

# 8.1

## Multiplication Properties of Exponents

*What you should learn*

**GOAL 1** Use properties of exponents to multiply exponential expressions.

**GOAL 2** Use powers to model **real-life** problems, such as finding the area of crop irrigation circles in **Example 5**.

*Why you should learn it*

▼ To solve **real-life** problems such as calculating the power generated by a windmill in **Ex. 77**.



### GOAL 1 MULTIPLYING EXPONENTIAL EXPRESSIONS

To multiply two powers that have the same base, you add exponents. Here is an example.

$$a^2 \cdot a^3 = \underbrace{a \cdot a}_{2 \text{ factors}} \cdot \underbrace{a \cdot a \cdot a}_{3 \text{ factors}} = a^5 = a^{2+3}$$

5 factors

To find a power of a power, you multiply exponents. Here is an example.

$$(a^2)^3 = \underbrace{a^2 \cdot a^2 \cdot a^2}_{3 \text{ factors}} = \underbrace{a \cdot a \cdot a \cdot a \cdot a \cdot a}_{6 \text{ factors}} = a^6 = a^{2 \cdot 3}$$

These two rules for exponents and the rule for raising a product to a power are summarized below.

#### CONCEPT SUMMARY

#### MULTIPLICATION PROPERTIES OF EXPONENTS

Let  $a$  and  $b$  be numbers and let  $m$  and  $n$  be positive integers.

##### PRODUCT OF POWERS PROPERTY

To multiply powers having the same base, add the exponents.

$$a^m \cdot a^n = a^{m+n}$$

$$\text{Example: } 3^2 \cdot 3^7 = 3^{2+7} = 3^9$$

##### POWER OF A POWER PROPERTY

To find a power of a power, multiply the exponents.

$$(a^m)^n = a^{m \cdot n}$$

$$\text{Example: } (5^2)^4 = 5^{2 \cdot 4} = 5^8$$

##### POWER OF A PRODUCT PROPERTY

To find a power of a product, find the power of each factor and multiply.

$$(a \cdot b)^m = a^m \cdot b^m$$

$$\text{Example: } (2 \cdot 3)^6 = 2^6 \cdot 3^6$$

### EXAMPLE 1 Using the Product of Powers Property

$$\begin{aligned} \text{a. } 5^3 \cdot 5^6 &= 5^{3+6} \\ &= 5^9 \end{aligned}$$

$$\begin{aligned} \text{b. } x^2 \cdot x^3 \cdot x^4 &= x^{2+3+4} \\ &= x^9 \end{aligned}$$

$$\begin{aligned} \text{c. } 3 \cdot 3^5 &= 3^1 \cdot 3^5 \\ &= 3^{1+5} \\ &= 3^6 \end{aligned}$$

$$\begin{aligned} \text{d. } (-2)(-2)^4 &= (-2)^1 \cdot (-2)^4 \\ &= (-2)^{1+4} \\ &= (-2)^5 \end{aligned}$$

#### STUDENT HELP

##### Look Back

For help with exponential expressions, see page 9.

**STUDENT HELP****HOMEWORK HELP**

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for extra examples.

**EXAMPLE 2** *Using the Power of a Power Property*

$$\begin{aligned}\text{a. } (3^5)^2 &= 3^5 \cdot 2 \\ &= 3^{10}\end{aligned}$$

$$\begin{aligned}\text{b. } (y^2)^4 &= y^{2 \cdot 4} \\ &= y^8\end{aligned}$$

$$\begin{aligned}\text{c. } [(-3)^3]^2 &= (-3)^{3 \cdot 2} \\ &= (-3)^6\end{aligned}$$

$$\begin{aligned}\text{d. } [(a + 1)^2]^5 &= (a + 1)^{2 \cdot 5} \\ &= (a + 1)^{10}\end{aligned}$$

.....

When you use the power of a power property, it is the quantity within the parentheses that is raised to the power *not* the individual terms.

Correct:  $(a + 1)^3 = (a + 1)(a + 1)(a + 1)$  ← 3 factors  
Incorrect:  ~~$(a + 1)^3 = a^3 + 1^3$~~  ← 2 terms

**EXAMPLE 3** *Using the Power of a Product Property*

$$\begin{aligned}\text{a. } (6 \cdot 5)^2 &= 6^2 \cdot 5^2 \\ &= 36 \cdot 25 \\ &= 900\end{aligned}$$

**Raise each factor to a power.**

**Evaluate each power.**

**Multiply.**

$$\begin{aligned}\text{b. } (4yz)^3 &= (4 \cdot y \cdot z)^3 \\ &= 4^3 \cdot y^3 \cdot z^3 \\ &= 64y^3z^3\end{aligned}$$

**Identify factors.**

**Raise each factor to a power.**

**Simplify.**

$$\begin{aligned}\text{c. } (-2w)^2 &= (-2 \cdot w)^2 \\ &= (-2)^2 \cdot w^2 \\ &= 4w^2\end{aligned}$$

**Identify factors.**

**Raise each factor to a power.**

**Simplify.**

$$\begin{aligned}\text{d. } -(2w)^2 &= -(2 \cdot w)^2 \\ &= -(2^2 \cdot w^2) \\ &= -4w^2\end{aligned}$$

**Identify factors.**

**Raise each factor to a power.**

**Simplify.**

**STUDENT HELP****Study Tip**

In parts (c) and (d) of Example 3, notice that the two expressions are different. In  $(-2w)^2$ , the negative sign is part of the base. In  $-(2w)^2$ , the negative sign is not part of the base.

**EXAMPLE 4** *Using All Three Properties*

Simplify  $(4x^2y)^3 \cdot x^5$ .

**SOLUTION**

$$\begin{aligned}(4x^2y)^3 \cdot x^5 &= 4^3 \cdot (x^2)^3 \cdot y^3 \cdot x^5 \\ &= 64 \cdot x^6 \cdot y^3 \cdot x^5 \\ &= 64x^{11}y^3\end{aligned}$$

**Power of a product**

**Power of a power**

**Product of powers**

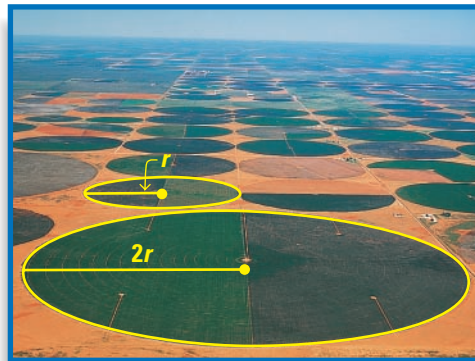
## GOAL 2 USING POWERS IN REAL LIFE



### EXAMPLE 5 Using the Power of a Product Property

Some farmers use *center-pivot irrigation*, which is a sprinkler system that revolves around a center pivot. The large irrigation circles help farmers to conserve water, maximize crop yield, and reduce the cost of pesticides.

- Find the ratio of the area of the larger irrigation circle to the area of the smaller irrigation circle.
- Write a general statement about *doubling* the radius of an irrigation circle.



#### SOLUTION

a. Ratio =  $\frac{\pi(2r)^2}{\pi r^2} = \frac{\pi \cdot 2^2 \cdot r^2}{\pi \cdot r^2} = \frac{\pi \cdot 4 \cdot r^2}{\pi \cdot r^2} = \frac{4}{1}$

- b. Doubling the radius of an irrigation circle makes the area *four* times as large.

### EXAMPLE 6 Using the Product of Powers Property

**PROBABILITY CONNECTION** A true-false test has two parts. There are  $2^{10}$  ways to answer the 10 questions in Part A. There are  $2^{15}$  ways to answer the 15 questions in Part B.

- How many ways are there to answer all 25 questions?
- If you guess each answer, what is the probability you will get them all right?

#### SOLUTION

- a. For each of the  $2^{10}$  ways to answer the questions in Part A, there are  $2^{15}$  ways to answer the questions in Part B. Use the counting principle to find the total number of ways to answer for both parts. The number of ways to answer the 25 questions is the product of  $2^{10}$  and  $2^{15}$ .

$$2^{10} \cdot 2^{15} = 2^{10+15}$$

Use product of powers property.

$$= 2^{25}$$

Add exponents.

$$= 33,554,432$$

Use a calculator.

- The number of ways to answer the 25 questions is 33,554,432.

b. Probability =  $\frac{\text{Ways to get all right}}{\text{Ways to answer}} = \frac{1}{33,554,432}$

- The probability of guessing and getting all answers correct is about 0.00000003.

#### STUDENT HELP

##### Skills Review

For help with the counting principle, see page 788.

# GUIDED PRACTICE

## Vocabulary Check ✓

## Concept Check ✓

## Skill Check ✓

1. In the expression  $a^5$ ,  $a$  is called the    of the expression.
2. How are the expressions  $x^7 \cdot x^3$  and  $(x^7)^3$  different? Explain your answer.
3. Can  $a^3 \cdot b^4$  be simplified? Explain your answer.

Use the product of powers property to simplify the expression.

- |                        |                    |                    |
|------------------------|--------------------|--------------------|
| 4. $c \cdot c \cdot c$ | 5. $m \cdot m^2$   | 6. $2^2 \cdot 2^3$ |
| 7. $3^2 \cdot 3^5$     | 8. $a^4 \cdot a^6$ | 9. $x^4 \cdot x^5$ |

Use the power of a power property to simplify the expression.

- |               |               |               |
|---------------|---------------|---------------|
| 10. $(3)^2$   | 11. $(-2)^2$  | 12. $(2^4)^3$ |
| 13. $(4^3)^3$ | 14. $(y^4)^5$ | 15. $(m^4)^8$ |

Use the power of a product property to simplify the expression.

- |                  |                  |                 |
|------------------|------------------|-----------------|
| 16. $(2m^2)^3$   | 17. $(ab^2)^2$   | 18. $(5x)^2$    |
| 19. $(x^3y^5)^4$ | 20. $(x^3y^8)^5$ | 21. $(-2x^3)^3$ |

# PRACTICE AND APPLICATIONS

## STUDENT HELP

**Extra Practice**  
to help you master  
skills is on p. 804.

**SIMPLIFYING EXPRESSIONS** Simplify, if possible. Write your answer as a power or as a product of powers.

- |   |   |                              |                     |
|---|---|------------------------------|---------------------|
| 22. $3^4 \cdot 3^6$                         | 23. $5^8 \cdot 5^3$                         | 24. $(2^3)^2$                | 25. $(7^4)^2$       |
| 26. $x \cdot x^6$                           | 27. $(3 \cdot 7)^2$                         | 28. $(2x)^2$                 | 29. $(-5a)^3$       |
| 30. $(-2m^4n^6)^2$                          | 31. $[(-4)^2]^3$                            | 32. $[(-5xy)^2]^5$           | 33. $[(5 + x)^3]^6$ |
| 34. $[(2x + 3)^3]^2$                        | 35. $(3b)^3 \cdot b$                        | 36. $5^3 \cdot (5a^4)^2$     |                     |
| 37. $4x \cdot (x \cdot x^3)^2$              | 38. $(-3a)^5 \cdot (4a)^2$                  | 39. $-(3x)^2 \cdot (7x^4)^2$ |                     |
| 40. $2x^3 \cdot (3x)^2$                     | 41. $3y^2 \cdot (2y)^3$                     | 42. $(-ab)(a^2b)^2$          |                     |
| 43. $(-rs)(rs^3)^2$                         | 44. $(-2xy)^3(-x^2)$                        | 45. $(-3cd)^3(-d^2)$         |                     |
| 46. $(5b^2)^3\left(\frac{1}{2}b^3\right)^2$ | 47. $(6a^4)^2\left(\frac{1}{4}a^3\right)^2$ | 48. $(2t)^3(-t^2)$           |                     |
| 49. $(-w^3)(3w^2)^2$                        | 50. $(-y)^3(-y)^4(-y)^5$                    | 51. $(-x)^4(-x)^3(-x)^2$     |                     |
| 52. $(abc^2)^3(a^2b)^2$                     | 53. $-(r^2st^3)^2(s^4t)^3$                  | 54. $(-3xy^2)^3(-2x^2y)^2$   |                     |

## STUDENT HELP

### HOMEWORK HELP

**Example 1:** Exs. 22–62  
**Example 2:** Exs. 22–62  
**Example 3:** Exs. 22–62  
**Example 4:** Exs. 36–62  
**Example 5:** Exs. 77, 78  
**Example 6:** Ex. 79

**EVALUATING EXPRESSIONS** Simplify. Then evaluate the expression when  $a = 1$  and  $b = 2$ .

- |                       |                   |                 |                                     |
|-----------------------|-------------------|-----------------|-------------------------------------|
| 55. $a^2 \cdot a^3$   | 56. $b \cdot b^4$ | 57. $(a^3)^2$   | 58. $(-b)^3 \cdot b^2$              |
| 59. $(a \cdot b^2)^2$ | 60. $(a^2b)^4$    | 61. $-(ab^3)^2$ | 62. $(b^2 \cdot b^3) \cdot (b^2)^4$ |

**WRITING INEQUALITIES** Complete the statement using  $>$  or  $<$ .

63.  $(5 \cdot 6)^4$   $\underline{\hspace{1cm}}$   $5 \cdot 6^4$

64.  $5^2 \cdot 3^6$   $\underline{\hspace{1cm}}$   $(5 \cdot 3)^6$

65.  $(3^6 \cdot 3^{12})$   $\underline{\hspace{1cm}}$   $3^{72}$

66.  $4^2 \cdot 4^8$   $\underline{\hspace{1cm}}$   $4^{16}$

67.  $(7^2)^3$   $\underline{\hspace{1cm}}$   $7^5$

68.  $(6^2 \cdot 3)^3$   $\underline{\hspace{1cm}}$   $6^5 \cdot 3^3$



**EVALUATING POWERS** In Exercises 69–74, simplify the expression. Then use a calculator to evaluate the expression. Round the result to the nearest tenth when appropriate.

69.  $(2.1 \cdot 4.4)^3$

70.  $6.5^3 \cdot 6.5^4$

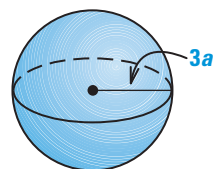
71.  $2.6^4 \cdot 2.6^2$

72.  $(5.0 \cdot 4.9)^2$

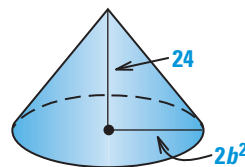
73.  $(3.7^3)^5$

74.  $(8.4^2)^4$

75. **GEOMETRY CONNECTION** The volume of a sphere is given by  $V = \frac{4}{3}\pi r^3$ , where  $r$  is the radius and  $\pi$  is approximately 3.14. What is the volume of the sphere in terms of  $a$ ?



76. **GEOMETRY CONNECTION** The volume of a cone is given by  $V = \frac{1}{3}\pi r^2 h$ , where  $r$  is the radius of the base,  $h$  is the height, and  $\pi \approx 3.14$ . What is the volume of the cone in terms of  $b$ ?



**WINDMILLS** In Exercises 77 and 78, use the following information. The power generated by a windmill can be modeled by the equation  $w = 0.015s^3$ , where  $w$  is the power measured in watts and  $s$  is the wind speed in miles per hour.

77. Find the ratio of the power generated by a windmill when the wind speed is 20 miles per hour to the power generated when the wind speed is 10 miles per hour.
78. **Writing** Write a general statement about how doubling the wind speed affects the amount of power generated by a windmill.
79. **PROBABILITY CONNECTION** Part A of a test has 10 true-false questions. Part B has 10 multiple-choice questions. Each of the multiple-choice questions has 4 possible answers. There are  $2^{10}$  ways to answer the 10 questions in Part A. There are  $4^{10}$  ways to answer the 10 questions in Part B.
- How many ways are there to answer all 20 questions?
  - If you guess the answer to each question, what is the probability that you will get them all right?

**PROBABILITY CONNECTION** In Exercises 80 and 81, suppose you put one red marble, one green marble, and one blue marble in each of six bags. There are  $3^6$  possible orderings of the colors of the marbles you can get when you choose one marble from each bag.

80. If you use 8 bags, there are  $3^8$  possible orderings of colors. What is the probability that the marbles you choose will all be red?
81. If you use 14 bags, how many different orderings of colors are there? What is the probability that the marbles you choose will all be red?

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## Test Preparation



**MULTI-STEP PROBLEM** Use the results of Exercise 82 for Exercise 83.

82. a. Copy and complete the table of values.

$x$	0	1	2	3	4
$2x$	?	?	?	?	?
$2^x$	?	?	?	?	?

b. Sketch the graphs of  $y = 2x$  and  $y = 2^x$  in the same coordinate plane.

c. Compare the graphs. How are they the same? How are they different?

83. **CRITICAL THINKING** You are offered a job that pays  $2x$  dollars or  $2^x$  dollars for  $x$  hours of work. Assuming you must work at least 2 hours, which method of payment would you choose? Explain your reasoning.

## ★ Challenge

84. **LOGICAL REASONING** Fill in the blanks and give a reason for each step to complete a convincing argument that the power of a power property is true.

$$\begin{aligned}(a^2)^3 &= a^2 \cdot \underline{\quad} \cdot \underline{\quad} \\ &= \underline{\quad} \cdot \underline{\quad} \cdot \underline{\quad} \cdot \underline{\quad} \cdot \underline{\quad} \cdot \underline{\quad} \\ &= \underline{\quad}\end{aligned}$$

85. **LOGICAL REASONING** Write a convincing argument to show that the power of a product property is true.

### EXTRA CHALLENGE

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## MIXED REVIEW

**EXPONENTIAL EXPRESSIONS** Evaluate the expression. (Review 1.3 for 8.2)

86.  $b^2$  when  $b = 8$

87.  $(5y)^4$  when  $y = 2$

88.  $\frac{1}{2}n^3$  when  $n = -2$

89.  $\frac{1}{y^2}$  when  $y = 5$

90.  $\frac{24}{x^3}$  when  $x = 2$

91.  $\frac{45}{a^2}$  when  $a = 2$

**GRAPHING EQUATIONS** Use a table of values to graph the equation. (Review 4.2)

92.  $y = x + 2$

93.  $y = -(x - 4)$

94.  $y = \frac{1}{2}x - 5$

95.  $y = \frac{3}{4}x + 2$

96.  $y = 2$

97.  $x = -3$

**GRAPHING INEQUALITIES** Sketch the graph of the inequality. (Review 6.1)

98.  $x < 4$

99.  $x > 15$

100.  $x \geq -9$

101.  $x \leq 3$

**SOLVING INEQUALITIES** Solve the inequality. (Review 6.2)

102.  $-x - 2 < -5$

103.  $8 + 3x \geq -2$

104.  $2 < 2x + 7$

105. **PRICE OF MILK** In 1910, a quart of milk cost \$.07. In 1994, a quart of milk cost \$.71. After 1910, the price of milk increased steadily, never falling below \$.07 per quart again. During the period 1910–1994, the maximum price of a quart was \$.71. Write a compound inequality that represents the possible costs of a quart of milk between 1910 and 1994. (Review 6.3)