# Mystery maths

Mike Mudge consults the book Unsolved Problems for his teasers this month, including distances and square numbers and circles. Plus, roll up for a number theory conference.

This is inspired by *Unsolved Problems in Geometry* by HT Croft, KJ Falconer and RK Guy, ISBN-0-387-97506-3, Springer Verlag 1991.

Question A. What is the maximum diameter of n equal circles that can be packed into a unit square? How should n points be arranged in a unit square so the minimum distance between them is greatest? These problems are equivalent: if a collection of points in a unit square are at a distance of at least d from each other, the points can serve as the centres of a collection of circles of diameter d that will pack into a square of side 1 + d.

Consider the second version of this problem and denote by $d_n$ the greatest minimum distance between n points in a unit square. Exact results are known for n less than or equal to 9; also for n = 14, 16, 25 and 36. For n lying between 2 and 5 these are "easy" to obtain. Graham has established the result for n = 6, the results for n = 7, 8 & 9 are due to Schaer and Meir, those for 14, 16 & 25 & 36 are attributed to Wengerodt & Kirchner. Examples of both exact results and conjectual bounds are given in Fig 1 *(above)*.

Fig 1

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| n | 2 | 3 | 4 | 7 | 10* | 13* | 17* | 26* |
| $d_n$ | $2^{1/2}$ | $6^{1/2} - 2^{1/2}$ | 1 | $2(2-3^{1/2})$ | 0.421 | 0.366 | 0.306 | 0.239 |

*Indicates a conjecture as far as the writer is aware.

Up to which square number is the square lattice packing the best? Certainly for up to 36. Wengerodt has found a packing of 64 points that is denser than the square lattice packing, but he conjectures that for 49, the square lattice packing is best.

Are there any values of n such that $d_n = d_{n+1}$? The problem can be asked for packing an equilateral triangle. Oler has shown that if n is a triangular number, of the form m(m+1)/2, the obvious configuration is the extremal one. The natural question is, can one do better if n is 1 less than a triangular number? Note that n spheres have been packed into a cube and certain other polyhedra, but even for a cube, exact results are only known for n less than eleven. A great deal of work is stilll to be done in this area!

Question B. **Spreading points in a circle**

The analog of the previous problem for the circle can be posed in a few equivalent ways:

1. What is the maximum radius of a disk, n copies of which can be packed into a circle of radius 1?
2. What is the radius of the smallest circle into which n unit disks can be packed?
3. What is the radius of the smallest circle containing n points, no pair of these points being a distance of less than 1 apart?
4. How large can the least distance between a pair chosen from n points in the circle be?

The last formulation yields the result that for n between two and six...inclusive...the least distance is 2*sin(PI/n) and for n = 7,8 & 9 the least distance is 2*sin(PI/(n-1)) with the obvious configurations. This is straightforward for n less than eight and was proved by Pirl for n = 8 & 9. He also solved the case of n = 10 and conjectured the values for n less than 20.

Suggested approaches to the problems include randomly generated points with analysis of large samples to estimate bounds, and the use of computer graphics to draw the optimum configurations, in the cases where these are known, and to examine and conjecture solutions for higher n.

Any investigations of the above problems may be sent to Mike Mudge at 22 Gors Fach, Pwll-Trap, St Clears, Carmarthenshire SA33 4AQ, tel 01994 231121, to arrive by 1st September 1997. All material received will be judged using suitable subjective criteria and a prize will be awarded by *PCW* to the best entry arriving by the closing date (SAE for return of entries, please). Each contribution should contain details of run times and a summary of the results obtained.

Comments on the topics would be appreciated. The topics included here range from tiling and dissection through packing and covering to include nets of polyhedra and lattice point problems.

*Details of the winner of November 1996 Numbers Count will appear next month.*

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**PCW Contact**

Mike Mudge welcomes correspondence from readers on any subject within the areas of number theory and computational maths, together with suggested subject areas or specific problems for future articles. Email numbers@pcw.co.uk.