers. One way to do this consistently is to define the columns, m, of $^2DS_{n+1}(x)$ to be equal to ds_1(row m) as is illustrated by the following example:

$^2DS_1(0) = 1_0$ (because $ds_1(0) = 10$) thus

$^2DS_2(0) = 11_{10}$ (because $ds_1(1) = 11$ and $ds_1(0) = 10$) repeated iteration leads to:

$$^2DS_5(0) = \begin{matrix} 4 & 1 & 2 & 1 \\ 1 & 0 & 1 & 1 \\ & & 3 & 1 \\ & & 1 & 2 \end{matrix}$$

Jonathan Ayres has failed to discover any two-dimensional self-descriptive or amicable descriptive sequences, having investigated up to $^2DS_{1000}(0)$ and beyond. However, he observes that $^2DS(\ )$ must lead to a recurrent sequence because it is fixed in size. The biggest $^2DS(\ )$ gets in size is 19 rows by 19 columns and since each position contains a digit 0..9 then there are 10^{361} possible values for $^2DS(\ )$, but half the possible positions on average are spaces and half the remaining numbers are fixed because they are the digit number, so maximum period is about 10^{81}.

Problem F. Investigate two-dimensional descriptive sequences with a view to finding self-descriptive or amicable descriptive patterns.

Any investigations of the above problems may be sent to Mike Mudge, 22 Gors Fach, Pwll-Trap, Carmarthenshire SA33 4AQ, tel 01994 231121, to arrive by 1st October 1996. All material received will be judged using suitable subjective criteria and a prize will be awarded by Mike Mudge, to the "best" entry arriving by the closing date.

Feedback: November 1995 — Squambling

This proved to be a remarkably popular topic. Why? Gareth Suggett established the answer to the original *Sunday Times* problem as 46, for which one iteration of the squambling function gives 232, and a second gives 47. He found all of George Sassoon's loops and lists a 105-step loop, 40372656... whose smallest entry is 5 and largest entry is 43055027. He found modsquam less interesting, being monotonic decreasing and ending (always) with 1. Nigel Hodges proved that squambling sequences and their various generalisations cannot diverge. However, this month the prize is awarded to G.D. Williams of 18 Mawnog Fach, Bala, Gwynedd LL23 7YY, who displays an awareness of the problems of integer overflow even when programming in Turbo C++. Mr Williams has noted the basic difference in the behaviour (as he perceives it) between sqm( ) & modsqm( ).

There is scope for further investigation of this function, in particular when the number base is different from ten.

PCW Contributions Welcome

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Sequence of events

Descriptive Number Sequences, presented by **Mike Mudge**.

THIS APPARENTLY NEW AND certainly fascinating topic has been suggested by Jonathan Ayres of Leeds. The sequences are denoted by $ds_n(m)$ where n is the index of the sequence and m is the original number. There is a simplified version of the GLEICHNISZHLEN-RIEHE sequence with the property that the next number in the sequence describes the number of each digit in the previous number.

So, taking the case of ds(0) in *Fig 1*.

Fig 1

Gleichniszhlen-Riehe sequence for ds(0)

$ds_1(0)$ = 10 (1 zero in previous number, 1 is the digit number and 0 is the occurrence number)
$ds_2(0)$ = 1011 (1 zero and 1 one in previous number)
$ds_3(0)$ = 1031 (1 zero and 3 ones in previous number)
$ds_4(0)$ = 102113 (NB. Because there are no twos in previous number the 0 twos are not listed, so $ds_4(0)$=102113 instead of 10210213. (I will deal with this case later.)
$ds_5(0)$ = 10311213
$ds_6(0)$ = 10411223
$ds_7(0)$ = 1031221314
$ds_8(0)$ = 1041222314
$ds_9(0)$ = 1031321324
$ds_{10}(0)$ = 1031223314
$ds_{11}(0)$ = 1031223314, and so on

After ds11(0) all further numbers in the sequence are equal to 1031223314. This is a *self-descriptive number*, i.e. it describes itself. For example, 1031223314 is composed of 1 zero, 3 ones, 2 twos, 3 threes and 1 four = 1031223314.

From my investigations the self-descriptive numbers are:

22
10311233
21322314, 21322315, 21322316, 21322317, 21322318, 21322319
31123314, 31123315, 31123316, 31123317, 31123318, 31123319 *
1031223314, 1031223315, 1031223316, 1031223317, 1031223318, 1031223319 *
3122331415, 3122331416, 3122331417, 3122331418, 3122331419 *

The asterisked lines are related families because the final 1n is not important as n is not involved with the rest of the number.

This leads to my first question:

(1) Is this an exhaustive list of self-descriptive numbers?

Sequences which do not lead to self-descriptive numbers instead lead to *amicable descriptive sequences*. For example, in the case of ds(4), see *Fig 2*.

Fig 2

Amicable descriptive sequences for ds(40)

$ds_1(40)$ = 1014
$ds_2(40)$ = 103114
$ds_3(40)$ = 10311214
$ds_4(40)$ = 1041121314
$ds_5(40)$ = 1051121324
$ds_6(40)$ = 104122131415
$ds_7(40)$ = 105122132415
$ds_8(40)$ = 104132131425
$ds_9(40)$ = 104122232415
$ds_{10}(40)$ = 103142132415
$ds_{11}(40)$ = 104122232415
$ds_{12}(40)$ = 103142132415, and so on

$ds_{10}(40) = ds_{12}(40)$, so this sequence has entered into a *recurring sequence* of numbers with a *period* of 2, because $ds_n(40) = ds_{n+2}(40)$, $n \Rightarrow 10$.
104122232415 and 103142132415
are known as an amicable descriptive pair of numbers, because
ds_1(104122232415) = 103142132415
and ds_1(103142132415) = 104122232415

There are also amicable descriptive triplets such as
10414213142516 - 10512213341516 - 10412223142516
which have a period of 3. The amicable descriptive sequences are shown in *Fig 3*.

From this I define ds(x) to be the lowest recurring value of dsn(x), so that ds(x) is either a self-descriptive number or ds(x) is the lowest member of an amicable sequence, i.e. ds(0) = 1031223314.

Fig 3

Amicable descriptive sequences for triplets

Period 2

103142132415 - 104122232415

314213241516 - 412223241516,
314213241517 - 412223241517,
314213241518 - 412223241518,
314213241519 - 412223241519

41421314251617 - 51221334151617,
41421314251618 - 51221334151618,
41421314251619 - 51221334151619

1051421314152617 - 1061221324251617,
1051421314152618 - 1061221324251618,
1051421314152619 - 1061221324251619

5142131415261718 - 6122132425161718,
5142131415261719 - 6122132425161719
106142131415162718 - 107122132415261718,
106142131415162719 - 107122132415261719,
614213141516271819 - 712213241526171819,
10714213141516172819 - 10812213241516271819

Period 3

10414213142516 - 10512213341516 - 10512223142516
10414213142517 - 10512213341517 - 10512223142517
10414213142518 - 10512213341518 - 10512223142518
10414213142519 - 10512213341519 - 10512223142519
41421314251617 - 51221334151617 - 51222314251617
41421314251618 - 51221334151618 - 51222314251619
41421314251619 - 51221334151619 - 51222314251619

(2) Is this a complete list of the amicable descriptive sequences?

(3) Are there any of higher period?

Any investigations of these three questions may be sent to Mike Mudge, 22 Gors Fach, Pwll-Trap, St. Clears, Carmarthenshire SA334 AQ, tel 01994 231121, to arrive by 1st September 1996. All material received will be judged using suitable subjective criteria and a prize will be awarded to the "best" response arriving by the closing date.

PCW Contributions welcome

Mike Mudge welcomes readers' correspondence on any subject within the areas of number theory and computational mathematics, together with suggested subject areas and/or specific problems for future Numbers Count articles.