What Math Should We Teach When We Teach Math With CAS?

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Introduction

„Youth is the wealth of a nation“ says Sheikh Zayed Bin Sultan Al Nahyan, president of the United Arab Emirates. This is a remarkable statement for the leader of a country which is famous for owing their wealth to their mineral resources. But it goes without saying that mineral resources are limited – while „human resources“ are unlimited. The method of producing and increasing human resources is *education*.

Let's hear/read what one of the brightest minds of history said about education. The following is a quote from a speech Albert Einstein gave to educators in 1936: *"Sometimes one sees in the school simply the instrument for transferring a certain maximum quantity of knowledge to the growing generation. But that is not right. ... Thus the wit was not wrong who defined education in this way: 'Education is that which remains, if one has forgotten everything he learned in school.'"* [Einstein 1956].

Albert Einstein's quote could not be any more topical. In mathematics education, computer algebra systems (CAS) have sparked off a worldwide discussion about the value and reasonableness of what we teach in a traditional mathematics class. CAS point us to the fact that we focus too much on the teaching of the craftsmanship of performing operations, i.e. skills which a computer can so easily replace. Hence CAS make us think about WHAT we teach.

Here is what another famous person, William Shakespear, said: *"Nothing is either good or bad – only thinking makes it so."* In the context of CAS for teaching and learning this translates into: *"CAS are neither good nor bad teaching tools – only using them makes them so."* There are thousands of ways of using CAS for teaching – good ones and bad ones. Bad or, better, improper approaches often come from technology freaks among the teachers, who use the CAS just because it exists and because it is possible to use it in a certain way. But CAS should never drive the math we teach – our math (teaching goals) should drive the (use of) CAS! I fully agree with Helmut Heugl, the director of the Austrian Derive and TI-89/92 Projects (which involved almost 6,000 students), who said: *"If it is not pedagogically justified to use CAS, it is pedagogically justified not to use CAS."* So: CAS also make us think about HOW we teach.

A good reflection on the HOW is [Kutzler 2000]. In this article we focus on the WHAT.

What Mathematics Should We Teach?

An answer to this requires an answer to the question "What is mathematics?". There are many answers to this, following are two which complement each other in a perfect way:

1) Mathematics is the principal language of science and technology. As such it is the key for understanding and shaping the world around us.
2) Mathematics is the principal means of educating the human mind (quote C. F. Gauss). As such it is the key for mental wellness and health, it is an important contribution to a good all-round education, and it is also an indispensable prerequisite for (1).

Both aspects reflect important and worthwhile facets of mathematics, hence our mathematics teaching should support both. In the context of mathematics education I suggest to refer to the first aspect as "problem solving training" and to the second as "mental training". This Two-Quality-Model, which I abbreviate with the simple equation

School Mathematics = Mental Training + Problem Solving Training

is explained in more detail in this paper. It helps during teaching with or without technology, it helps to develop „technology-save" curricula, and it helps to plan the proper use of technology, in particular CAS, in the classroom.

Mental Training

I borrow an analogy from physical activities: Most people in the industrialized nations don't use their muscles any more to earn a living. But, unused muscles degenerate. Hence we do (physical) sports such as jogging and playing tennis or soccer so that we stay physically fit and healthy.

Due to a massive increase of (computer) automation over the past years, many of our intellectual skills are in jeopardy. In the past we needed to memorize phone numbers – today we use memory keys or even voice-dialing. In the past we had to memorize how to program a video recorder – today we swipe a bar code reader over the TV program. It goes without saying that all this makes life so much more comfortable, BUT it leads us to losing what I call *mental wellness and fitness*. Many school teachers complain about students's lack of concentration and their weak memories. These are typical symptoms of a diminishing mental wellness. If you haven't use your legs for 100 days, you will not be able to walk any more. If you haven't used your brain for 100 days, you will not be able to understand the evening news. It is not that bad yet, but we are heading there ...

Physicians use definitions of what a healthy person must be able to do physically. After a heart attack, for example, a patient must learn to walk a certain distance and to climb a certain number stairs before being released to home care. We need something similar

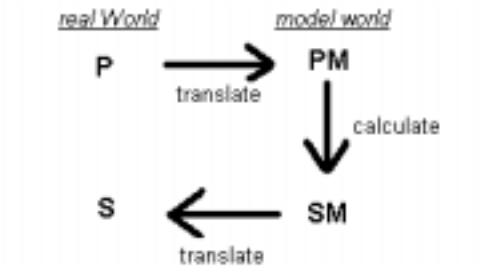
for our mental capabilities, i.e. a definition of what a person who has a healthy mind must be able to do in terms of, for example, memorization and mental calculation. A provocative attempt to give such a definition for mathematics teaching is [Herget etal 2000].

We introduced *sports* into our schools in order to fight a further deterioration of physical fitness. We need to take similar steps regarding our mental capabilities; we need a subject *mental sports* (or *mental training*) in our schools. I believe that this subject should be an integral part of mathematics teaching.

Another important aspect of mental training is that it is an indispensable preparation for problem solving. The one who knows something about linear equations and how to solve them will be the better "button-pusher" and problem solver – simply because (s)he knows better what (s)he is doing when using a CAS.

Problem solving

In mathematics I consider problem solving to be the ability to use mathematical tools for solving real world technical problems. Characteristic of problem solving are the three steps shown in Figure 1.



Today, problem solving is treated at school only half-heartedly. The main emphasis is put on the second step, calculation, and its execution with paper and pencil. Let's look at optimization problems. Typically we treat them in a manner so that we can do, may be, three optimization problems in an hour using 80% of the time for (hand) calculations so that only 20% remain for modeling and translating. Hence, most problem solving exercises turn into exercises for practicing the required calculation skills. And this we call "problem solving training"?! Since translations from the real world into mathematics and vice versa rarely are taught explicitly, it is understandable that a majority of students don't develop this ability. Hence, they are afraid of exercises requiring such translations. With the (extensive) use of powerful technology such as CAS for the calculation step, we can dedicate a lot more time to teach the choice of models and how to translate problems and results. We may be able to treat ten or more optimization problems in an hour spending 80% of the time on modeling

and translation and only 20% on calculations. This would be the proper "problem solving training"!

Mathematics = Mental Training + Problem Solving Training

Curricula should aim at educating students in the disciplines of *mental training* and *problem solving training*. The goals of *mental training* are the building of a healthy and fit mind and a preparation for problem solving. It goes without saying that at most technology should be a training tool ("mathematical home trainer", see [Kutzler 2000] for an elaboration of this) and must not be used when testing mental abilities. The goal of *problem solving training* is the ability to use the appropriate tools for solving problems. The use of technology should be not only welcomed but recommended and supported here.

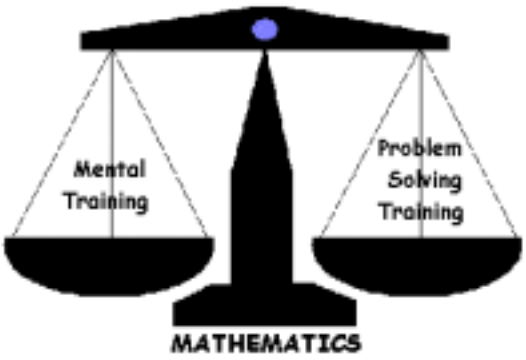
Employing an analogy, problem solving compares with a moving problem such as "tomorrow noon I need to be in London". It is unimportant what transportation means I use to get there, it only matters that I be there at the given time. Mental training is comparable to a jogger who runs along a track in order to gain physical fitness. Only the jogging counts, it is unimportant where the track is or where it leads to. Transferred to mathematics this means the following: In problem solving only the appropriate result counts, it is irrelevant how the calculations were performed. In intellectual sports only performing the calculation counts, while the meaning of the result is unimportant.

It goes without saying that most concrete activities in mathematics teaching possess both qualities in varying proportions. (This is similar to a widely known approach used by Eastern philosophies which teach us that everything in the cosmos possesses the elementary qualities Yin and Yang in varying proportions.)

The usefulness of this model becomes obvious already with the following: When talking to teachers about the use of technology, in particular CAS, we encounter two groups. One group is made up of teachers who prefer to use the most powerful tools, such as CAS, in the belief that students should *"focus on solving problems rather than spoil their time on tedious hand calculations"*. The other group consists of the teachers who oppose the use of these systems, because the scientific calculators already *"made our students lose most of their manual calculation skills"* and the use of even more powerful tools may cause students to have even more deficiencies. What looks like a non-resolvable situation at first glance can well be understood by our model, within which each group represents an extreme position. For the first group "school mathematics = problem solving training + ε", i.e. the mental training is of minor or no importance for them. For the second group "school mathematics = mental training + ε", i.e. mental training is the central goal for them while problem solving plays only a subordinate role. This is easily resolved with the following compromise which satisfies the demands of both groups.

Balance

What mathematics should we teach? The magic word is *balance*. Good mathematics teaching includes both qualities in a well-balanced manner, see Figure 2.



The Solution: Two-Tier Exams

Since assessment is an important pedagogical instrument, it must reflect this balance too. A logical solution is a *two-tier exam*:

- When assessing mental fitness, no tools are allowed. This includes even a simple four-function calculator.
- When assessing problem solving capabilities, all tools are allowed (better: recommended). This includes in particular CAS.

If the split is not manageable within a single exam, one should assess the two disciplines at different times. I draw a parallel with ice skating: Mental training compares with the compulsory exercise, in which the athlete demonstrates a mastery of the basic required techniques. Problem solving compares with the voluntary exercise (freestyle), in which the athlete demonstrates the ability to combine the basic techniques into a choreographed presentation. The total score depends on the scores of both the compulsory and the voluntary part.

Concluding remarks

Mathematics education is a key for producing a maximum of human resources. With the proper mathematics education students will

- develop their mental talents to an optimum
- learn to understand and shape the world around us.

The proper use of technology plays a crucial role in getting the most out of mathematics education.

Since exams determine a great deal what we teach, exams need to incorporate CAS. Two-tiered exams would be a well-balanced compromise meeting both the desires of technology supporters and the reservations of those who are concerned about the use of technology in the classroom. Hence, they are a way to let technology in.

In the end technology should play a secondary role in both disciplines. In mental training the goal is a performance with a minimum of tools. In problem solving the goal is to learn those skills and abilities that are needed for problem solving and are independent of technology. A good mathematics education will use CAS like language education uses dictionaries and phrase books: as (perfectly) natural tools which are used all now and then.

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