



Loose ends

Mike Mudge bids farewell to *PCW* and brings his column to as tidy a close as possible.

After fourteen and a half years, *Numbers Count* has come to an end. To conclude in the tidiest possible manner, I will cover a selection of responses to recent columns. All responses to last month's problem will be welcome, although no feedback will appear in *PCW*.

■ August 1997

Despite the errors — (1) the top row of high numbers contains only 25, 50, 75 and 100 (no 10); and (2) the target is an integer in the inclusive range 100 - 999 — a number of working programs have been received, including: a Borland C++ 3.5 version which works in Windows, from John Blyte; and a DOS program written in C, by Graem Yeandle, which is running on a 133MHz Pentium in under 2 seconds and has "found answers missed by Carol Vorderman".

■ July 1997

This was the most popular column, ever. Why? A variety of programs containing only two, and even one, semi-colon have been received. However, the expression of primes in the form $x^2 + ny^2$, $n = 1(1)$ max prompted Gareth Suggett to discover that 1201 is not the smallest. 1009 and 1129 also have the same property, although these could be seen as imperfect because for one of the values of n , $y = n$, this does not happen for 1201. Reference: DA Cox, *Primes of the form $x^2 + dy^2$* (Wiley, 1989 — now out of print) and Beiler's *Recreations in the Theory of Numbers* (chapter XXIII).

■ June 1997

This brought to light a research group at Helsinki University of Technology, PO Box 1100, FIN-02015 HUT, Finland (email Kari Nurmela at kjnu@vipunen.hut.fi or Patric.Ostergard@hut.fi) which has papers pending in *Discrete Mathematics* and

Discrete & Computational Geometry. See also, Kari J Nurmela and Patric R J Ostergard *Graph Theory, Combinatorics, Algorithms, and Applications 1996*, entitled *Optimal Packings of Equal Circles in a Square*.

A much simpler and experimental approach is being developed by Bruce Halsey using a 100MHz 486 with Turbo Pascal 7.0 and "several Pentium 133s...for some of the number crunching".

■ May 1997

So far, this has been unpopular. I would nevertheless welcome correspondence from any reader who has attempted to experiment in this area and perhaps decided that the results (or lack of) did not warrant a submission. Remember that a number of readers have succeeded in publishing their results in one of the learned journals, so please respond if you have anything at all in these areas, particularly relating to non-amorphic numbers.

■ April 1997

George Sassoon investigated the number of non-occurrences in a JAMS sequence and found 95 between 40×10^9 and 50×10^9 all of which are even. He was continuing the analysis up to 10^{10} . Mike Bennett, as reported in the July issue, solved $X(n) = 876$ to obtain $n = 34732165539$ in 2hrs 11mins 3secs on an Acorn RISC PC with a StrongARM processor.

Nigel Backhouse obtained confirmation of this in about four and a half days in UBASIC on a Pentium 133. What does the comparison of these times tell us about the hardware, the software and the individuals?

■ March 1997

This set of problems produced an investigation by Eddie Clough using a

freeware version of "J" on a 486DX40 (8Mb RAM). He found speed to prove a problem when identifying large numbers of large primes. How large is large? The proof that $1^{**2} + 2^{**2} + 3^{**2} + \dots + n^{**2} = N^{**2}$ has only two solutions (namely 2 and 24) and is provided in an accessible form by WS Anglin, *The Square Pyramid Puzzle*, *American Mathematical Monthly* (vol 97, pp120-124, Feb 1990). Copies on request.

■ February 1997

As for the general Smarandache-type mathematics, I recommend the recent publication *Surfing on the Ocean of Numbers — A Few Smarandache Notions and Similar Topics* by Henry Ibstedt (Ehru University Press, 1997, ISBN 1-879585-57-x). The author made a substantial response to the problems $S(n) = 1(1)7$. The prizewinner is Nigel Hodges.

■ Finale

Please submit requests for further details of solutions referred to or hinted at above, to me at the address below. I am seeking another vehicle to encourage empirical number theory research and any suggestions for reviving the concept of *Numbers Count* outside *PCW* would be welcome. One idea would be a Numbers Count Club, whereby a monthly newsletter would be posted, faxed or emailed to members. Does this have any support?

Thank you, to those readers who have communicated with me via this column over the years. Here's hoping this is *au revoir* and not goodbye.

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What comes next?

Mike Mudge reads from the Book of Numbers and throws down a challenge to readers.

The LOOK and SAY Sequence appears in *The Book of Numbers* by John H Conway (of *The Life Game* fame) and Richard K Guy, ISBN 0-387-97993-X, Springer-Verlag, 1996.

Consider the sequence

1, 11, 21, 1211, 111221, 312211,
13112221, 1113213211,
31131211131221...

Have you guessed the general rule? The first term is one "one" so the second is "one one". This consists of two "ones" so say the third is "two one". This in turn is seen as one "two" and one "one" and so say for the next term "one two one one", and so on.

The question is, how many digits are there in the n^{th} of this sequence? Now using those epic words, it can be shown that the number of digits in the n^{th} term is roughly proportional to $(1.3035772\dots)n$, where Conway and Guy quote this constant to 49 decimal places followed by the 71st degree polynomial equation with integer coefficients less than 20, of which it is a root. I shall be amazed if any PCW readers can reproduce this result, but the associated Numbers Count problem is as follows.

L&S Sequence

Design and implement an algorithm to generate consecutive terms of the LOOK and SAY Sequence up to the available system limitation; fit x^n to the length of the n^{th} term by any empirical method. Is the resulting x anything like the above constant?

Introducing the Pseudo-Smarandache Function

This concept is due to Kenichiro Kashihara (private communication).

As a first step, readers are asked to recall the definition of the Smarandache Function $S(n)$, as the smallest integer m such that n evenly divides $m!$ (factorial $m = m! =$

$1 \cdot 2 \cdot 3 \dots n$), for any integer n greater than or equal to 1. The Pseudo-Smarandache Function has a similar definition where the multiplication in the definition of the factorial function is replaced by addition — it is denoted by $Z(n)$.

n	1	5	10	15	20	25	30	35
Z(n)	1	4	4	5	15	24	15	14

Problem: Z(n)

(a) Find all values of n such that:

(a) $Z(n) = z(n+1)$

(b) $Z(n)$ divides $Z(n+1)$

(c) $Z(n+1)$ divides $Z(n)$.

(b) Is there an algorithm that can solve:

(a) $Z(n) + Z(n+1) = Z(n+2)$

(b) $Z(n) = Z(n+1) + Z(n+2)$

(c) $Z(n) * Z(n+1) = Z(n+2)$

(d) $Z(n) = Z(n+1) * Z(n+2)$

(e) $2 * Z(n+1) = Z(n) + Z(n+2)$

(f) $Z(n+1) * Z(n+1) = Z(n) - Z(n+2)$

(c) For a given natural number M how many solutions are there to $Z(x) = m$?

(d) Are there any instances where FOUR CONSECUTIVE POSITIVE integers yield a monotonic (increasing or decreasing) sequence of Z -values?

Harmonic Numbers

The n^{th} harmonic number H_n is defined as the sum of the first n terms of the harmonic series, thus $H_n = 1 + 1/2 + 1/3 + 1/4 + 1/5 \dots + 1/n$.

Problem Harmonic

Design and implement an algorithm for generating harmonic numbers. Use it to discuss that: "the n^{th} harmonic number is about one n^{th} of the n^{th} prime number."

Investigations of the above problems may be sent to Mike Mudge — the address is in the box alongside: a prize to the best entry received by 1 December 1997. Please include an SAE if you want your entry back.

Review of Numbers Count 164, December 1996, Close Relations

(Correction — Page 294, column 3, line -1: *Scientific American* should, in fact, read *American Scientist*.)

Jim Parker jim@jebp.demon.co.uk

(address on request) has sent a late submission, 2 June 1997, advising of a program operating in a particular form of arithmetic which provides the full results of the snub dodecahedron. Some knowledge of abstract algebra (field theory) is needed to fully appreciate this work.

John Sharp saw recurrence relation $T_n = 2T_{n-1} - T_{n-4}$ associated with $t^4 = 2t^3 + 1$ (number E) yields: 0, 1, 1, 1, 2, 3, 5, 9, 16, 29, ... where ratio of successive terms converges, very slowly, to the Tribonacci Number.

Duncan Moore, Nigel Hodges and others found simple algebraic functions for A through I and partial results for the snub-dodecahedron, e.g.

$$I = (t(T+2))^{**1/2},$$

$$G = (2t+3)^{**1/2}$$

Nigel succeeded in proving, in relation to problem SL, that: $T(2) = 128$, $T(\text{cubes}) = 12758$, $T(\text{fourth powers}) = 5134240$, $T(\text{fifth powers}) = 67898771$, while $T(\text{sixth powers})$ greater than 500 million while $T(\text{triangular numbers}) = 33$.

After much heart-searching, the very worthy winner is Paul Richter of Flat 2, 21 Queens Road, Tunbridge Wells, Kent TN4 9LL, for a non-sophisticated approach to this investigation. Details can be obtained from Mike Mudge, or John Sharp at The Glebe, Watford WD2 6LR.

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A Countdown conundrum

Carol Vorderman replaced by a machine? The very idea. But here, Daniel Norris-Jones and Julian Sweeting dare to ponder, as Mike Mudge sorts the vowels from the consonants.

This month's project is proposed by Daniel Norris-Jones and Julian Sweeting of Wheldrake, Yorkshire (Dan@akqa.com).

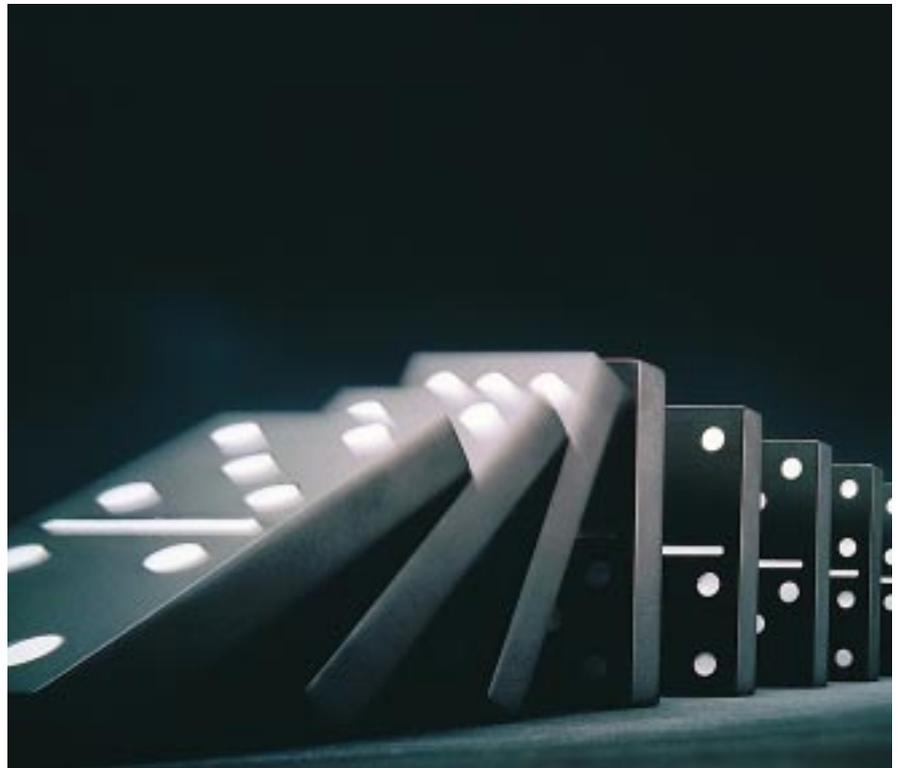
Carol Vorderman or machine?

Those of you who have watched the TV show, Countdown, will know that Carol Vorderman is not infallible when it comes to the numbers game. Now and then she is unable to solve the problem and this raises the question: "Is it *always* possible?"

The Countdown numbers game requires the contestants to pick six cards. Each card has a number on the back. The cards are arranged into four rows. The top row contains the numbers 10, 25, 50, 75 and 100. The other three rows have cards from one to ten.

A random number generator then creates a target number (an integer between 0 and 999) and the contestants must then use the six numbers and the four operators (+, -, x, ÷) to create a number as close to the target as possible. The contestants have 30 seconds in which to achieve this target, using only pencil, paper and mental power. Carol, on the other hand, gets a little longer because the contestants prove their solutions first.

There are two ways to approach this problem: intelligence, or brute force and ignorance. The Artificial Intelligence solution may only be as good as Carol Vorderman, and until the ADI Dynamic Link Library is available it would prove difficult to implement. So this results in the solution at which computers are best. Try every possible combination of numbers and operators and then you will know whether it is possible to achieve the target.



As a schoolboy, Julian Sweeting attempted this on an Atari 8-bit home computer. Naturally he suffered from low computing power and limited knowledge. He did, however, identify the problem of parenthesis which complicated the number of permutations and combinations of operators and numbers. Initial estimates of the number of potential calculations were in the order of tens of millions, far beyond the home computing power of the eighties.

Five years on, while learning to program LISP, Sweeting came across Reverse Polish Notation and recognised that it removed the need to consider parenthesis. Obviously this was the way to tackle the

problem. Some time later, during a road trip around America, he happened to discuss the problem with his fellow traveller, programmer Daniel Norris-Jones. Their appetite for solving this problem was whetted and the project sparked into life. Travelling to LA from Las Vegas, the two applied the limited processing power they had available (two Psion 3a organisers) to parts of the problem.

The essential aspects of the problem are as follows: there are six numbers and so there are $6! = 6 * 5 * 4 * 3 * 2 * 1 = 720$ possible ways of ordering the numbers, i.e.

1 -	1, 2, 3, 10, 25, 50
2 -	2, 1, 3, 10, 25, 50

```
3-      2,3,1,10,25,50
      . . . . .
719-    . . .
720-    50,25,10,3,2,1
```

Ignoring parenthesis, an operator may be placed between each number pair. Hence five operators, each of which can take four values, 4 to power 5 or $4 * 4 * 4 * 4 * 4 = 1,024$. That is:

```
1      +++++
2      +++++-
3      +++++*
.      . . . .
1,023  *////
1,024  /////
```

The Psion Organiser was used to attempt simple problems not requiring parenthesis. Because limited battery life made it possible to calculate only a few tens of numbers per second, the 700,000 was out of the question.

The road trips continued and in the desert near Roswell, Sweeting and Norris-Jones had their first inclination of whether the problem could be solved within 30 seconds. They required a compiled language (100 times faster than the interpreted OPL from Psion) run on a fast computer, perhaps a few hundred times faster than a Psion. The parenthesis problem proved simple. Reverse Polish Notation showed there are only ten ways to arrange the operators and operand, which brings the total to approximately seven million numbers to calculate in 30 seconds.

The two programmers arrived in Albany, New York, where they had access to some "real" computers (IBM RISC 6000 workstations). Not all solutions to the numbers game require all six numbers to be used. It is therefore necessary to

check intermediate results to determine whether you have the answer. Successive calculations differ only slightly, so only the change requires calculation. This allows intermediate results to throw a helping hand to the floundering processor. For example, given the numbers 1, 2, 3, 10, 25, 50, the calculations may be done as follows:

```
check 50 + 25 + 10 + 3 + 2 - 1
check 50 + 25 + 10 + 3 + 1 - 2
                        10 calculations
```

As can be seen, the calculations are reduced if the intermediate result is stored.

```
i = 50 + 25 + 10 + 3
check i
check i + 2 - 1
check i + 1 - 2
                        7 calculations
```

This may be applied simply in the nesting of the code and reduces the number of calculations per combination from five to between two and five.

At this stage, the solution was within reach. The code had been rewritten in C and was ready to go. The interpreter of the Psion had flagged overflows and these were dealt with easily. However, the Unix C compiler was not so accommodating. Overflows could go unnoticed and hence reproduce spurious results. Sweeting and Norris-Jones had come across a question which must have been asked by every serious programmer: "How do I detect integer overflow?"

Fortunately, at 3am that night they found Marcus, a diehard programmer, in an Albany bar. When the whole Countdown problem was explained to him, he suggested they use assembler. They claimed they required "machine independence" (the best excuse when you want to avoid using assembler, which nobody really does). Marcus gave an answer that was both robust and fast at instruction level: two single precision integers (except zero) when

operated on with +, -, * or / cannot be larger than a double precision integer. So use single precision integers throughout and if the answer requires any of the bits of double precision, the operation has overflowed. The code was complete.

The program executed and found solutions within five seconds. When given a problem that was impossible to calculate, the solution required inspection of all the seven million combinations. In these cases the RISC 6000 completed the job in 25 seconds: a complete success.

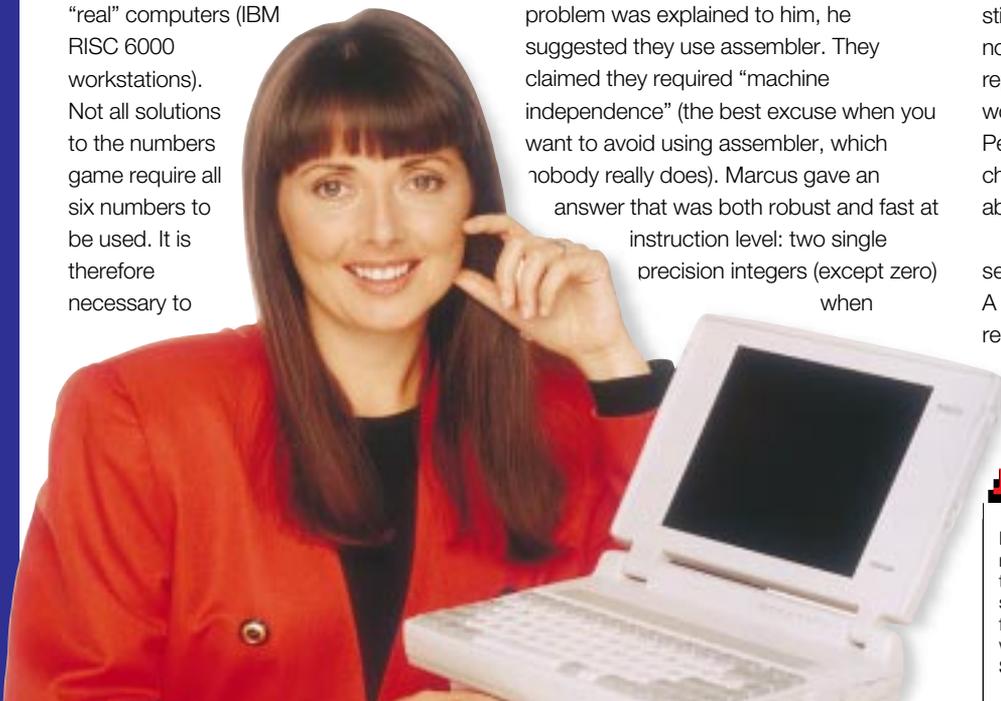
The performance of the program was of interest and a small routine was created to simulate the picking of numbers. A reasonable sample would be required to analyse the performance properly, but at up to 25 seconds a game, such a sample would require a few hours of runtime. Unfortunately, as these machines were for university use, a different kind of access was required. Fortunately access was available, but not in the physical sense. The code was sent to a machine back in LA using ftp, and this machine was remotely set up to execute the program for 3,678 seconds every night from midnight. Generally, Unix machines are on day and night, so exclusive processor use could almost be guaranteed at that time. The NY workstations were set up for remote viewing of the LA machine's processor occupancy. The scrolling bar chart changed from a thin line to a solid black rectangle. The processor was running flat out.

That was October 1995. The program is still running with a log file of a few megabytes: nobody likes an idle computer. We can now replace Carol Vorderman if we wish. A Unix workstation does the trick, but nowadays a Pentium should be sufficient and a pretty cheap replacement. The next task? How about trying to replace Richard Whiteley?

Investigations of this problem should be sent to Mike Mudge at the address below. A prize will be awarded to the best entry received by 1st November. (SAE for return of entries, please.)

PCW Contact

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Detector work

A prime number detector requiring only semi-colons? Pretty tight code, you may think. It lends Mike Mudge inspiration to set another poser for investigative readers.

Once upon a time, when Jonathan Cochrane started “messing about” with prime numbers, the first thing he did was to write a function to test whether a number is prime or not. It was easy enough, basically a function of the form:

```
int prime (int x)
{
algorithm;
return PRIME or NOT_PRIME
}
```

While getting the routine working and thinking of what to do next, he decided to try to optimise the prime number routine as much as possible (*haven't we all made this decision? — MM*) and he came up with a prime number detector that requires only semi-colons: pretty tight code, he thought! Can any readers implement a prime number detector satisfying the following specifications?

1. Use any amount of C code you want, but only two semi-colons are to be present.
2. Only allowed to pass one variable to the function, that is, the number to be tested.
3. No pointers are allowed.
4. The function must return a 1 or a 0

depending on prime or composite.

5. Semi-colons within the brackets of a for loop do not count, i.e. for

```
(x=2.3;x<99;x++)
```

^-----^----- don't count.

6. The following style is also excluded, define semi_colon; Jonathan claims to have tried this on a number of colleagues without finding any solutions (other than his own!).

A different style of investigation, the responses from Numbers Count readers, will be examined with interest. Perhaps other code-based optimisation criteria might be applied?

An exercise in change of number base

Mr P Cowen of Middlesbrough has extended the recent result of JJ Clessa, viz. to find a number using the digits 1 to 9 once each only, such that the leading N digits of the number be divisible by N - to different number bases.

His first observation, that the number base must be even (why?) was followed by the use of a Pentium Pro 200 with 64Mb ROM “*which constipated with hard disk over-use at base 34,*” he tells us, but found results for bases 2, 4, 6, 8, 10 and 14. Can any readers extend this investigation, and, if



possible, find an underlying theory which can be used to dramatically reduce the amount of computation needed to discover such numbers?

Any investigations of the above problems may be sent to me at the address below (see "PCW Contacts"), to arrive by 1st October 1997. All material received will be judged using suitable subjective criteria and a prize will be awarded by PCW to the best entry arriving by the closing date. (SAE for return of entry if required, please.) Each contribution should contain brief descriptions of the hardware and software used, together with coding, run times and a summary of the results obtained. General comments on the topics, with references to published or unpublished work in these general areas, would be appreciated.

JAMS: a result

Further to my column which dealt with the subject of JAMS (*Numbers Count*, April) the result $X(34732165539) = 876$ has been reported by both Mike Bennet (2hr, 11min, 3 sec on an Acorn Risc PC with a StrongARM processor) and by Nigel Backhouse (4½ days on a Pentium 133). So, do not become despondent at the lack of output from this investigation!

Appeals for reference material

Alexander Slack at 106431.2710@compuserve.com would like an *elementary* introduction to Mandelbrot Sets and wonders if there is any software available in QBASIC? Help for a 14-year-old embryo computer scientist would be appreciated.

Perhaps this is not quite in the spirit of Numbers Count, but the author would be interested to receive references to the problem of Tessellations in two dimensions. These need not involve any aspects of computing, although this is clearly a subject where computer graphics skills can be exploited both before and after the underlying maths has been understood.

Close relations

Going back to Numbers Count, December '96, John Sharp observes that the recurrence relation $T_n = 2T_{n-1} - T_{n-4}$ associated with $t^4 = 2t^3 + 1$ (number E) yields the sequence: 0, 1, 1, 1, 2, 3, 5, 9, 16, 29, ... for which the ratio of successive terms converges, albeit very slowly, to the Tribonacci Number. Duncan Moore, Nigel Hodges and others found simple algebraic functions for A through I and partial results

for the snubdodecahedron, e.g.

$$I = (t(t+2))^{**1/2}, G = (2t+3)^{**1/2}$$

NH also proved, in relation to Problem SL, that: $T(2)=128$, $T(\text{cubes})=12758$, $T(\text{fourth powers})=5134240$, $T(\text{fifth powers})=67898771$, while $T(6\text{th powers})$ greater than 500 million and $T(\text{triangular numbers}) = 33$.

The worthy prizewinner, however, is Paul Richter of Tunbridge Wells, for a non-sophisticated approach to this investigation. Details from John Sharp at 20 The Glebe, Watford WD2 6LR (or from me).

Going back to your roots

This item in the November '96 column proved to be very popular. The Problem Function lead to a great deal of analysis. Ultimately, Nigel Hodges printed out the two roots to 700 places of decimals showing them to differ in the 647th place. Other analyses included using a program called "Mercury" on a 486DX by Martin Sewell. Duncan Gray refers to p3 of the Excel workbook, *Solutions*. James Lea cites *Numerical Recipes in C* (2nd edition), so this section is very well known.

RF Tindall has been aware of a very fast converging method of approximating to square roots, which is exactly equivalent to the algorithm given, for some time. But he observes that if N is at all large, there are difficulties finding the initial solution.

The worthy prizewinner is Matthew Davies of Luton, who offers an error estimate for the iteration scheme, a generalisation to rational rather than integers, a list of (m_0, n_0) seeds generated using a Turbo Pascal version 6.0 program in the range (1,100). And there's a concluding observation that "If this technique were to be used as the basis of root calculations on something like an embedded system, I'd be inclined to compile a look-up table of $N \dots (m_0, n_0)$ pairs rather than determine them on-the-fly."

■ *Correction: Dec '96 issue, p294, col.3 — for "Scientific American" read "American Scientist".*

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Mystery maths

Mike Mudge consults the book *Unsolved Problems* for his teasers this month, including distances and square numbers and circles. Plus, roll up for a number theory conference.

This is inspired by *Unsolved Problems in Geometry* by HT Croft, KJ Falconer and RK Guy, ISBN-0-387-97506-3, Springer Verlag 1991.

Question A. What is the maximum diameter of n equal circles that can be packed into a unit square? How should n points be arranged in a unit square so the minimum distance between them is greatest? These problems are equivalent: if a collection of points in a unit square are at a distance of at least d from each other, the points can serve as the centres of a collection of circles of diameter d that will pack into a square of side $1 + d$.

Consider the second version of this problem and denote by d_n the greatest minimum distance between n points in a unit square. Exact results are known for n less than or equal to 9; also for $n = 14, 16, 25$ and 36. For n lying between 2 and 5 these are "easy" to obtain. Graham has established the result for $n = 6$, the results for $n = 7, 8$ & 9 are due to Schaer and Meir, those for 14, 16 & 25 & 36 are attributed to Wengerodt & Kirchner. Examples of both exact results and conjectural bounds are

points that is denser than the square lattice packing, but he conjectures that for 49, the square lattice packing is best.

Are there any values of n such that $d_n = d_{n+1}$? The problem can be asked for packing an equilateral triangle. Oler has shown that if n is a triangular number, of the form $m(m+1)/2$, the obvious configuration is the extremal one. The natural question is, can one do better if n is 1 less than a triangular number? Note that n spheres have been packed into a cube and certain other polyhedra, but even for a cube, exact results are only known for n less than eleven. A great deal of work is still to be done in this area!

Question B. Spreading points in a circle

The analog of the previous problem for the circle can be posed in a few equivalent ways:

1. What is the maximum radius of a disk, n copies of which can be packed into a circle of radius 1?
2. What is the radius of the smallest circle into which n unit disks can be packed?
3. What is the radius of the smallest circle containing n points, no pair of these points being a distance of less than 1 apart?

least distance is $2 \cdot \sin(\pi/n)$ and for $n = 7, 8$ & 9 the least distance is $2 \cdot \sin(\pi/(n-1))$ with the obvious configurations. This is straightforward for n less than eight and was proved by Pirl for $n = 8$ & 9. He also solved the case of $n = 10$ and conjectured the values for n less than 20.

Suggested approaches to the problems include randomly generated points with analysis of large samples to estimate bounds, and the use of computer graphics to draw the optimum configurations, in the cases where these are known, and to examine and conjecture solutions for higher n .

Any investigations of the above problems may be sent to Mike Mudge at 22 Gors Fach, Pwll-Trap, St Clears, Carmarthenshire SA33 4AQ, tel 01994 231121, to arrive by 1st September 1997. All material received will be judged using suitable subjective criteria and a prize will be awarded by PCW to the best entry arriving by the closing date (SAE for return of entries, please). Each contribution should contain details of run times and a summary of the results obtained.

Comments on the topics would be appreciated. The topics included here range from tiling and dissection through packing and covering to include nets

of polyhedra and lattice point problems.

■ *Details of the winner of November 1996 Numbers Count will appear next month.*

Fig 1

n	2	3	4	7	10*	13*	17*	26*
d_n	$2^{1/2}$	$6^{1/2} - 2^{1/2}$	1	$2(2 - 3^{1/2})$	0.421	0.366	0.306	0.239

*Indicates a conjecture as far as the writer is aware.

given in Fig 1 (above).

Up to which square number is the square lattice packing the best? Certainly for up to 36. Wengerodt has found a packing of 64

Conference on Smarandache-type Notions in Number Theory

21st-24th August 1997, Craiova, Romania. Bringing together those interested in Smarandache-type functions, sequences, algorithms, operations, criteria, theorems.

4. How large can the least distance between a pair chosen from n points in the circle be?

The last formulation yields the result that for n between two and six...inclusive...the

Contributed papers or plenary lectures are invited from all areas of Number Theory. For info: Dr C Dumitrescu, Mathematics Dept, University of Craiova, R-1100 Romania. Tel (40) 51-125302. Fax (40) 51-413728. ketyprod@topedge.com, research37@aol.com.

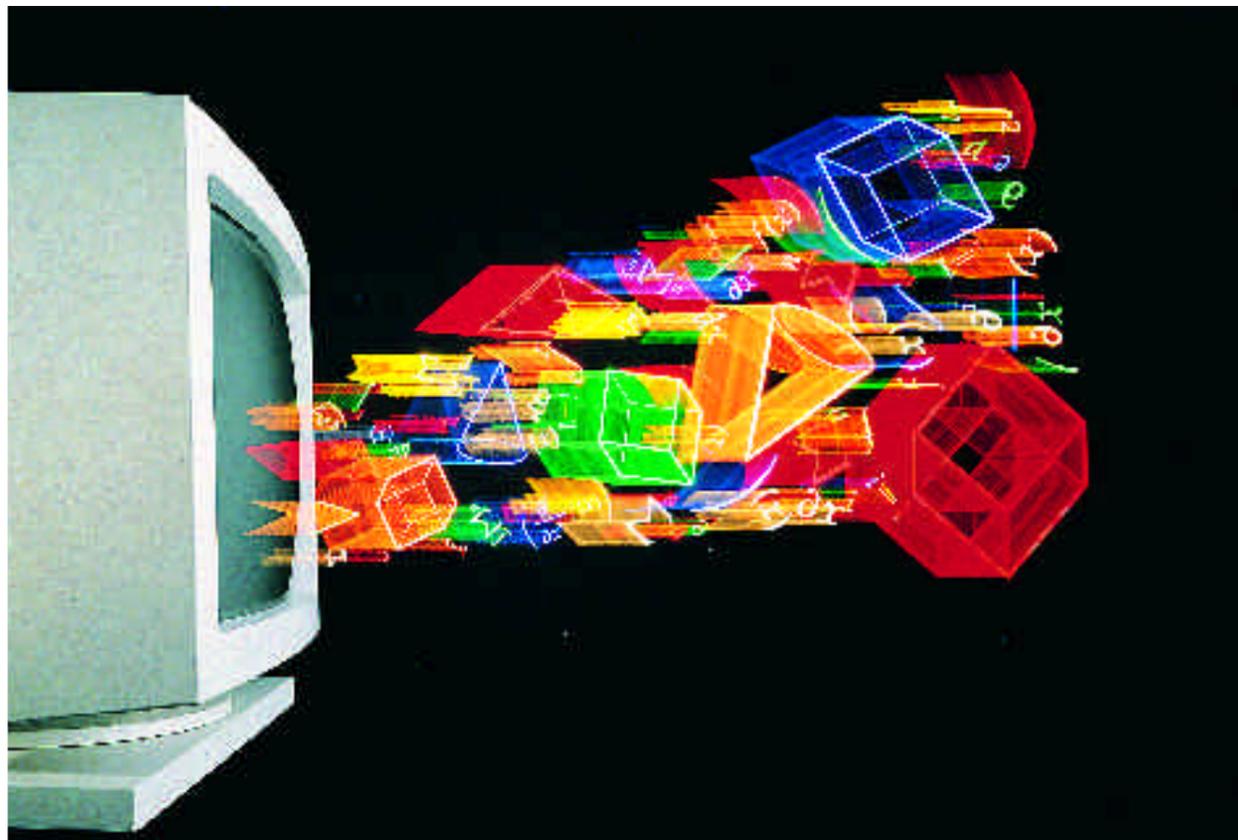
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Morph code

Instead of dots and dashes, Mike Mudge checks his figures to find out whether numbers are nonamorphic or nonagonal. He also wonders why readers have been slow to respond.



Once upon a time... In the *Journal of Recreational Mathematics* Vol. 20(2), 1988, Charles W Trigg, of San Diego, addressed the problem of which primes had the sums of the squares of their digits also prime, e.g. if Prime (P) = 9431, then $9^2 + 4^2 + 3^2 + 1^2 = 107$ (Q) which is also prime. Among the 1229 prime numbers less than 10^4 , Charles found 237 primes with this property... five two-digit, 47 three-digit and 185 four-digit primes. He observed that among the generating primes were the nine palindromes:

11, 101, 131, 191, 313, 353, 373, 797 & 919
 The smallest of these is the sole prime repunit $P = 11$. For further study of repunits see *Repunits and Repetends* by Samuel Yates, Library of Congress Catalog Card Number 82-502451 (Star Publishing Co, Boynton Beach, Florida 33435, in 1982).
 There are also two near repunits, 223 and 8887. Among other structures present are members of the 25 reversal prime pairs such as 3169 and 9613. The smallest numbers of the pairs include

113, 179, 199... 3389, 3583, 7187, 7457, 7949 and 9479.
 There are also some cases where the sums of the digits and the generating prime are equal, e.g. any prime permutation of 1136 giving 47 and 11, a prime permutation of 337, 1741 or 3037 giving 67 and 13, a prime permutation of 119 or 1019 giving 83 and 11. The most complex structure observed by Charles showed ten chains of primes wherein each Q is a P for the next link in the chain, e.g;
 191, 83, 73, :443, 41, 17, :463,

61, 37, :1699, 199, 163 :6599, 223, 17, : 6883, 173, 59, : 467, 101, 2, :883, 137, 59 : 449, 113, 11, 2, : 797, 179, 131, 11, 2, :

Problem CWT
 Extend this analysis to both squares of digits of integers greater than 10^4 , the cubes and higher powers of the digits of such prime numbers... and also address the problem to other "well-known" classes of integers like Fibonacci Numbers, Triangular Numbers, Tetrahedral Numbers etc. There may be underlying structures that deserve attention? Finally on this particular topic, the MM special: how do these results extend to other number bases? (Is there anything particular about base ten, from a number theoretic viewpoint? And if so, why?).

Nonamorphic numbers
 Charles Trigg, the author cited above, introduced this terminology in the *Journal of Recreational Mathematics*, 13:1, pp 48-49 (1980-81). Definition: Nonagonal Numbers have the form $N(n) = n(7n - 5)/2$. A number is said to be nonamorphic if it terminates its nonagonal number.

Clearly, 1 is trivially nonamorphic in any number base. With this exception there are no nonamorphic numbers in bases two, three, four, five, eight and nine. In base ten there are five nonamorphic numbers less than 10^4 , namely

$N(1)=1, N(5)=75, N(25)=2125, N(625)=1365625$ and $N(9376)=307659376$.

In base six there are five nonamorphic numbers less than 10^4 , namely

$N(1)=1, N(4)=114, N(13)=1113, N(213)=253213$ and $N(5344)=302505344$.

Now, in base seven there are 42 such numbers!

Problem CWT nonamorph
 Extend the above statistics to number bases greater than seven, and investigate any structure within these nonamorphs.

Finally, generate further "agonal" with associated "amorphs" and attempt to find an underlying general theory relating to their distributions within a given number base, and in particular the number bases in which non-trivial "amorphs" do not occur.

Can we consider "almost amorphs", where the termination differs from the input number in only one digit (by only one digit in that place)? Are we losing sight of number

theory here and just playing with patterns? An underlying theory would say no.

Send any investigations of the above problems to Mike Mudge (see "PCW Contact", below) to arrive by 1st August, 1997. All material received will be judged using suitable subjective criteria and a prize will be awarded by PCW to the best entry arriving by the closing date (SAE for the return of entries, please). Each contribution should contain brief descriptions of the hardware and coding used, together with run times and a summary of the results obtained, and general comments on the topics. References to published or unpublished work in these areas would be appreciated.

Stop Press
 In the March issue of PCW I requested a proof that $1^2 + 2^2 \dots + n^2 = N^2$ had no solutions other than $n = 1$ and $n = 24$. The reference has been supplied by Robin John Chapman of the University of Exeter to WS Anglin, The Square Pyramid Puzzle, *American Mathematical Monthly* Vol. 97, pp 120-124 (February 1990). Thanks, Robin.

George Sassoon has investigated $x^2 = ny^2 = p$ and has so far (10/2/97) found that the value $p = 316234801$ leads to integer solutions for $n = 1(1)30$. He wonders what percentage of possible n values give solutions and suggests that there is no upper bound on values for p yielding such solution sets? Your comments, please.

Review of "Prime candidate", (Numbers Count 162, Oct '96)
 For reasons totally beyond my comprehension, this did not prove to be a popular hunting ground for PCW readers. The worthy prizewinner is therefore the originator of the problem: Jonathon Ayres, 59 Watson Road, Leeds LS14 6AE.

Are there any readers with at least partial results to Jonathon's questions? If so, please contact him directly. There is also a fourth question to consider: What happens if you use different functions such as the highest Alliot Hailstone function, so that HAHF = highest alliot function ($a \cdot x + b$)?

PCW Contact
 Mike Mudge welcomes correspondence from readers on any subject within the areas of number theory and computational maths, together with suggested subject areas or specific problems for future articles. Email numbers@pcw.vnu.co.uk or write to 22 Gors Fach, Pwll-Trap, St Clears, SA33 4AQ (tel 01994 231121).



Mods and rockers

Mike Mudge JAMS with mod sequences. No, he hasn't joined a retro band; here he presents a stimulating exercise in occurrences to get your feet tapping and your calculators clicking.

JAMS, or Jonathon Ayres Mod Sequences, are believed to have their origins in Leeds in the autumn of 1996. I am indebted to Jonathon for the following presentation of the idea which both he, and I hope readers of this column, will find interesting and stimulating.

Mod sequences

The mod sequence is defined as $X(n) = (2^*X(n-1)+1) \text{ mod } n$

where n starts at 1 and x(0) equals 0. The first few numbers in the mod sequence are 0, 1, 0, 1, 3, 1, 3, 7, 6 and 3.

1. Occurrence of X

When does a number occur in this sequence? The first occurrence of the numbers 0 to 19 in the mod sequence

are shown in Fig 1.

All numbers less than 1,000 occur in this sequence, for n less than 10,000,000, with the exception of 204, 344, 614, 622, 876 and 964. These first occur at:

X(n)= 614,	n= 10629529
X(n)= 204,	n= 15245143
X(n)= 344,	n= 26713415
X(n)= 622	n= 47286732
X(n)= 964	n= 67815823

I have not been able to find the first occurrence of X(n)=876, but if it does occur n is bigger than 75,000,000.

2. Special values of X(n)

- X(n)=0 for n = 1, 3, 79, 35, 431, 1503, 2943, 6059, 6619, 18911 and 54223.
- X(n)=n-1, for n=1, 2, 8, 32, 46, 392, 12230, 155942, 659488, 1025582, 10471228 and 3437088
- X(n)=n/2, for n=2, 78, 234, 430, 1502, 2942, 6058, 6618, 18910 and 54222
- X(n) and n end in the same last four digits for n=34875, 52363, 54975 and four others less than 100,000, and with the last five digits of both the same, the only values of n less than 1,000,000 are n=389103, 469599 and 742955.

3. Distribution of X(n)

- The most common occurring values of X(n) are of the form 2^*p-1 , so that for n less than 1,000,000, the number 63 occurs 47 times.
- The average value of x(n) is about n/4.
- There are no values of n greater than 1 so that X(n)=X(n+1), but for X(n)=X(n+2) this is true for n=6, 7, 12, 13, 24, 25, 174, 175, 2448, 2449, 3072, 3073, 6768 and 6769.
- X(n)+1=X(n+1) is true for the values of n,

Fig 1

First occurrences of the numbers Y, Y=0 to 19, so that X(N)=Y

Y	N	Y	N
0	0	10	149
1	2	11	27
2	53	12	91
3	5	13	18
4	71	14	21
5	26	15	17
6	9	16	43
7	8	17	20
8	19	18	29
9	72	19	50

Fig 2

First values of n so that X(n)+a = X(n+1)

A	X	A	X
1	3	11	151
2	6	12	29
3	55	13	93
4	9	14	64
5	73	15	29823
6	28	16	33
7	63	17	45
8	18	18	42
9	21	19	71
10	74	20	52

n=3, 5, 81, 237, 433, 1505, 2945...

Fig 2 shows the first values of n so that X(n)+a = X(n+1). All values of a, less than 500, occur for n less than 10,000,000 except for 205, 215, 345 and 391.

- For pairs of numbers x and y, y is at most 2x+1. The values of x where y has values other than 2x+1, are x=1,3,6,7,13,14,15, 16,17,18,20,23...

Numbers Count (PCW, September '96) — 'Fraction Action'

■ Gareth Suggett obtained successive length records for the period of the continued fractions of the square roots of the non-square integers up to d=10,000, terminating with d=9,949 having cycle length 217. However, Gareth discovered a program called "CALC", written by KR Matthews of the University of Queensland. The MSDOS version is available from the Mathematics Archives ftp site: <ftp://archives.math.utk.edu/software/msdos/number.theory/krm-calc>. On a 25MHz 386 PC, each of the 10-digit results quoted in the original article can be obtained in about 20 minutes. The final 11-digit result was confirmed on a 133MHz Pentium in 15 minutes, producing a 6.8Mb output file!

John Borland observed that at some time, "continued fractions were a standard topic in higher mathematics". Readers' experiences of instruction in this topic would be most interesting, together

with their personally recommended reference books both for numerical and function approximation theory applications.

This month's prizewinner, however, is Duncan Moore of Birkenhead for his major contribution to "Something Different", spread over August 1993 and January 1997. The total number of solutions now known is 30.

Also in relation to this problem, Henry lbstedt reported (November '96) finding one with three of p, q, r, s, t sharing one factor and the other two sharing a different factor. This solution is p=286, q=154 sharing the factor 2, and r=s=t=11 sharing the factor 11 with (2, 11) = 1.

Henry points out that p and q also share the factor 11 but that this was not excluded from the question — there is still a great deal of work to be done before this problem is fully understood.

Questions

- Do all numbers occur in this sequence, and also, do they occur an infinite number of times?
- Is there always a value of n, for every a (positive or negative) so that X(n)+a = X(n+1)?
- Is there a way of predicting when a number will occur in the sequence?

■ Is there a formula which gives the nth value of the sequence, without calculating the rest of the series?

■ What happens for other sequences, such as $x(n)=ax(n-1)+b \text{ mod } n$ or $x(n)=(x(n-1)+x(n-2)) \text{ mod } n$?

Something different

This item was taken from *Computer Weekly*

(19th January edition, 1989).

Following up on the observation that $15226_{10} = 62251_7$ and further that $99481_{10} = 18499_{16}$ (where the subscript denotes the base in which the number is represented), find the lowest five-digit number (in any base). Generalise this process to n-digit integers.

Answering back...

Please send any investigations of the above problems to Mike Mudge at 22 Gors Fach, Pwll-Trap, St Clears, Carmarthenshire, SA33 4AQ (tel 01994 231121), to arrive by 1st July, 1997. All material received will be judged according to suitable criteria and a prize will be awarded by PCW to the best entry arriving by the closing date (an SAE is required for the return of entries). Each contribution should contain brief descriptions of the hardware and coding used, together with run times and a summary of the results obtained.

General comments on the topic of JAMS would be welcome, together with any practical (or unusual) applications of integer arithmetic in number bases other than 2 and 10.

PCW Contact

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Power points

Mike Mudge faces a stiff challenge in proving a solution, and this leads him to considering a number of related problems concerned with the power sums of separate digits.

I was asked (by Cyprian Stockford) for a proof that the only solution to

$$1^2 + 2^2 + \dots + n^2 = N^2$$

is $n = 24$ when $N = 70$, viz. positive integer solution of

$$n(n+1)(2n+1) = 6N$$

is unique as asserted in *The Penguin Book of Curious and Interesting Numbers* (David Wells, 1987) and elsewhere. Being unable to provide such a proof (can any readers help?) my attention was caught by a number of notionally related problems involving the power sums of the separate digits or the partitions of a given positive integer.

■ **1:** 1201 seems to be the smallest prime number which can be represented by the expression $x^2 + ny^2$ for all values of n from 1 to 10. Is this true? What other prime numbers can be so represented, and what happens if the range of values of n is increased to 1 to M for an arbitrary M ?

■ **2:** It is clear that $1233 = 12^2 + 33^2$ while $8833 = 88^2 + 33^2$. Under what circumstances is a given integer equal to the sums of the squares of its partitions into pairs? How does this result extend to the cases of higher powers (i.e. cubes) and also to the cases of partitions into ordered triples, 4-tuples, etc? Does this lead to a sensible problem in number bases other than 10?

■ **3:** $3435 = 3^3 + 4^4 + 3^3 + 5^5$ while it is said that (Wells, p.190) 438579088 is the only other number exhibiting this behaviour when powers of a single digit are considered. Can this result be generalised to pairs, i.e. $abcdef\dots = (ab)^{ab} + (cd)^{cd} + \dots$ or even to triples, etc? What happens in other number bases?

■ **4:** By inspection, $175 = 1^1 + 7^2 + 5^3$; when, in general, does

$$a_1^1 + a_2^2 + a_3^3 + \dots + a_n^n = a_1 a_2 \dots a_n$$

where the right-hand side is understood to

mean the integer so written in any number base? It is more natural to reverse the powers and even to start at zero, thus requiring

$$b_0^0 + b_1^1 + b_2^2 + \dots + b_n^n = b_n b_{n-1} \dots b_2 b_1 b_0$$

The Subfactorial Function is defined as

$$!N = N! (1 - 1/1! + 1/2! - 1/3! + 1/4! \dots (-1)^N/N!)$$

where

$$N! = 1.2.3\dots N \text{ e.g. } !5 = 5! (1 - 1/1! + 1/2! - 1/3! + 1/4! - 1/5!) = 44$$

while $!7 = 1854$. It is stated that 148349 is the only number equal to the sum of the subfactorials of its digits.

■ **5:** Prove this result and attempt to generalise it to other number bases. Try replacing subfactorial by factorial and/or replacing sum by product. Comment on the function obtained from the subfactorial function by introducing only positive signs into the definition.

■ **6:** Regarding the individual digits of an integer: is it possible to get a prime number from any given number by changing one of its digits? The answer is "No". The smallest integer for which this is not possible is 200. Is it possible to get a prime number from any given integer by changing two of its digits? If not, what is the smallest number for which this is not possible?

Investigations of the above problems should be sent to Mike Mudge, 22 Gors Fach, Pwll-Trap, St Clears, SA33 4AQ, by 1st June 1997. All material will be judged using suitable subjective criteria and a prize will be awarded to the best entry arriving by the closing date (SAE for return of entries).

Golomb Rules, OK (PCW, Aug '96)

This problem produced a large and varied response. In the problem P1 seeking a solution greater than 7 to $n! + 1 = N^2$, Alan

Cox extended Kraitchik's lower bound from 1020 to 2500 using MAPLE V release 4 on a Dell 486D DX33 with 8Mb RAM and about 250Mb hard disk, in about six hours.

Problem P2 is solved completely.

Dr John Cohen gave the reference to *Finkelstein & London* in *J. Number Th.* 2 (1970), pp 310-321, together with references to work on $y^2 + k = x^3$ for a large range of k by Josef Gebel. Nigel Backhouse obtained a list of Golomb Rulers up to order 15, the final length being 151 with an example (0, 4, 20, 30, 57, 59, 62, 76, 100, 111, 123, 136, 144, 145, 151).

Gareth Suggett indicates that a group from Duke University have obtained optimum rulers up to 19 marks (*New Algorithms for Golomb Rulers Derivation and Proof of the 19 Mark Ruler*, Dollas, Rankin & McCracken, Nov '95). Gareth speculated on the metric result for measuring all distances in centimetres from 1 to 100 on a metre rule. He refers to The *Dipole* column in *The IEE News* some years ago with the best known solution as 15 marks at 1, 2, 8, 14, 25, 36, 47, 58, 69, 80, 85, 90, 95, 98, 99. Is this minimal and/or unique?

Our prizewinner is RF Trindall, of Cambridge, for his extension to circular Golomb Rulers with $n(n-1) + 1$ points spaced round a circle uniformly and n of them marked to measure every distance from 1 to $n(n-1)$. This was accompanied by analysis of P2 and P3 and some (accepted) criticism of their difficulty... sorry, readers!

PCW Contact

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Not numerology but numeralogy!

There's a world of difference between the o and the a, as Mike Mudge explains.

Numerology is variously defined as the study of numbers as supposed to show future events or the relationship between numbers and the occult. However, the term *numeralogy*, supplied by P Castini of Arizona, USA, is defined (by him) as "Properties of the Numbers": his proposal for a Numbers Count column includes some 37 sequences each with a rule of generation and some associated queries for investigation.

There follows a (random?) sample of these. Others may be included at a later date depending on the popularity of such research areas.

The **PROBLEM CAS. (n)**. is the same in every case, viz. implement a computer algorithm to generate the defined sequence and hence, or otherwise, investigate the associated queries.

S(1). Non-arithmetic Progression. General definition: If m_1 & m_2 are the first two terms of the sequence, then m_k for k greater than 2 is the smaller number such that no 3-term arithmetic progression is in the sequence, i.e. we do not find

$$m_p - m_q = m_q - m_r$$

for distinct p, q & r .

e.g. if

$$m_1 = 1 \text{ \& } m_2 = 2$$

we generate

1, 2, 4, 5, 10, 11, 13, 14, 28, 29, 31, ...

Generalised S(1) Same initial conditions, but no t -term arithmetic progression in the sequence for t greater than 3.

Query How does the density of such a sequence, i.e. the fraction of the integers less than N which it contains, vary with N , (m_1, m_2) & t ?

S(2). Prime-product sequence Here T_n is one greater than the product of the first n primes with the proviso that $T_1=2$.

Sequence begins

2, 7, 31, 211, 2311, 30031, ...

since $2 \times 3 \times 7 \times 11 \times 13 + 1 = 30031$.

Query How many members of this sequence are prime numbers?

S(3). Square-product sequence As S(2)

above with primes replaced by squares, viz.

2, 5, 37, 577, 14401, 518401, ...

since

$$1^2 \times 2^2 \times 3^2 \times 4^2 \times 5^2 \times 6^2 + 1 =$$

518401

Query How many members of this sequence are prime numbers?

Generalised S (3) Replace squares by cubes, fourth powers, etc. and investigate the same query. May also be generalised using the products of the factorial numbers

1, 2, 6, 24, 120, 720, ...

Now let (T_n) be a sequence defined by a property P and screen this sequence, selecting only those terms whose individual digits hold the property P to obtain the S . P -digital subsequence. e.g. the S . square-digital subsequence

0, 1, 4, 9, 49, 100, 144, ...

is obtained from

0, 1, 4, 9, 16, 25, 36, 49, ...

by selecting the terms whose digits are all perfect squares — only 0, 1, 4 & 9 allowed.



Numbers Count, June 1996

"Sequence of events", Descriptive Number Sequences Part (1), *PCW* June 1996, proved very popular. It is intended to review at length the two parts of this topic in the next issue. Suffice it to announce the prizewinner as Jean Flower of The Mathematics Centre, Chichester IHE, Upper Bognor Road, Bognor Regis, West Sussex PO21 1HR, who used Mathematica on a Pentium 120 and (eventually) was able to find all cycles of length less than 17, with a greater than 1 and n greater than 13. All of this was accomplished in about five minutes of processor time and was accompanied by a fascinating alphabetic version of the same problem. Consider the sequence of sentences. "This sentence contains three hundred and seventeen occurrences of the letter 'e'", the next term being a sentence which describes the previous one etc. What about carrying this analysis on a computer?

More to come on this topic.

Similarly for the S. cube-digital subsequence and higher powers.

S (4). Consider the S. prime-digital subsequence

2, 3, 5, 7, 37, 53, 73, . . .

Query Is this sequence infinite?

S (5). The S. odd sequence

1, 13, 135, 1357, 13579, 1357911, . . .

Query How many terms are prime?

S (6). The S. even sequence

2, 24, 246, 2468, 246810, . . .

Query How many terms are the nth powers of a positive integer?

S (7) The S. prime sequence

2, 23, 235, 2357, 235711, . . .

Query How many terms are prime?

For further study of S(4) through (7) see: Sylvester Smith, *Bulletin of Pure and Applied Sciences*, vol. 15. E (no. 1) 1996. pp101-107. A set of conjectures on Smarandache* Sequences.

*All the sequences discussed this month have appeared in print under Smarandache Notions.

For further information on this area of work see *Smarandache Notions Journal*, vol. 7 no. 1-2-3, August 1996. ISSN 1084-2810. Department of Mathematics, University of Craiova, Romania.

Something totally different

Eric Adler has drawn my attention to the approximate sizes of elements in the Mathematica 3.0 Software Package where "Front end etc. 6.0Mb, Kernel etc. 18,5Mb, MathLink Libraries 0.5Mb and Fonts 4.5Mb total 27.5Mb whilst Standard Add-on Packages at 9.0Mb together with The Mathematica Book of 36Mb, Listing of Built-In Functions at 5.5Mb, Standard Add-on Packages occupying 11.0Mb and Additional Documentation of 15.0Mb (the latter four items totalling 66Mb) yield 74.5Mb. The total size of storage (again approximate) is quoted as 96Mb whilst strict addition yields 106.0Mb."

Eric asks: "How do they get that?" and offers ten IBM format 3.5in 1.44Mb floppy disks as first prize, with 40 IBM-format 3.5in 1.44Mb floppy disks with UBASIC as runners-up prizes. Facetious answers such as "They used a Microsoft Calculator" or "They are measuring using Microsoft Drive Space" will not be eligible for the first prize!

Stop press!

Would Duncan Moore please let me have his address as I have some information for him. Sorry, Duncan, for the inefficiency of my filing system!

Following on from the study of "Golomb rulers" in the August 1996 issue of *PCW*, at least one reader has expressed an interest in the "Circular Golomb Ruler". Here, the problem is essentially the same except that the points are spaced around the circumference of a circle and distances measured along the circumference also. Apparently solutions are known for some n (maximum distance to be measured); it is further known that for certain n, no solution is possible. What happens if the distance is measured in a straight line!?

Any investigations of this month's queries may be sent to Mike Mudge, 22 Gors Fach, Pwll-Trap, St Clears, Carmarthenshire SA33 4AQ, tel. 01994 231121, to arrive by 1st May 1997. All material received will be judged using suitable criteria and a prize will be awarded by *PCW* to the best entry (SAE for return of entries, please).

•PCW Contributions Welcome

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