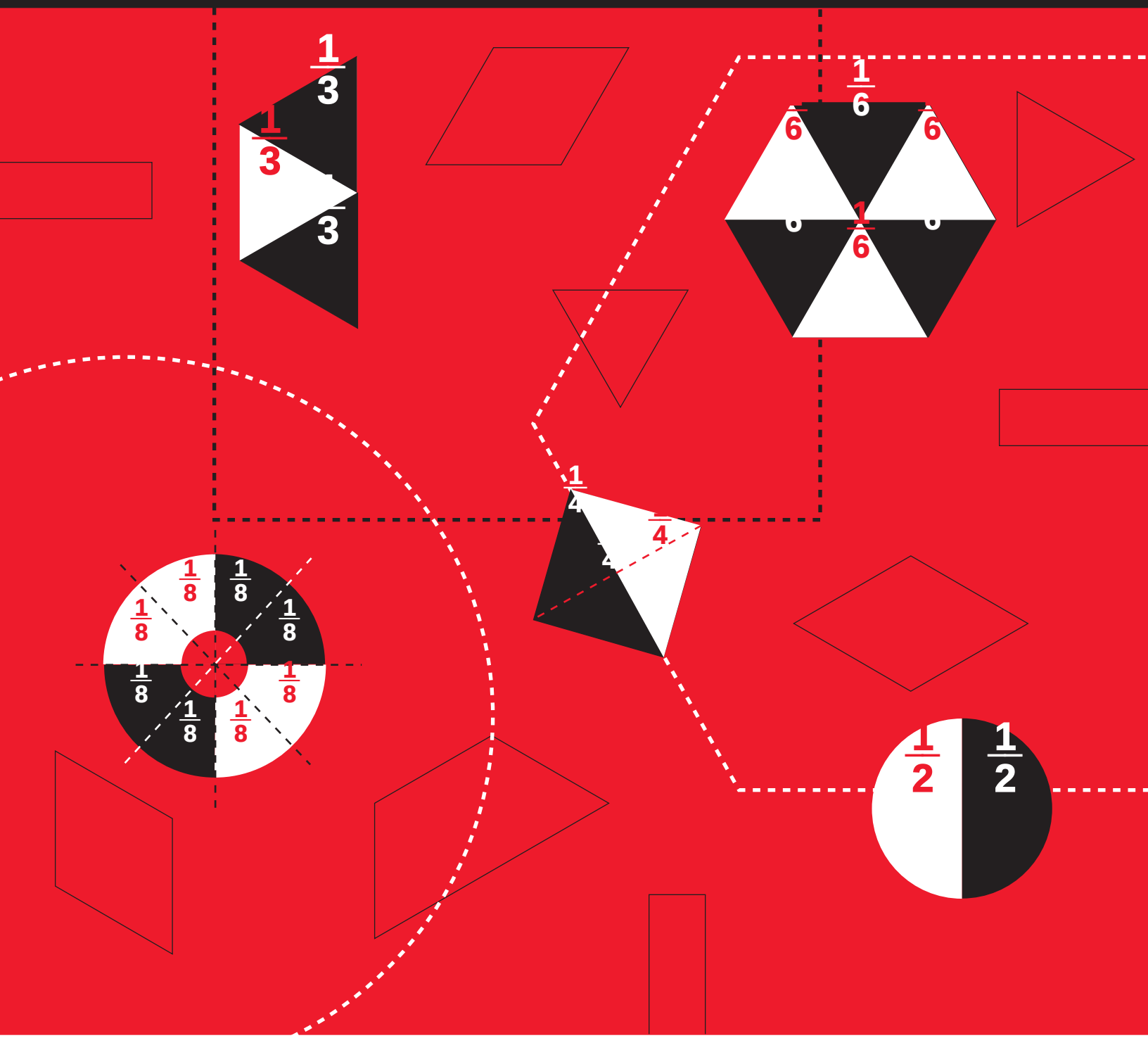
Understanding Fractions

BOOK 3



NAME: _____

CURRICULUM ASSOCIATES®, Inc.

Welcome to ***Understanding Fractions.*** In this book, you will learn what fractions are all about, why they are important to math, and how they are used to solve real-life problems. Building an understanding of fractions is the focus of this book. You will learn how fractions relate to other areas of mathematics. You will work with hands-on material to build your understanding of fractions, one step at a time. And, as you carefully think about and complete the activities, your mind will open to a clearer understanding of the basic operations.

Your teacher will determine how you will work to complete fractions activities. Sometimes, you will work alone. Other times, you will work with a partner or in a small group. As you work with other students, you will find ways of looking at fractions that you might not uncover while working alone. Use the materials on pages 35 and 37, the math tiles and the pattern blocks. Cut them out, and place them in an envelope until you need them. Or, you may use math materials provided by your teacher, as well as items that you find in your home.

Draw pictures or make charts to help you arrive at solutions to the fractions problems. Also, make up similar problems to find out if each solution works in all situations. Where indicated in the materials, you may use a calculator to solve problems.

You will find math terms in the book that are in bold type. Look in the glossary on page 33 for the meaning of these terms. Refer to the glossary each time you come across such a word. You can also add your own math terms and definitions on page 34.

Enjoy your adventure with fractions.

This ***Understanding Fractions*** was prepared for students by Chet Delani.

ISBN 0-7609-0485-5
©1999—Curriculum Associates, Inc.
North Billerica, MA 01862
No part of this book may be reproduced by any means
without written permission from the publisher.
All Rights Reserved. Printed in USA.
15 14 13 12 11 10 9 8 7 6 5 4 3 2 1

LESSON **14**

FOCUS: Discover the relationship among money, fractions, and the concept of $\frac{1}{10}$

MATERIALS: Red, yellow, blue, and green crayons or markers; play money (or real pennies)

1. Circle $\frac{1}{2}$ of the 10 pennies. $\frac{1}{2} \times 10$ pennies = _____

2. a. How many pennies are in 1 dime? _____

   b. What is $\frac{1}{10}$ of 1 dime? _____

3. Color $\frac{1}{10}$ of the 10 pennies red. Color $\frac{2}{10}$ yellow, $\frac{2}{10}$ blue, and $\frac{5}{10}$ green. You may use real or play money to help. Then use the coins to find the answers.

   a. $\frac{1}{10}$ of 1 dime = _____ penny

   b. $\frac{4}{10}$ of 1 dime = _____ pennies

   c. $\frac{2}{10}$ of 1 dime = _____ pennies

   d. $\frac{3}{10}$ of 1 dime = _____ pennies

   e. $\frac{5}{10}$ of 1 dime = _____ pennies

4. You can write $\frac{1}{10}$ as 0.1, which is a **decimal.** Both are read as "one tenth." Write each fraction as a decimal.

   a. $\frac{2}{10}$ = _____ c. $\frac{3}{10}$ = _____ e. $\frac{4}{10}$ = _____ g. $\frac{5}{10}$ = _____

   b. $\frac{6}{10}$ = _____ d. $\frac{7}{10}$ = _____ f. $\frac{8}{10}$ = _____ h. $\frac{9}{10}$ = _____

5. Find the answer.

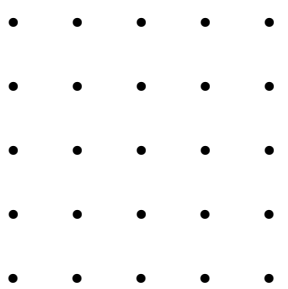
   a. $\frac{1}{2} \times 1$ dime = _____ pennies b. 0.5×1 dime = _____ pennies

   c. Which is more, $\frac{1}{2} \times 1$ dime or 0.5×1 dime? Explain your answer.

LESSON 19

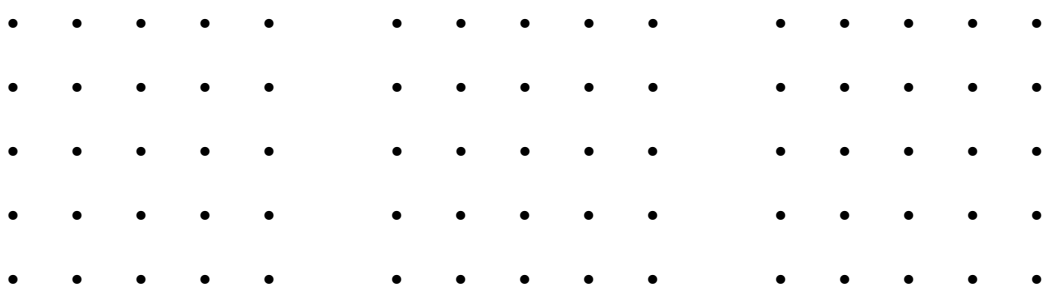
FOCUS: Explore spatial measurement with fractions
MATERIALS: Red crayons or markers, pencils

1. a. Draw a red line to connect all the outside dots in each of the squares.
   How many dots did you connect? _____

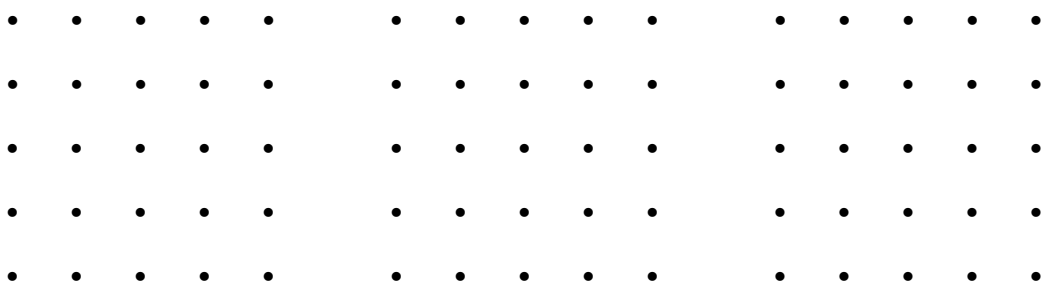


   b. Now use a pencil to draw lines connecting all the dots inside the red line.
   The lines should connect the dots in rows going down and across.
   How many little squares are inside the larger square? _____

2. Draw a red line to connect all the outside dots in each of the three squares.
   Then draw a line to divide each square into halves. Divide each square
   in a different way.



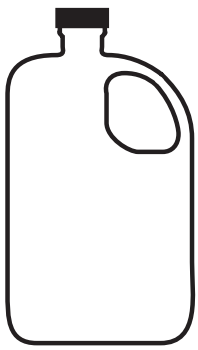
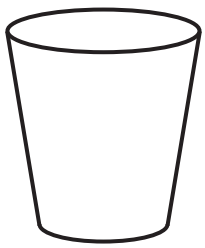
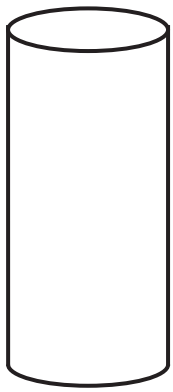
3. Draw a red line to connect all the outside dots in each of the three squares.
   Then draw lines to divide each square into fourths. Divide each square
   in a different way.



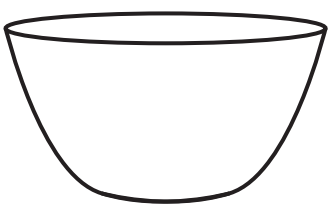
LESSON 22

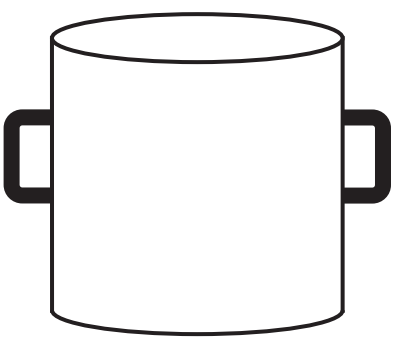
FOCUS: Explore fractions in measurement
MATERIALS: None

1. Shade each container so that it looks to be about $\frac{1}{2}$ full.

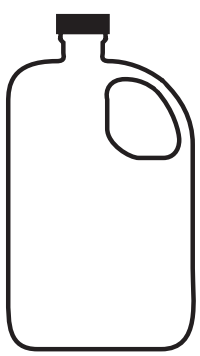
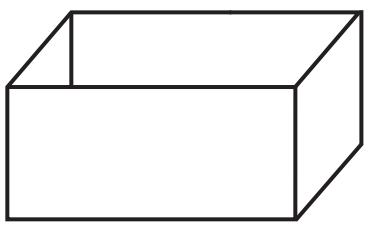




2. Shade each container so that it looks to be about $\frac{1}{3}$ full.



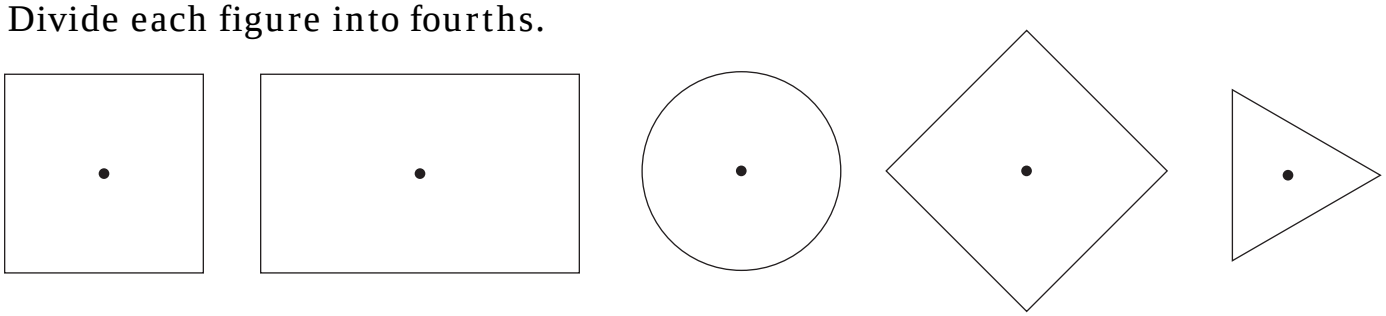


3. Shade each container so that it looks to be about $\frac{1}{4}$ full.

FINAL QUIZ

1. Divide each figure into fourths.



2. Find the answers.

a. $\frac{1}{2} \times 18 =$ _____

b. $\frac{1}{3}$ of 12 = _____

c. $\frac{1}{2} \times (\frac{1}{2} \times 20) =$ _____

d. $\frac{1}{2}$ of 32 = _____

e. $\frac{1}{4}$ of 12 = _____

f. $\frac{1}{2} \times 900 =$ _____

g. $\frac{1}{4}$ of 800 = _____

h. $\frac{1}{2} \times (\frac{1}{2} \times 200) =$ _____

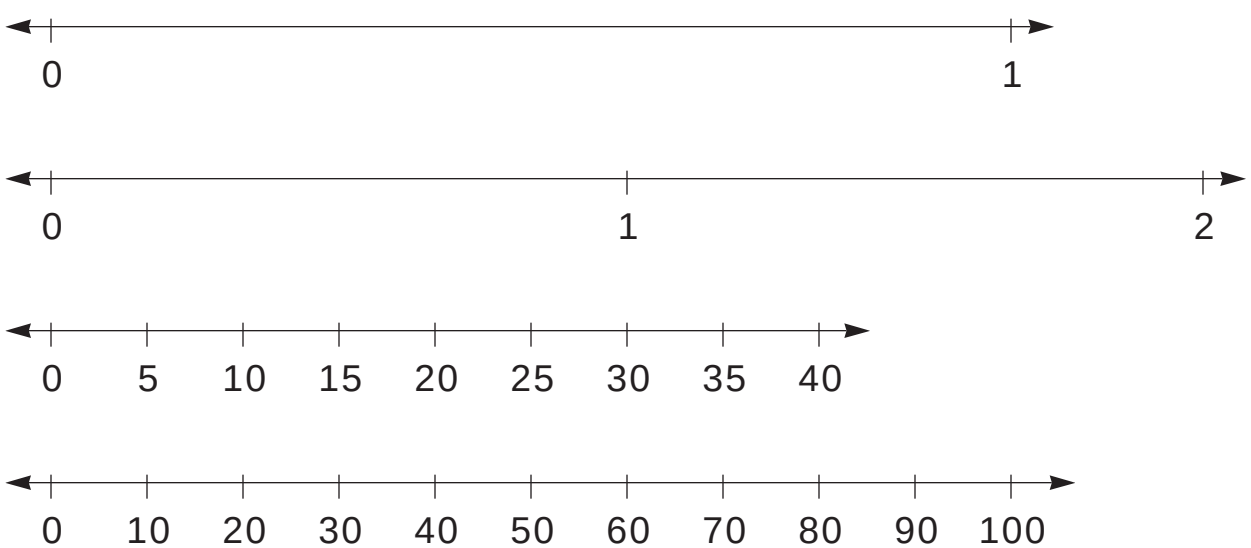
3. Circle the greatest amount.

$\frac{1}{2}$ $1\frac{1}{2}$ $\frac{1}{4}$ $2\frac{1}{4}$ $\frac{1}{3}$

4. Circle the smallest amount.

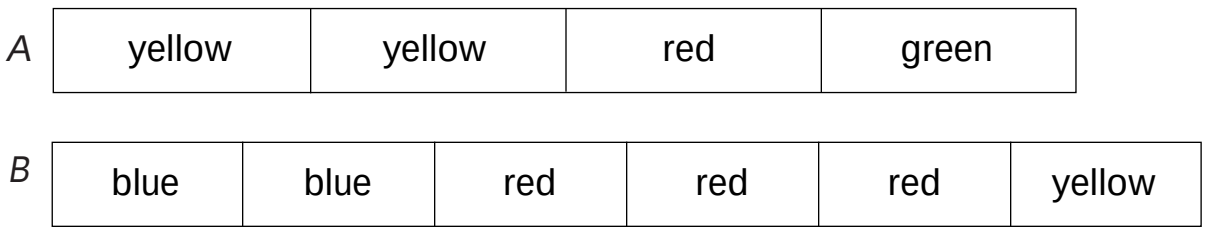
$\frac{1}{2} \times 4$ 0.5×4 0.5×2

5. Put an *X* at the halfway point of each number line.



FINAL QUIZ

6. Use the diagram to answer the questions.



a. Which bar is $\frac{1}{2}$ red? _____

b. Which bar is $\frac{1}{6}$ yellow? _____

c. Which bar is $\frac{1}{4}$ red? _____

d. Which bar is $\frac{1}{3}$ blue? _____

e. Which bar is $\frac{1}{4}$ green? _____

7. Find the answers.

a. 0.1 × 1 dollar = _____

b. 0.5 of 1 dollar = _____

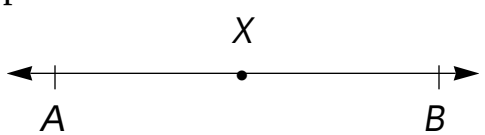
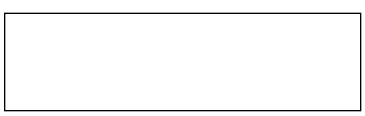
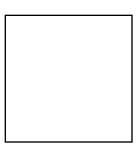
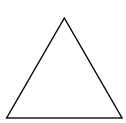
c. 0.3 of 1 dollar = _____

d. $\frac{1}{2}$ × 1 dollar = _____

8. Solve the problem. Draw a picture, and explain how you solved the problem.

Nick brought in 15 muffins to share with the class. There are 20 students in the class. How can Nick share the muffins so that everyone gets an equal number?

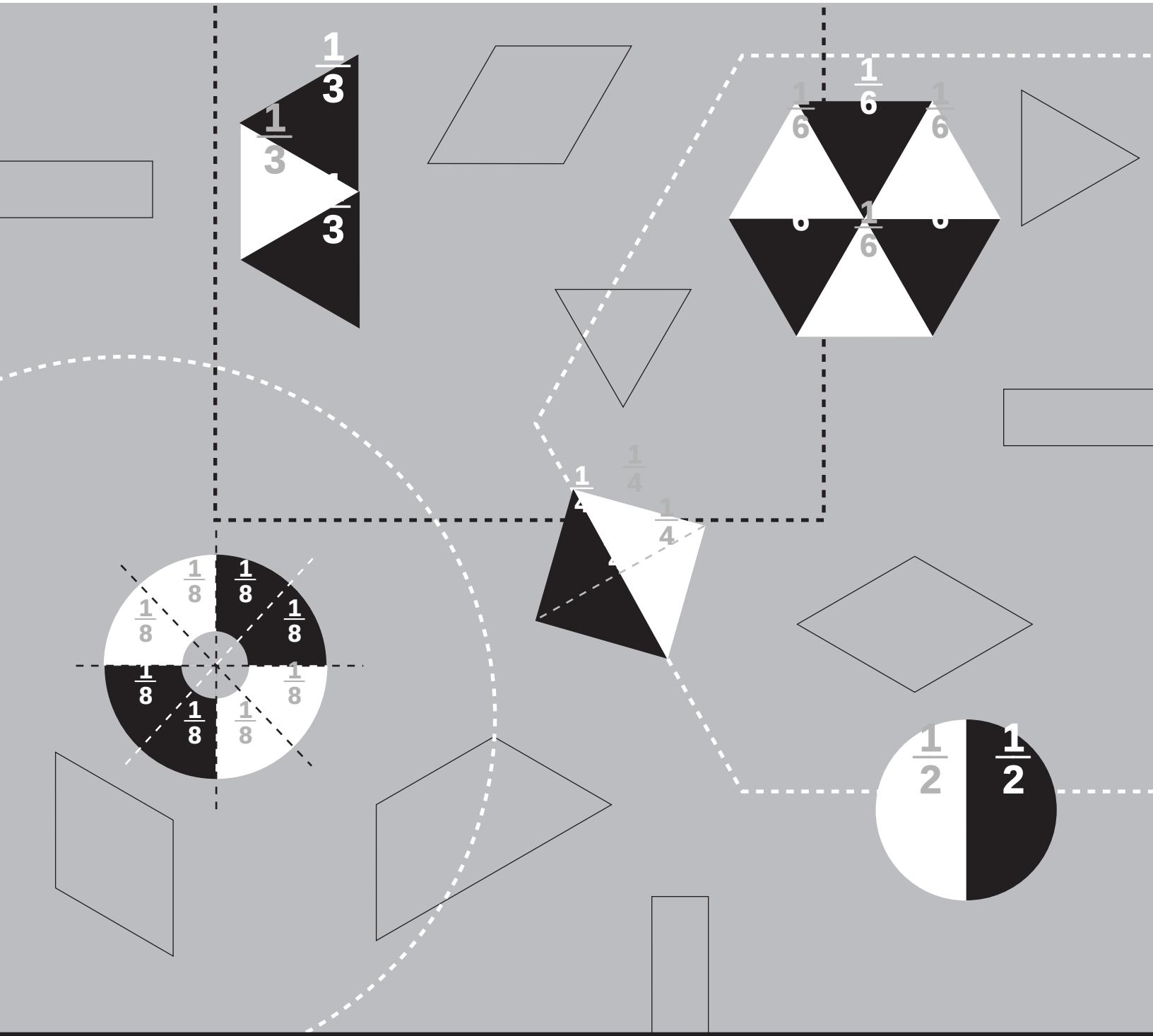
GLOSSARY

WORD	DEFINITION	EXAMPLE
decimal	Any number expressed in a base ten and written with a decimal point	0.1 and 0.05 are **decimals;** their values are $\frac{1}{10}$ and $\frac{5}{100}$
fraction	A number that expresses a part of a whole	$\frac{1}{2}$ and $\frac{3}{4}$ are **fractions**
halfway point	A point that divides a line into halves	Point *X* is the halfway point on line *AB* 
rectangle	A parallelogram that has four right angles and opposite sides that are equal in length	
square	A parallelogram with four equal sides and four right angles	
triangle	A polygon with three sides	

Understanding Fractions

BOOK 3

TEACHER GUIDE



CURRICULUM ASSOCIATES®, Inc.

INTRODUCTION

Understanding Fractions is a series of books for grades 1–8. Each book's activities are designed to increase students' understanding of fractions, and have been developed with the conviction that students construct their own understanding through rich, hands-on mathematical experiences. Although the activities in each book are for a specific grade, they all connect to the core of mathematics learning that is important to every elementary-school and middle-school student.

This book in the series is designed for grade 3. However, depending upon the experiences and background of your students, these activities may be used in grade 2 or 4. They should not be considered remedial in the upper grade levels. Many students who study fractions gain little or no understanding. The activities allow students to explore and to play with fractional relationships and to come to their own understanding. Students can actually begin to analyze shapes, relationships, and number, and in the process, develop their own sense of mathematical power.

The National Council of Teachers of Mathematics has had a significant impact on the nation's classrooms. In the past, the role of the math teacher focused on presenting rules and procedures, followed by seemingly endless drills for practice. The student's role was practice, practice, practice, until mastery of algorithms was proven on the weekly test. The N.C.T.M. standards have drastically de-emphasized this computational focus. Emphasis is now on communication, reasoning, and problem solving. Teachers now foster student thinking for the problem solving and for alternative, original algorithms. Much of today's math instruction is rooted in the work of Piaget, Suppes, Gattegno, Wirtz and Botel, Dienes, and Burns.

ORGANIZATION

Understanding Fractions is not a recipe series for teaching fractional concepts. Rather, it is a series to be used by you and your students to study and explore the dynamic field of fractions. The experiences presented are intended to allow students to gradually develop a systematic awareness of fractional parts and their relationships. Whereas the earlier levels in this series depend primarily upon visual input for students to construct a personal framework for understanding fractions, Book 3 directs students to self-discovery of the operational algorithms and the move from simpler to more complex fractions. Their response to diagrams in this book requires students to develop their mathematical writing ability. Except for the two quizzes, all work may be done in pairs or in small groups of three or four students. The student exchange of ideas increases the power of learning as it uncovers a variety of strategies for problem solving. Working together also allows students to answer one another's questions about the directions and concepts.

You are not asked to correct each page, but rather to discuss the students' results. Encourage students to focus more on their strategies than just on getting the right answer. The specific answers to each page are not as important as the students' awareness of a developing sense of fractions and an ability to apply previous understandings.

Try to be open to a multitude of correct strategies. Some may not be the strategies that you had in mind, but may demonstrate a student's own sense of the question at hand. As the teacher, you may be surprised at the students' sophisticated awareness of numbers and relationships. School for most students has been an exercise in getting the right answer in order to get a good mark, even if what they have done makes no sense to them. They have not had the opportunity to move from their baseline of knowledge to more complex thinking. This past approach undermined the student's belief in his or her ability. The teaching of mathematics today requires a tremendous shift in thinking on your part, as you learn to ask questions that are powerful catalysts for student exploration. Relax and have fun with this book. Mathematics can turn out to be the most exciting study of the day.

MATERIALS

Provide students with an ample supply of paper, pencils, rulers, colored markers, and an assortment of manipulatives. Manipulatives can be as simple as buttons, plastic counters, egg cartons, marbles, and toothpicks. Commercially available manipulatives—such as color tiles, pattern blocks, geoboards, color squares, and Cuisenaire Rods®—are powerful tools for recreating a three-dimensional representation. Various manipulatives are referred to and recommended in the materials section. You may replace these specific manipulatives with comparable objects from the classroom environment or made from construction paper and laminated for durability. Some activities don't list any specific materials, but you should encourage students to use any available manipulatives that will help with the learning procedure. Calculators are optional, but may prove to be useful with many activities.

Using already-constructed objects whenever possible will allow you more time for planning math content and instructional strategies. You may want to look through the book ahead of time and instruct students to create their own manipulatives packet. Students may cut out the math tiles and pattern blocks from pages 35 and 37 in the student book and color them appropriately.

Or you may choose to make copies of Reproducible B (page 17) and Reproducible C (page 18) and distribute them to students. If possible, have the materials laminated. There are other reproducible pages in the back of this teacher guide that can be copied and used as manipulatives. Provide additional common classroom objects for students to add to their packet.

Because students need physical models to construct meaningful representations of their solutions, a variety of readily available hands-on materials will increase the opportunity each student has to bring his or her perspective to the problem at hand. Whereas some materials may seem more appropriate to you for the solution to a problem, allowing students to select their own materials increases individual, diverse thinking.

Encourage your students to use manipulatives to interpret data. Develop a math center that contains varieties of math manipulatives. Recreating printed data with manipulatives provides a kinesthetic opportunity for understanding and more options for solutions. As with any manipulative materials, the thinking activity generated from the physical manipulations is the focus.

GROUPING

The grouping recommended for this series is individual, partners, or a small collaborative groups of three or four members. Although there are many approaches to learning, and some students need to work without outside distractions, most students need and benefit from access to one another's thinking. Students who are allowed to help one another will minimize misunderstandings or confusion from directions or adult interpretations. Encourage your students to discuss the directions together and make sure that their partner understands what to do before they begin. Partners should help each other and should not be afraid to say that they do not understand. Students need to know that this is how learning occurs. Suggest that students try to do the work on their own, checking with a partner as they go along to make sure that they are moving in the right direction. Students need to learn how to work as a team, being responsible for their own work, yet not doing the work for one another. When students finish their work, the group provides a powerful opportunity for immediate feedback and self-correction. Encourage students to use this checking-in period to adjust their responses. Explain that the goal is on learning, not on determining how many problems the students got right or wrong. In a group, students learn to accept the diversity of responses, which fosters creative mathematical thinking.

Sometimes, you will work together with all students. Often, this entire-class work follows the paired-group or collaborative-group sessions in which the students write and check their work together. Bringing all groups together for a focused lesson and classwide sharing permits you to correct any misunderstandings and to expand on concepts that students might bring up. See the teacher guide for reproducibles to use during some full-class-focused lessons.

THE LESSONS AND INSTRUCTION

The lessons in ***Understanding Fractions*** involve problem solving, communication, reasoning, and mathematical connections—the first four N.C.T.M. performance standards. Each activity focuses on at least one of the curriculum strands of number, geometry, measurement, and patterns/functions. Working each lesson along with your students provides a powerful opportunity for you to develop a sense of the concepts and understandings that are emerging for them. Students are often eager to explain ideas to their teachers, especially if the teacher acknowledges his or her own difficulties. This experience makes the classroom a community of learners. Any preconceived notions of fractions that you might have will be no more important than those of the emerging mathematicians in your classroom. The lessons are meant to be exploratory and are sequentially connected. When individual lessons are presented in isolation, they become more dependent upon adult interpretation and formal teaching.

Mathematics does not lend itself to solitary pursuit, so your role as teacher in the development of a deep and powerful understanding is crucial. The majority of students are dependent upon their instructors to lead the exploratory journey through the different strands. The social interaction of the students; the artful and timely questioning by you; and the latitude to probe, question, and discover are all critical elements in developing the mathematical minds of the students.

The lessons in ***Understanding Fractions*** are designed to enable students to increase their own mathematical power. The intent is for you to set the stage but not steal the show. Some activities may seem challenging, and you may be tempted to teach by modeling the solutions. However, this would not provide students with the opportunity to try the activity in whatever way they can. As long as the students have a way of beginning an activity, give them the opportunity to work it through. As members of a pair or a collaborative group, most students will meet with success. Through trial and error and discussion, students will learn.

ASSESSMENT

Because assessment is multifaceted, this series encourages a range of strategies for assessing student progress with the purpose of modifying instruction, not judging ability. Each mathematics lesson, as well as the quizzes, is a part of the assessment process. Also, you can gather much information by listening to students' explanations, observing their thinking as manifested by the manipulation of physical objects, and examining their writing and reflections. These assessment tools are some ways of probing student understanding for the purpose of determining how to modify instruction. Encourage students to keep a portfolio, which will allow both you and them to see growth over time and identify successful problem-solving strategies.

LESSON 14 page 20

FOCUS: Discover the relationship among money, fractions, and the concept of $\frac{1}{10}$

MATERIALS: Red, yellow, blue, and green crayons or markers; play money (or real pennies)

DIRECTIONS: The most important objective of this lesson is for students to understand that $\frac{1}{2}$ is the same as $\frac{5}{10}$ and 0.5. Once students can interchange these mentally, they will be ready to use them in operational procedures. The term *decimal* is introduced in this lesson, but is not developed at this point. Recognition of a decimal is the objective.

The money system provides a natural opportunity for students to use and understand tenths. You can provide physical examples of how a penny is $\frac{1}{10}$ of a dime or a dime is $\frac{1}{10}$ of a dollar. Use both fractions and decimals in discussions of this type. Money is also an ideal subject for extending the ideas to real-world applications and problem solving.

RESPONSES:

1. Check students' work; 5 pennies
2. a. 10
   b. 1 penny
3. Check students' work.
   a. 1
   b. 4
   c. 2
   d. 3
   e. 5
4. a. 0.2
   b. 0.6
   c. 0.3
   d. 0.7
   e. 0.4
   f. 0.8
   g. 0.5
   h. 0.9
5. a. 5
   b. 5
   c. $\frac{1}{2} \times 1$ dime = 5 pennies, and 0.5×1 dime = 5 pennies. Since $\frac{1}{2}$ and 0.5 are equal, both $\frac{1}{2}$ of a dime and 0.5 of a dime must also be equal.

LESSON 15 pages 21 and 22

FOCUS: Explore the relationship among fractions, money, and decimals

MATERIALS: Crayons or markers, play or real money

DIRECTIONS: This lesson provides more practice with fractions, decimals, and money. Students shouldn't need much of an introduction, although you may want to review money equivalencies (1 dollar = 10 dimes = 100 cents, etc.).

RESPONSES:

1. a. 5
   b. 5
2. a. 1
   b. 1
   c. 3
   d. 3
   e. 6
   f. 6
   g. 2
   h. 2
   i. 4
   j. 4
   k. 7
   l. 7
   m. 9
   n. 9
   o. 5
   p. 5
   q. 8
   r. 8
3. 0.7 of a dollar
4. 0.3×1 dollar

5., 6. Students' drawings should include 5 dimes.

LESSON 18 page 25

FOCUS: Collect and organize data; use fractions to study data

MATERIALS: None

DIRECTIONS: This lesson requires students to collect data about the kind of footwear that students in the class are wearing on a particular day. This works well as a whole-class activity. As each student describes his or her footwear, keep a tally on the board. Students can keep the same tally in their book. Then they can work alone to complete the chart and answer the questions.

RESPONSES:

Answers will vary. Sample answers are provided.

Type of Footwear	Tally	Number of Pieces of Footwear
Shoes	𝍸 ||||	18
Sneakers	𝍸 𝍸 ||	24
Boots	|||	6
Other	||	4

1. 52
2. 26
3. 12
4. Explanations will vary.

LESSON 19 page 26

FOCUS: Explore spatial measurement with fractions

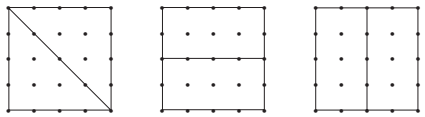
MATERIALS: Red crayons or markers, pencils; Optional: Reproducible A (page 16 of teacher guide)

DIRECTIONS: In Lessons 19, 20, and 21, students work with fractions and a variety of pattern blocks that are on dot paper. Students practice dividing the figures into different fractional parts. In some problems, students count the number of squares inside the figure. You may want to expand upon this as a lead-in to concepts about area.

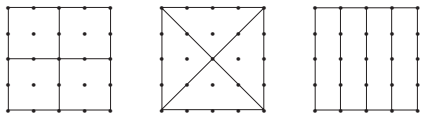
You may want to use Reproducible A to make an overhead-projector overlay of the dot paper. This can be used to show students examples of the type of work they will be doing in the three lessons. You can also use this opportunity to have students count individual squares inside the parts of the figures to emphasize the fact that fractions represent equal parts.

RESPONSES:

1. a. Check students' work; 16
   b. 16
2. Students' answers will vary. A sample answer is provided.



3. Students' answers will vary. A sample answer is provided.



LESSON 22 page 29

FOCUS: Explore fractions in measurement

MATERIALS: None

DIRECTIONS: In this lesson, students study a variety of containers and then shade the figures so that they appear to be $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$ full. You might use this activity for an assessment. Students have completed a variety of activities involving $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$. If they truly understand the concepts, they should have little difficulty completing the lesson. The focus is on estimating, both with measurements and fractions. Discuss with students when it might be appropriate to estimate and when it's important to have precise measurements.

RESPONSES:

1.–3. Check students' work.

LESSON 23 page 30

FOCUS: Solve problems with fractions

MATERIALS: Tiles, counters, crayons or markers

DIRECTIONS: This lesson requires students to use what they have learned about fractions to solve a real-world word problem. Provide students with a variety of manipulatives. Encourage students to use the manipulatives and drawings to help find the solution and check the answer.

RESPONSES:

1. Check students' work. Answers may vary. Sample answer: Each student will get 1 whole cookie and $\frac{1}{3}$ of another cookie.
2. Students' explanations will vary.

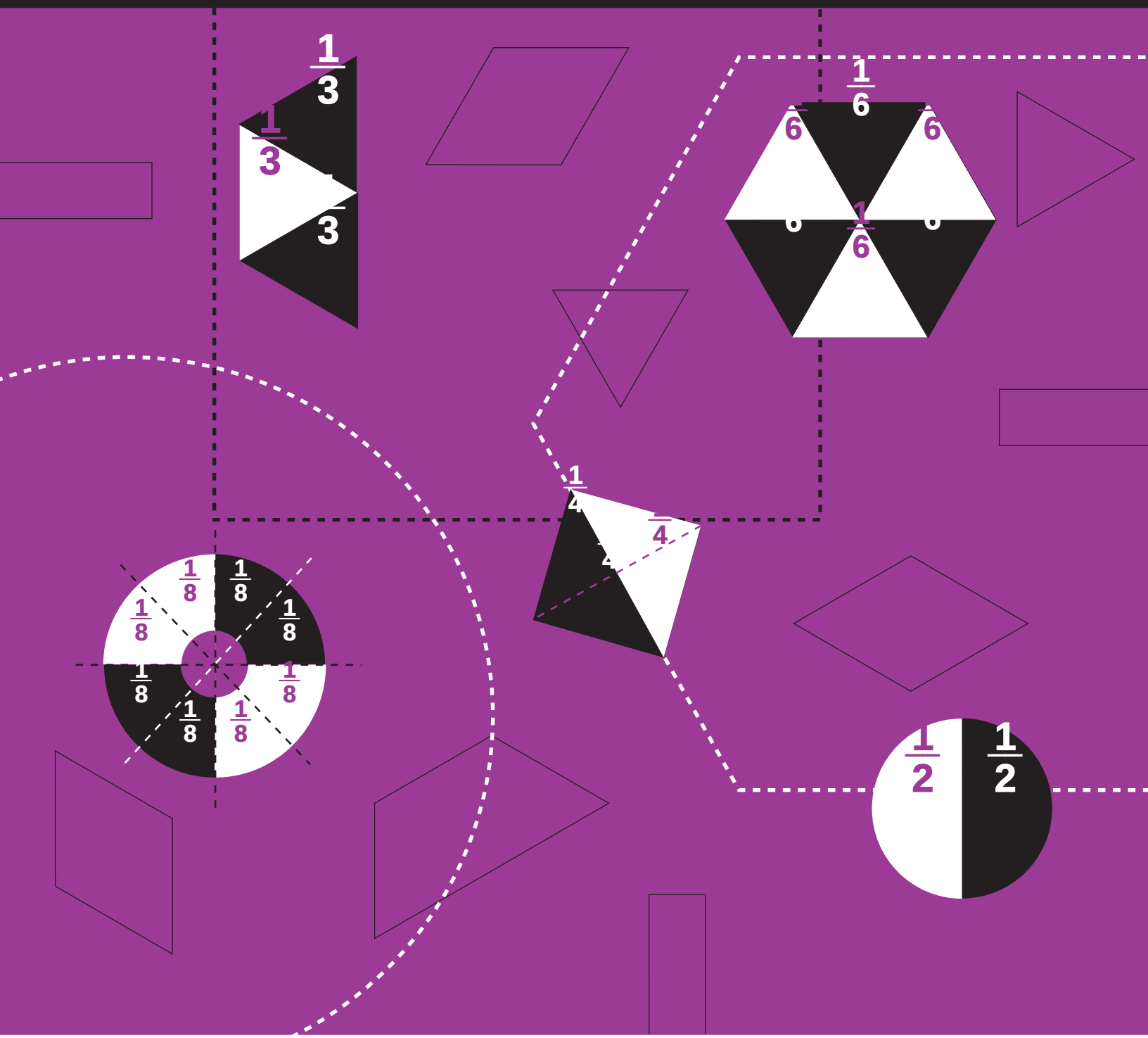
FINAL QUIZ pages 31 and 32

RESPONSES:

1. Answers may vary. Check students' work.
2. a. 9
   b. 4
   c. 5
   d. 16
   e. 3
   f. 450
   g. 200
   h. 50
3. $2\frac{1}{4}$
4. 0.5×2
5. Check students' work. Students should mark halfway between 0 and 1 on the first number line. On the other number lines, students should mark 1, 20, and 50 as the halfway points.
6. a. bar *B*
   b. bar *B*
   c. bar *A*
   d. bar *B*
   e. bar *A*
7. Students' answers may vary. Accept any appropriate combination of coins. Sample answers are provided.
   a. 10 pennies
   b. 5 dimes
   c. 3 dimes
   d. 2 quarters
8. Students' drawings and explanations may vary. Sample answer: Divide 10 muffins into halves so that there are 20 halves. Then divide the remaining 5 muffins into quarters so that there are 20 quarters. Each student would get $\frac{1}{2}$ and $\frac{1}{4}$, or $\frac{3}{4}$, of a muffin.

Understanding Fractions

BOOK 5



NAME: _____

CURRICULUM ASSOCIATES®, Inc.

Welcome to ***Understanding Fractions.*** In this book, you will learn what fractions are all about, why they are important to math, and how they are used to solve real-life problems. Building an understanding of fractions is the focus of this book. You will learn how fractions relate to other areas of mathematics. You will work with hands-on material to build your understanding of fractions, one step at a time. And, as you carefully think about and complete the activities, your mind will open to a clearer understanding of the basic operations.

Your teacher will determine how you will work to complete the fractions activities. Sometimes, you will work alone. Other times, you will work with a partner or in a small group. As you work with other students, you will find ways of looking at fractions that you might not uncover while working alone. Use the materials on pages 35 and 37, the math tiles and the pattern blocks. Cut them out, and place them in an envelope until you need them. Or, you may use math materials provided by your teacher, as well as items that you find in your home.

Draw pictures or make charts to help you arrive at solutions to the fractions problems. Also, make up similar problems to find out if each solution works in all situations. Where indicated in the materials, you may use a calculator to solve problems.

You will find math terms in the book that are in bold type. Look in the glossary on page 33 for the meaning of these terms. Refer to the glossary each time you come across such a word. You can also add your own math terms and definitions on page 34.

Enjoy your adventure with fractions.

This ***Understanding Fractions*** book was prepared for students by Chet Delani.

ISBN 0-7609-0489-8
©1999—Curriculum Associates, Inc.
North Billerica, MA 01862
No part of this book may be reproduced by any means without written permission from the publisher.
All Rights Reserved. Printed in USA.
15 14 13 12 11 10 9 8 7 6 5 4 3 2 1

LESSON 7

FOCUS: Practice with equivalent fractions and cross-products
MATERIALS: Calculator, tiles

1. Study the pattern below. Answer the questions to complete the pattern.

$$\frac{1}{3} = \frac{2}{6} = \frac{3}{9} = \frac{4}{12} = \frac{5}{15} = \frac{?}{?}$$

   a. What fraction comes after $\frac{5}{15}$? _____
   b. Continue the pattern. Write the next five fractions after $\frac{5}{15}$.

   _____ = _____ = _____ = _____ = _____

2. Use the rule of cross-products to check your work. Write your cross-products. You may want to use your calculator. Use your tiles or any other manipulatives that help you picture your work.

3. Study the pattern below. Answer the questions to complete the pattern.

$$\frac{1}{4} = \frac{2}{8} = \frac{3}{12} = \frac{4}{16} = \frac{5}{20} = \frac{?}{?}$$

   a. What fraction comes after $\frac{5}{20}$? _____
   b. Continue the pattern. Write the next five fractions after $\frac{5}{20}$.

   _____ = _____ = _____ = _____ = _____

4. Use the rule of cross-products to check your work. Write your cross-products. You may want to use your calculator.

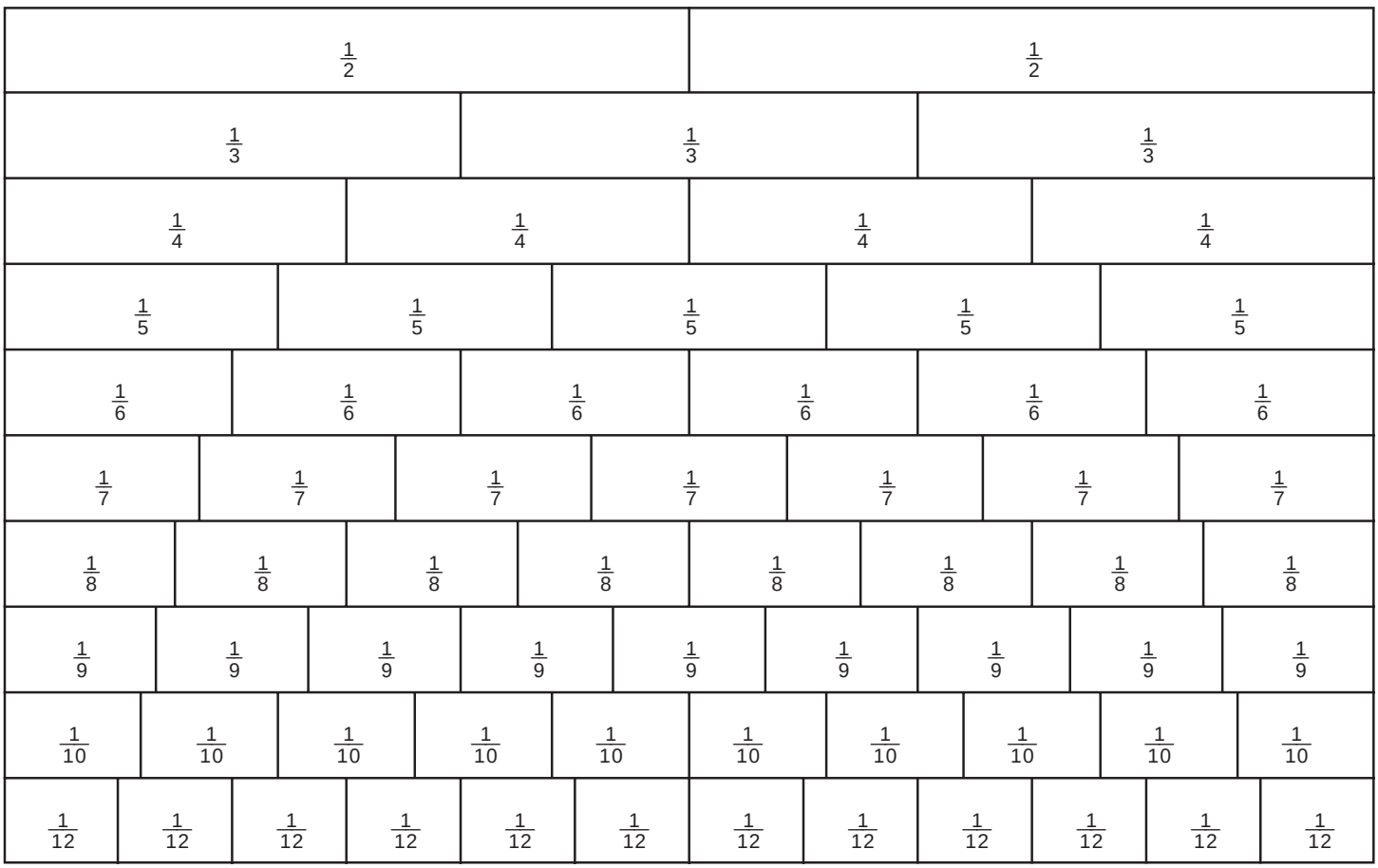
5. Circle the equivalent fractions. Write the cross-products to prove that your choices are correct.

   a. $\frac{1}{2} = \frac{2}{4}$ _____
   b. $\frac{1}{3} = \frac{2}{4}$ _____
   c. $\frac{1}{3} = \frac{3}{0}$ _____
   d. $\frac{1}{5} = \frac{2}{10}$ _____
   e. $\frac{3}{6} = \frac{5}{10}$ _____
   f. $\frac{4}{8} = \frac{25}{50}$ _____
   g. $\frac{4}{12} = \frac{1}{4}$ _____
   h. $\frac{50}{100} = \frac{3}{6}$ _____
   i. $\frac{4}{16} = \frac{1}{3}$ _____
   j. $\frac{1}{10} = \frac{10}{100}$ _____
   k. $\frac{1}{5} = \frac{3}{4}$ _____
   l. $\frac{2}{4} = \frac{1}{3}$ _____

LESSON 8

FOCUS: Practice with equivalent fractions
MATERIALS: None

Below are pictures of strips divided into a variety of parts.



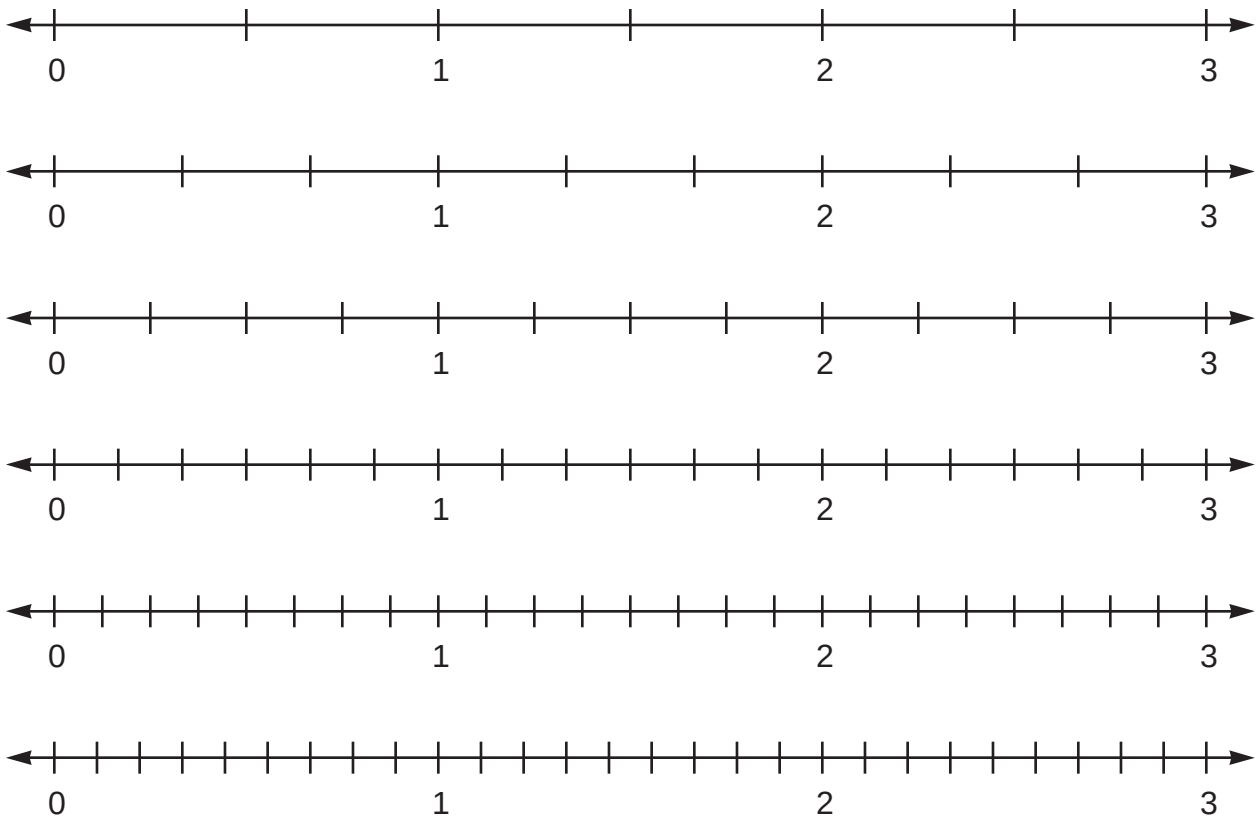
List all the fractions that are equivalent to the fractions shown in 1-8.

1. $\frac{1}{2}$	3. $\frac{1}{4}$	5. $\frac{2}{3}$	7. $\frac{4}{9}$
2. $\frac{1}{3}$	4. $\frac{1}{5}$	6. $\frac{2}{2}$	8. $\frac{2}{7}$

LESSON

15

FOCUS: Practice addition and multiplication of fractions
MATERIALS: None



1. Find the answers. You may use the number lines.
   a. $\frac{1}{2} + \frac{1}{2} + \frac{1}{2} =$ _____ $3 \times \frac{1}{2} =$ _____
   b. $\frac{1}{3} + \frac{1}{3} + \frac{1}{3} =$ _____ $3 \times \frac{1}{3} =$ _____
   c. $\frac{2}{4} + \frac{1}{4} + \frac{2}{4} =$ _____ $5 \times \frac{1}{4} =$ _____
   d. Study the pairs of completed number sentences in *a–c*. Explain how they are related.

2. Use the number lines. List all the fractions that are equivalent to $\frac{1}{2}$.

3. Use the information from problems 1 and 2 to solve these problems.
   a. $\frac{1}{2} + \frac{2}{4} =$ _____
   b. $\frac{1}{2} + \frac{3}{6} =$ _____
   c. $\frac{1}{2} + \frac{4}{8} =$ _____
   d. $\frac{2}{4} + \frac{4}{8} =$ _____
   e. $\frac{3}{6} + \frac{2}{4} =$ _____
   f. $\frac{3}{6} + \frac{4}{8} =$ _____
   g. Explain how you solved these problems.

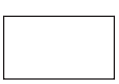
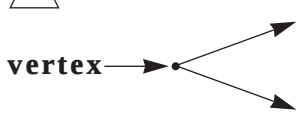
4. Use the number lines. List all the fractions equal to $\frac{1}{3}$.

5. Use the number lines to solve these problems.
   a. $\frac{1}{3} + \frac{1}{3} =$ _____ $\frac{1}{3} + \frac{2}{6} =$ _____
   b. $\frac{1}{3} + \frac{0}{3} =$ _____ $\frac{2}{6} + \frac{0}{3} =$ _____
   c. $\frac{1}{3} + \frac{2}{3} =$ _____ $\frac{1}{3} + \frac{4}{6} =$ _____
   d. Explain how you solved these problems.

4. Chet hiked a trail where every $\frac{1}{2}$ mile was marked. He counted 11 markers. How far did Chet hike? _____ Draw a picture to show your solution.

5. Explain your solution.

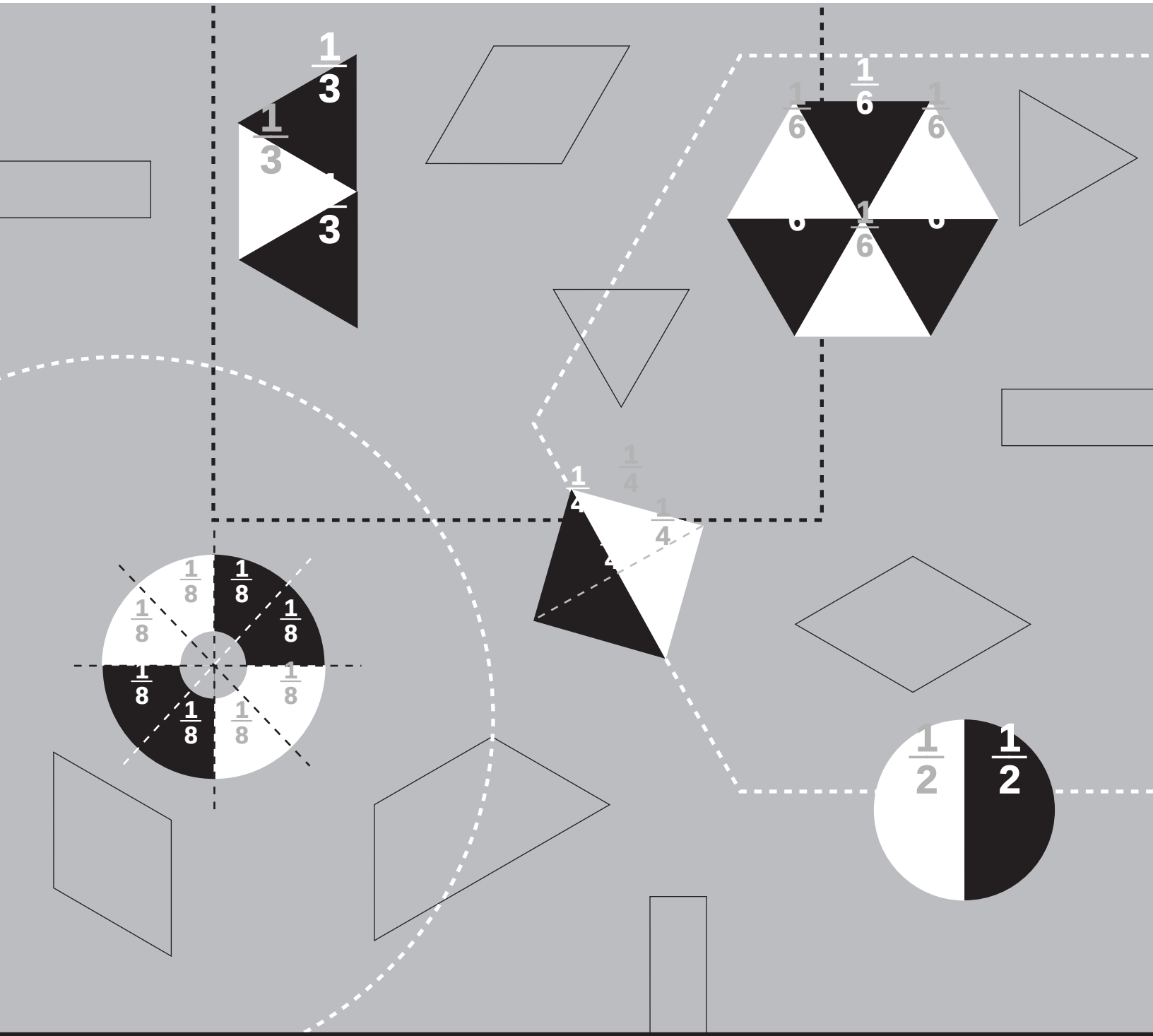
GLOSSARY

WORD	DEFINITION	EXAMPLE
circle	The set of all points in a plane that are the same distance from the center point	
common factor	A number that is a factor of all the numbers in a given set	2 is a **common factor** of 4 and 6
decimal	Any number expressed in a base ten and written with a decimal point	0.1 and 0.05 are **decimals;** their values are $\frac{1}{10}$ and $\frac{5}{100}$
denominator	The number written below the line in a common fraction; tells the number of parts in a whole	The 3 in $\frac{2}{3}$ tells the number of parts in the whole
divisible	Can be divided evenly with no remainder	28 is **divisible** by 7
equivalent fractions	Fractions that have the same value	$\frac{1}{2}$, $\frac{2}{4}$, and $\frac{3}{6}$ are **equivalent fractions**
factor	One of two or more numbers that when multiplied together produces a product	6 and 3 are **factors** of 18
fraction	A number that expresses a part of a whole	$\frac{1}{2}$ and $\frac{3}{4}$ are **fractions**
greatest common factor	The greatest number that evenly divides into a given set of numbers	6 is the **greatest common factor** of 12 and 18
hexagon	A polygon with six sides	
mixed number	A number made up of a whole number and a fraction	$1\frac{1}{2}$ and $6\frac{2}{3}$ are **mixed numbers**
multiple	A number that is a product of a given number and a whole number	6 is a **multiple** of 2 and 3
numerator	The number written above the line in a fraction; tells the number of parts that are used	The 2 in $\frac{2}{3}$ tells the number of thirds that are used
octagon	A polygon with eight sides	
rectangle	A parallelogram that has four right angles and opposite sides that are equal in length	
rhombus	A parallelogram with all four sides having equal length	
rule of cross-products	Two fractions are considered equivalent when the numerator of the first fraction times the denominator of the second fraction equals the product of the denominator of the first fraction and the numerator of the second fraction.	$\frac{2}{3} = \frac{4}{6}$ because $2 \times 6 = 3 \times 4$ $\frac{3}{6} = \frac{4}{8}$ because $3 \times 8 = 4 \times 6$
trapezoid	A quadrilateral with one pair of parallel sides	
triangle	A three-sided polygon	
vertex	The point at which the rays of an angle intersect	

Understanding Fractions

BOOK 5

TEACHER GUIDE



CURRICULUM ASSOCIATES®, Inc.

INTRODUCTION

Understanding Fractions is a series of books for grades 1–8. Each book's activities are designed to increase students' understanding of fractions, and have been developed with the conviction that students construct their own understanding through rich, hands-on mathematical experiences. Although the activities in each book are for a specific grade, they all connect to the core of mathematics learning that is important to every elementary-school and middle-school student.

This book in the series is designed for grade 5. However, depending upon the experiences and background of your students, these activities may be used in grade 4 or 6. They should not be considered remedial in the upper-grade levels. Many students who study fractions gain little or no understanding. The activities allow students to explore and to play with fractional relationships and to come to their own understanding. Students can actually begin to analyze shapes, relationships, and number, and in the process, develop their own sense of mathematical power.

The National Council of Teachers of Mathematics has had a significant impact on the nation's classrooms. In the past, the role of the math teacher focused on presenting rules and procedures, followed by seemingly endless drills for practice. The student's role was practice, practice, practice, until mastery of algorithms was proven on the weekly test. The N.C.T.M. standards have drastically de-emphasized this computational focus. Emphasis is now on communication, reasoning, and problem solving. Teachers now foster student thinking for the problem solving and for alternative, original algorithms. Much of today's math instruction is rooted in the work of Piaget, Suppes, Gattegno, Wirtz and Botel, Dienes, and Burns.

ORGANIZATION

Understanding Fractions is not a recipe series for teaching fractional concepts. Rather, it is a series to be used by you and your students to study and explore the dynamic field of fractions. The experiences presented are intended to allow students to gradually develop a systematic awareness of fractional parts and their relationships. Whereas the earlier levels in this series depend primarily upon visual input for students to construct a personal framework for understanding fractions, Book 5 directs students to self-discovery of the operational algorithms and the move from simpler to more complex fractions. Their response to diagrams in this book requires students to develop their mathematical writing ability. Except for the two quizzes, all work may be done in pairs or in small groups of three or four students. The student exchange of ideas increases the power of learning as it uncovers a variety of strategies for problem solving. Working together also allows students to answer one another's questions about the directions and concepts.

You are not asked to correct each page, but rather to discuss the students' results. Encourage students to focus more on their strategies than just on getting the right answer. The specific answers to each page are not as important as the students' awareness of a developing sense of fractions and an ability to apply previous understandings.

Try to be open to a multitude of correct strategies. Some may not be the strategies that you had in mind, but may demonstrate a student's own sense of the question at hand. As the teacher, you may be surprised at the students' sophisticated awareness of numbers and relationships. School for most students has been an exercise in getting the right answer in order to get a good mark, even if what they have done makes no sense to them. They have not had the opportunity to move from their baseline of knowledge to more complex thinking. This past approach undermined the student's belief in his or her ability. The teaching of mathematics today requires a tremendous shift in thinking on your part, as you learn to ask questions that are powerful catalysts for student exploration. Relax and have fun with this book. Mathematics can turn out to be the most exciting study of the day.

MATERIALS

Provide students with an ample supply of paper, pencils, rulers, colored markers, and an assortment of manipulatives. Manipulatives can be as simple as buttons, plastic counters, egg cartons, marbles, and toothpicks. Commercially available manipulatives—such as color tiles, pattern blocks, geoboards, color squares, and Cuisenaire Rods®—are powerful tools for recreating a three-dimensional representation. Various manipulatives are referred to and recommended in the materials section. You may replace these specific manipulatives with comparable objects from the classroom environment or made from construction paper and laminated for durability. Some activities don't list any specific materials, but you should encourage students to use any available manipulatives that will help with the learning procedure. Calculators are optional, but may prove to be useful with many activities.

Using already-constructed objects whenever possible will allow you more time for planning math content and instructional strategies. You may want to look through the book ahead of time and instruct students to create their own manipulatives packet. Students may cut out the math tiles and pattern blocks from pages 35 and 37 in the student book and color them appropriately.

Or you may choose to make copies of Reproducible D (page 20) and Reproducible E (page 21) and distribute them to students. If possible, have the materials laminated. There are other reproducible pages in the back of this teacher guide that can be copied and used as manipulatives. Provide additional common classroom objects for students to add to their packet.

Because students need physical models to construct meaningful representations of their solutions, a variety of readily available hands-on materials will increase the opportunity each student has to bring his or her perspective to the problem at hand. Whereas some materials may seem more appropriate to you for the solution to a problem, allowing students to select their own materials increases individual, diverse thinking.

Encourage your students to use manipulatives to interpret data. Develop a math center that contains varieties of math manipulatives. Recreating printed data with manipulatives provides a kinesthetic opportunity for understanding and more options for solutions. As with any manipulative materials, the thinking activity generated from the physical manipulations is the focus.

GROUPING

The grouping recommended for this series is individual, partners, or a small collaborative group of three or four members. Although there are many approaches to learning, and some students need to work without outside distractions, most students need and benefit from access to one another's thinking. Students who are allowed to help one another will minimize misunderstandings or confusion from directions or adult interpretations. Encourage your students to discuss the directions together and make sure that their partner understands what to do before they begin. Partners should help each other and should not be afraid to say that they do not understand. Students need to know that this is how learning occurs. Suggest that students try to do the work on their own, checking with a partner as they go along to make sure that they are moving in the right direction. Students need to learn how to work as a team, being responsible for their own work, yet not doing the work for one another. When students finish their work, the group provides a powerful opportunity for immediate feedback and self-correction. Encourage students to use this checking-in period to adjust their responses. Explain that the goal is on learning, not on determining how many problems the students got right or wrong. In a group, students learn to accept the diversity of responses, which fosters creative mathematical thinking.

Sometimes, you will work together with all students. Often, this entire-class work follows the paired-group or collaborative-group sessions in which the students write and check their work together. Bringing all groups together for a focused lesson and classwide sharing permits you to correct any misunderstandings and to expand on concepts that students might bring up. See the teacher guide for reproducibles to use during some full-class-focused lessons.

THE LESSONS AND INSTRUCTION

The lessons in ***Understanding Fractions*** involve problem solving, communication, reasoning, and mathematical connections—the first four N.C.T.M. performance standards. Each activity focuses on at least one of the curriculum strands of number, geometry, measurement, and patterns/functions. Working each lesson along with your students provides a powerful opportunity for you to develop a sense of the concepts and understandings that are emerging for them. Students are often eager to explain ideas to their teachers, especially if the teacher acknowledges his or her own difficulties. This experience makes the classroom a community of learners. Any preconceived notions of fractions that you might have will be no more important than those of the emerging mathematicians in your classroom. The lessons are meant to be exploratory and are sequentially connected. When individual lessons are presented in isolation, they become more dependent upon adult interpretation and formal teaching.

Mathematics does not lend itself to solitary pursuit, so your role as teacher in the development of a deep and powerful understanding is crucial. The majority of students are dependent upon their instructors to lead the exploratory journey through the different strands. The social interaction of the students; the artful and timely questioning by you; and the latitude to probe, question, and discover are all critical elements in developing the mathematical minds of the students.

The lessons in ***Understanding Fractions*** are designed to enable students to increase their own mathematical power. The intent is for you to set the stage but not steal the show. Some activities may seem challenging, and you may be tempted to teach by modeling the solutions. However, this would not provide students with the opportunity to try the activity in whatever way they can. As long as the students have a way of beginning an activity, give them the opportunity to work it through. As members of a pair or a collaborative group, most students will meet with success. Through trial and error and discussion, students will learn.

ASSESSMENT

Because assessment is multifaceted, this series encourages a range of strategies for assessing student progress with the purpose of modifying instruction, not judging ability. Each mathematics lesson, as well as the quizzes, is a part of the assessment process. Also, you can gather much information by listening to students' explanations, observing their thinking as manifested by the manipulation of physical objects, and examining their writing and reflections. These assessment tools are some ways of probing student understanding for the purpose of determining how to modify instruction. Encourage students to keep a portfolio, which will allow both you and them to see growth over time and identify successful problem-solving strategies.

LESSON 7 page 8

FOCUS: Practice with equivalent fractions and cross-products

MATERIALS: Calculator, tiles

DIRECTIONS: This lesson is a simple extension of Lesson 6. Students who did not discover previous patterns should be more successful here.

In a follow-up discussion, draw attention to the patterns of the numerators and denominators, as compared with the time tables.

RESPONSES:

1. a. $\frac{6}{18}$
   b. $\frac{6}{18}, \frac{7}{21}, \frac{8}{24}, \frac{9}{27}, \frac{10}{30}$
2. The students' work should show examples such as $\frac{1}{3} = \frac{3}{9}$. Each cross-product is 9.
3. a. $\frac{6}{24}$
   b. $\frac{6}{24}, \frac{7}{28}, \frac{8}{32}, \frac{9}{36}, \frac{10}{40}$
4. The students' work should show examples, such as $\frac{9}{36} = \frac{10}{40}$. Each cross-product is 360.
5. The answers that should be circled are
   a. (4 = 4)
   d. (10 = 10)
   e. (30 = 30)
   f. (200 = 200)
   h. (300 = 300)
   j. (100 = 100)

LESSON 8 page 9

FOCUS: Practice with equivalent fractions

MATERIALS: None; Optional: Reproducible C (page 16 of teacher guide)

DIRECTIONS: This lesson presents a different visual method for determining equivalent fractions. You may wish to follow up the lesson by having students apply the cross-product test to verify the answers. Since the focus of this lesson is equivalent fractions, not multiplication of whole numbers, you may want to allow students to use calculators. You may wish to make copies of the fraction strips for students to cut up and use with this and other lessons.

RESPONSES:

1. $\frac{2}{4}, \frac{3}{6}, \frac{4}{8}, \frac{5}{10}, \frac{6}{12}$
2. $\frac{2}{6}, \frac{3}{9}, \frac{4}{12}$
3. $\frac{2}{8}, \frac{3}{12}$
4. $\frac{2}{10}$
5. $\frac{4}{6}, \frac{8}{12}$
6. $\frac{3}{3}, \frac{4}{4}$, etc.
7. No fractions are equivalent.
8. No fractions are equivalent.

LESSON 14 page 17

FOCUS: Practice with factors and multiples; discover greatest common factor

MATERIALS: Crayons or colored markers, tiles

DIRECTIONS: Listing all the factors of different numbers in an orderly fashion will help students find common factors. Students can easily move among the numbers, visually comparing all of the factors in order to find the greatest common factor.

RESPONSES:

1. a. 1, 7
   b. 1, 3, 9
   c. 1, 2, 3, 4, 6, 12
   d. 1, 3, 5, 15
   e. 1, 2, 3, 4, 6, 8, 12, 24
   f. 1, 2, 3, 5, 6, 10, 15, 30
   g. 1, 2, 3, 4, 6, 9, 12, 18, 36
   h. 1, 3, 7, 21
2. a. 1, 7
   b. 1, 3
   c. 1, 2, 3, 4, 6, 12
   d. 1, 3, 5, 15
3. 7
4. a. 7
   b. 3
   c. 12
   d. 15
   e. 9
   f. 12
   g. 3
   h. 3
5. Students should use tiles or any other objects in the classroom to represent their 18 elephants and to show how they arranged the objects in equal-sized groups. Their groupings should include 1×18, 2×9, 3×6, 6×3, 9×2, and 18×1.

LESSON 15 pages 18 and 19

FOCUS: Practice addition and multiplication of fractions

MATERIALS: None

DIRECTIONS: Encourage students to draw from any previous knowledge or experiences as they try to figure out the solutions. The equations are related and sequenced in a way that encourages students to find answers from their previous work. It is important to accept either a mixed number or improper fraction as an answer at this time: the students are still developing an understanding of fractions. It is important for the students to solidify understanding of the relationship between addition and multiplication. If students understand how to add fractions, then they should be able to figure out how to complete an equation, even if they forget the algorithm.

RESPONSES:

1. a. $\frac{3}{2}$, or $1\frac{1}{2}$; $\frac{3}{2}$, or $1\frac{1}{2}$
   b. $\frac{3}{3}$, or 1; $\frac{3}{3}$, or 1
   c. $\frac{5}{4}$, or $1\frac{1}{4}$; $\frac{5}{4}$, or $1\frac{1}{4}$
   d. The number sentences are equal. Students should see the relationship between multiplication and repeated addition.
2. $\frac{1}{2}$, $\frac{2}{4}$, $\frac{3}{6}$, $\frac{4}{8}$
3. a. 1
   b. 1
   c. 1
   d. 1
   e. 1
   f. 1
   g. Responses should include the idea that $\frac{1}{2} + \frac{1}{2} = 1$, and all the different fractions in the problems are other ways to write $\frac{1}{2}$.
4. $\frac{1}{3}$, $\frac{2}{6}$, $\frac{3}{9}$
5. a. $\frac{2}{3}$, $\frac{2}{3}$
   b. $\frac{1}{3}$, $\frac{2}{6}$, or $\frac{1}{3}$
   c. $\frac{3}{3}$, or 1; $\frac{3}{3}$, or 1
   d. Responses should include the idea that $\frac{1}{3} + \frac{1}{3} = \frac{2}{3}$, and that most of these fractions are equivalent to $\frac{1}{3}$. Students may not understand that they must exchange $\frac{2}{3}$ for $\frac{4}{6}$ in the last problem.

LESSON 24 page 29

FOCUS: Practice multiplication and division of fractions

MATERIALS: Pattern blocks

DIRECTIONS: In this lesson, students are required to expand their thinking. Now, a whole figure is made up of three hexagons. Students can divide the figure into different numbers of parts to help solve the equations.

RESPONSES:

1. a. 2 e. 12 i. 3 m. 10 q. 6
   b. $\frac{2}{3}$ f. 9 j. 7 n. 2 r. 7
   c. 6 g. 2 k. 6 o. 6
   d. 12 h. 3 l. 5 p. 4
2. Answers and drawings may vary. Students might divide each hexagon into thirds and then count the number of thirds in $2\frac{1}{3}$ hexagons.

LESSON 25 page 30

FOCUS: Review operations with decimals

MATERIALS: Tiles

DIRECTIONS: This lesson is a review of the many concepts presented in this book. As you discuss this lesson with the students, stress the power of thinking mathematically to solve the equations. This type of thinking will serve them well in algebra and future mathematical explorations. Problem 4 asks the students to write related multiplication and division sentences from the equations given. Their work will provide invaluable insight into their thinking and should be so noted in any math portfolio that is maintained. The thinking involved here is an extension of the ideas with whole numbers. If students know that 2 + 2 = 4, then they can write it as 2 × 2 = 4. They can also reason that 4 ÷ 2 = 2. All these equations are related, and all students have to know is the simple addition fact.

RESPONSES:

1. $\frac{1}{10}$ or 0.1
2. Students label each part $\frac{1}{10}$ and 0.1.
3. a. 0.6, 0.6, 3 c. 0.9, 0.9, 3
   b. 0.6, 0.6, 2 d. 1, 1, 2
4. Dividends in related division sentences may vary.
   a. 0.4, 2 × 0.2 = 0.4, 0.4 ÷ 0.2 = 2
   b. 0.8, 2 × 0.4 = 0.8, 0.8 ÷ 0.2 = 4
   c. 0.4, 4 × 0.1 = 0.4, 0.4 ÷ 0.1 = 4
   d. 1.5, 3 × 0.5 = 1.5, 1.5 ÷ 0.5 = 3

FINAL QUIZ pages 31 and 32

RESPONSES:

1. a. 0.8 g. 2 m. 1
   b. $\frac{8}{10}$, or $\frac{4}{5}$ h. 4 n. 4
   c. 4 i. 3 o. 0.9
   d. 1 j. 6 p. 1.1
   e. 1 k. 4 q. $\frac{9}{10}$
   f. 0.4 l. 12 r. 4
2. 48; Drawings will vary.
3. Explanations will vary. Sample explanation: There are 24 hours in a day. There are 2 half-hours in 1 hour, so 24 × 2 = 48 half-hours in 1 day.
4. Chet hiked $5\frac{1}{2}$ miles. Drawings will vary.
5. Explanations will vary. Sample explanation: Ten markers would be 5 miles because 5 is half of 10. Since Chet hiked 1 more marker than 10, he hiked 5 miles + $\frac{1}{2}$ mile = $5\frac{1}{2}$ miles.

SCOPE AND SEQUENCE

CONCEPTS	Book 1	2	3	4	5	6	7	8
Meaning of fractions								
Concrete and pictorial models	•	•	•	•	•	•	•	•
Compare and order	•	•	•	•	•	•	•	•
Equivalent fractions			•	•	•	•	•	•
Part of a whole/part of a set	•	•	•	•	•	•	•	•
Number line	•	•	•	•	•	•	•	•
Lowest-term fractions					•	•	•	•
Greatest common factor					•	•	•	•
Least common multiple/denominator					•	•	•	•
Reciprocals							•	•
Related to measurement				•	•	•	•	•
Meaning of mixed numbers								
Number line				•	•	•	•	•
Improper fractions				•	•	•	•	•
Compare and order				•	•	•	•	•
Meaning of decimals								
Place value					•	•	•	•
Compare and order					•	•	•	•
Relationship to money				•	•	•	•	•
Convert decimals, fractions			•	•	•	•	•	•
Convert decimals, percents, fractions							•	•
Terminating and repeating decimals							•	•
Non-repeating decimals								•
COMPUTATION								
Fractions								
Estimation			•	•	•	•	•	•
Addition/same denominators		•	•	•	•	•	•	•
Addition/different denominators					•	•	•	•
Subtraction/same denominators			•	•	•	•	•	•
Subtraction/different denominators					•	•	•	•
Multiplication/whole numbers	•	•	•	•	•	•	•	•
Multiplication/fractions					•	•	•	•
Multiplication/mixed numbers					•	•	•	•
Division/whole numbers				•	•	•	•	•
Division/fractions				•	•	•	•	•
Decimals								
Estimation		•	•	•	•	•	•	•
Addition				•	•	•	•	•
Subtraction				•	•	•	•	•
Multiplication			•	•	•	•	•	•
Division				•	•	•	•	•
Mental math & number sense	•	•	•	•	•	•	•	•
Problem solving	•	•	•	•	•	•	•	•

Reorder No. CA7620.9—Single
CURRICULUM ASSOCIATES®, Inc.
North Billerica, MA 01862
Phone: 800 225-0248 (U.S. & Canada)
Fax: 800 366-1158 (U.S. & Canada)
E-mail: cainfo@curriculumassociates.com
Web: www.curriculumassociates.com