Festive fractions

Mike Mudge gets stuck into a feast of fractions for Christmas, and appeals for help on behalf of readers.

This month, I have a number of appeals to make:

1. Nigel Backhouse of the Division of Applied Mathematics at the University of Liverpool wonders if any readers would be interested in joining the Great Internet Mersenne Prime Search? He tells me that George Woltman is asking for volunteers with Pentiums and 486s and access to the web, to join a team searching for new, large Mersenne Primes. He provides free software and full instructions on how to use it. This can be downloaded from our world.compuserve.com/home pages/just for fun/prime.htm.

2. Alan Cox has been studying the paper by Artur Ekert and Richard Jozsa in *Reviews of Modern Physics*, July '96, pp1-28, entitled "Quantum Computation and Shor's Facturing Algorithm". In common with your columnist, he finds it difficult to understand but realises the importance of the subject area. Is anyone willing to produce a simple guide to the concepts involved? *PCW* may consider such material for publication, as it would be to the benefit of many readers and relate to the very frontiers of computational theory.

3. Caryl Takvorian is anxious to access a paper on the subject of NP-complete and intractable problems. Is any reader able to supply a suitable reference or offer such a paper to *PCW* and/or Caryl directly?

FRACTRAN: a simple universal programming language for arithmetic

Fractran: Due to JH Conway, Open Problems Commun. Comput, pp4-26, published in 1986.

To play the Fraction Game corresponding to a given list: $f_1, f_2, \ldots, f_k$ of fractions and a starting integer N, we repeatedly multiply the integer which is defined at any stage (initially N) by the earliest f_i in the list for which the answer remains an integer. Whenever there is no such f_i the game stops.

Formally: the sequence (N_n) is defined by $N_O = N$ (given) while $N_{n+1} = f_i N_n$ where i between 1 & k inclusive is the least i for which $f_i N_n$ is integral, providing such an i exists.

Experiment 1 Consider the list of fractions $^{17}/_{91}$ $^{78}/_{85}$ $^{19}/_{51}$ $^{23}/_{38}$ $^{29}/_{33}$ $^{77}/_{29}$ $^{95}/_{23}$ $^{77}/_{19}$ $^{1}/_{17}$ $^{11}/_{13}$ $^{13}/_{11}$ $^{15}/_{2}$ $^{1}/_{7}$ $^{55}/_{1}$: these define PRIMEGAME (after Conway). Choosing N = 2, the other powers of 2 which are generated are those whose indices are the Prime Numbers in ascending order.

Experiment 2 Consider the list of fractions $^{365}/_{46}$ $^{29}/_{161}$ $^{79}/_{575}$ $^{679}/_{451}$ $^{3159}/_{413}$ $^{83}/_{407}$ $^{473}/_{371}$ $^{638}/_{355}$ $^{434}/_{335}$ $^{89}/_{235}$ $^{17}/_{209}$ $^{79}/_{122}$ $^{31}/_{183}$ $^{41}/_{115}$ $^{517}/_{89}$ $^{111}/_{83}$ $^{305}/_{79}$ $^{23}/_{73}$ $^{73}/_{71}$ $^{61}/_{67}$ $^{37}/_{61}$ $^{19}/_{59}$ $^{89}/_{57}$ $^{41}/_{53}$ $^{833}/_{47}$ $^{53}/_{43}$ $^{86}/_{41}$ $^{13}/_{38}$ $^{23}/_{37}$ $^{67}/_{31}$ $^{71}/_{29}$ $^{83}/_{19}$ $^{475}/_{17}$ $^{59}/_{13}$ $^{41}/_{291}$ $^{1}/_{7}$ $^{1}/_{11}$ $^{1}/_{1024}$ $^{1}/_{97}$ $^{89}/_{1}$: these define PIGAME (after Conway). Choosing N as 2^n the next power of 2 to appear is $2^{p(n)}$ where p(n) is the n^{th} digit after the point in the decimal expansion of PI.

Experiment 3 Consider the list of fractions $^{583}/_{559}$ $^{629}/_{551}$ $^{437}/_{527}$ $^{82}/_{517}$ $^{615}/_{329}$ $^{371}/_{129}$ $^{1}/_{115}$ $^{53}/_{86}$ $^{43}/_{53}$ $^{23}/_{47}$ $^{341}/_{46}$ $^{41}/_{43}$ $^{47}/_{41}$ $^{29}/_{37}$ $^{37}/_{31}$ $^{299}/_{29}$ $^{47}/_{23}$ $^{161}/_{15}$ $^{527}/_{19}$ $^{159}/_{7}$ $^{1}/_{17}$ $^{1}/_{13}$ $^{1}/_{3}$: these define POLYGAME (after Conway). Define $f_c(n)$ +m if, when Polygame is started at $c2^{2^n}$, then it stops at 2^{2^m}, otherwise leave $f_c(n)$ undefined. Then every computable function appears among f_0, f_1, f_2… The number c is called the Catalogue Number and is "easily computed for some quite interesting functions". Conway gives f_c for any c whose largest odd divisor is less than 2^{10}.

Problem

Understand and implement FRACTRAN in the form of the first two experiments. Follow this with an initial investigation of Experiment 3… and comment upon this approach to computable functions.

■ Send any implementation of the above algorithms to Mike Mudge, by 1st April, 1997. All material received will be judged using suitable subjective criteria and a prize will be awarded for the best entry (SAE for return of entries, please).

■ Responses to the three appeals should also be sent to Mike Mudge (for forwarding). George Woltman can be contacted directly as indicated above.

Report on Numbers Count May '96

Nigel Hodges examined "Problem MM" and used x = m/n, y = a/b (in their lowest terms) to distinguish two cases p does/does not divide m: obtaining solutions for p = 5 involving integers of 15 & 16 digits for m&n and 22, 23, 24 digits for a and b. Note that A. Bremner and J. Cassels, *Mathematics of Computation*, vol. 42, no.165, Jan 1984, pp 257-264, cite "*a most startling generator of all solutions for p = 877 where 42 & 40 digit integers arise as m & n whilst a & b have 63 & 60 digits respectively*". However, the prizewinner this month is Patrick Moss, of 26 Hillside, Grays RM17 5SX. His submission, "Rational Points on a Cubic Curve", includes an arithmetic/algebraic section followed by a section dealing with geometrical arguments, and finally, a set of special cases and generalisations. The computational aspects were programmed in C++ on a Gateway 2000 P5-120, prompting Patrick to ask if any reader has access to some decent code or knows of a not-too-expensive piece of software for handling large integer-length arithmetic? He used Microsoft Excel to draw the graphs but wonders whether other software could have done the job?

Details of this work on request to Patrick. A number of his results were subsequently confirmed in *The Arithmetic of Elliptic Curves* by JH Silvermann.

PCW Contacts

Contributions welcome: Mike Mudge welcomes correspondence from readers on any subject within the areas of number theory and computational mathematics, together with suggested subject areas and/or specific problems for future *Numbers Count* articles. Write to him at **22 Gors Fach, Pwll-Trap, St Clears SA33 4AQ, or phone 01994 231121**.