Power points

Mike Mudge faces a stiff challenge in proving a solution, and this leads him to considering a number of related problems concerned with the power sums of separate digits.

I was asked (by Cyprian Stockford) for a proof that the only solution to

$1^2 + 2^2 + … + n^2 = N^2$

is n = 24 when N = 70, *viz.* positive integer solution of

$n(n+1)(2n+1) = 6N$

is unique as asserted in *The Penguin Book of Curious and Interesting Numbers* (David Wells, 1987) and elsewhere. Being unable to provide such a proof (can any readers help?) my attention was caught by a number of notionally related problems involving the power sums of the separate digits or the partitions of a given positive integer.

■ 1: 1201 seems to be the smallest prime number which can be represented by the expression x^2+ny^2 for all values of n from 1 to 10. Is this true? What other prime numbers can be so represented, and what happens if the range of values of n is increased to 1 to M for an arbitrary M?

■ 2: It is clear that $1233 = 12^2+33^2$ while $8833 = 88^2+33^2$. Under what circumstances is a given integer equal to the sums of the squares of its partitions into pairs? How does this result extend to the cases of higher powers (i.e. cubes) and also to the cases of partitions into ordered triples, 4-tuples, etc? Does this lead to a sensible problem in number bases other than 10?

■ 3: $3435 = 3^3+4^4+3^3+5^5$ while it is said that (Wells, p.190) 438579088 is the only other number exhibiting this behaviour when powers of a single digit are considered. Can this result be generalised to pairs, i.e. $abcdef… = (ab)^{ab} + (cd)^{cd} + …$ or even to triples, etc? What happens in other number bases?

■ 4: By inspection, $175 = 1^1 + 7^2 + 5^3$; when, in general, does

$a^1_1 + a^2_2 + a^3_3 + … + a^n_n = a_1a_2…a_n$

where the right-hand side is understood to mean the integer so written in any number base? It is more natural to reverse the powers and even to start at zero, thus requiring

$b^0_0 + b^1_1 + b^2_2 + … b^n_n = b_nb_{n-1} … b_2b_1b_0$

The Subfactorial Function is defined as

!N = N! (1 - 1/1! + 1/2! - 1/3! + 1/4! … (-1)N/N!)

where

N! = 1.2.3…N e.g. !5 = 5! (1- 1/1! + 1/2! - 1/3! + 1/4! - 1/5!) = 44

while !7 = 1854. It is stated that 148349 is the only number equal to the sum of the subfactorials of its digits.

■ 5: Prove this result and attempt to generalise it to other number bases. Try replacing subfactorial by factorial and/or replacing sum by product. Comment on the function obtained from the subfactorial function by introducing only positive signs into the definition.

■ 6: Regarding the individual digits of an integer: is it possible to get a prime number from any given number by changing one of its digits? The answer is “No”. The smallest integer for which this is not possible is 200. Is it possible to get a prime number from any given integer by changing two of its digits? If not, what is the smallest number for which this is not possible?

Investigations of the above problems should be sent to Mike Mudge, 22 Gors Fach, Pwll-Trap, St Clears, SA33 4AQ, by 1st June 1997. All material will be judged using suitable subjective criteria and a prize will be awarded to the best entry arriving by the closing date (SAE for return of entries).

Golomb Rules, OK (*PCW*, Aug ’96)

This problem produced a large and varied response. In the problem PI seeking a solution greater than 7 to $n! + 1 = N^2$, Alan Cox extended Kraitchik’s lower bound from 1020 to 2500 using MAPLE V release 4 on a Dell 486D DX33 with 8Mb RAM and about 250Mb hard disk, in about six hours.

Problem P2 is solved completely. Dr John Cohen gave the reference to *Finkelstein & London* in *J. Number Th. 2* (1970), pp 310-321, together with references to work on $y^2 + k = x^3$ for a large range of k by Josef Gebel. Nigel Backhouse obtained a list of Golomb Rulers up to order 15, the final length being 151 with an example (0, 4, 20, 30, 57, 59, 62, 76, 100, 111, 123, 136, 144, 145, 151).

Gareth Suggett indicates that a group from Duke University have obtained optimum rulers up to 19 marks (*New Algorithms for Golomb Rulers Derivation and Proof of the 19 Mark Ruler,* Dollas, Rankin & McCracken, Nov ’95). Gareth speculated on the metric result for measuring all distances in centimetres from 1 to 100 on a metre rule. He refers to The *Dipole* column in *The IEE News* some years ago with the best known solution as 15 marks at 1, 2, 8, 14, 25, 36, 47, 58, 69, 80, 85, 90, 95, 98, 99. Is this minimal and/or unique?

Our prizewinner is RF Trindall, of Cambridge, for his extension to circular Golomb Rulers with n(n-1) + 1 points spaced round a circle uniformly and n of them marked to measure every distance from 1 to n(n-1). This was accompanied by analysis of P2 and P3 and some (accepted) criticism of their difficulty… sorry, readers!

PCW Contact

Mike Mudge welcomes correspondence from readers on any subject within the areas of number theory and computational maths, together with suggested subject areas or specific problems for future articles. Email numbers@pcw.vnu.co.uk