10.4 Solving Polynomial Equations in Factored Form

What *you should learn*

GOAL 1 Solve a polynomial equation in factored form.

GOAL 2 Relate factors and x-intercepts.

Why *you should learn it*

▼ To solve **real-life** problems like estimating the dimensions of the Gateway Arch in **Exs. 55 and 56**.

REAL LIFE

GOAL 1 SOLVING FACTORED EQUATIONS

In Chapter 9 you learned how to solve quadratic equations in standard form.

Standard form: $2x^2 + 7x - 15 = 0$

In this lesson you will learn how to solve quadratic and other polynomial equations in *factored form*. A polynomial is in **factored form** if it is written as the product of two or more linear factors.

Factored form: $(2x - 3)(x + 5) = 0$

Factored form: $(2x - 3)(x + 5)(x - 7) = 0$

ACTIVITY Developing Concepts: Investigating Factored Equations

1. Copy and complete the table by substituting the x-values into the given expression.

Expression	x-value						
	-3	-2	-1	0	1	2	3
$(x - 3)(x + 2)$	$(-6)(-1) = 6$	?	?	?	?	?	?
$(x + 1)(x + 3)$	?	?	?	?	?	?	?
$(x - 2)(x - 2)$	?	?	?	?	?	?	?
$(x - 1)(x + 2)$	?	?	?	?	?	?	?

2. Use the table to solve each of the following equations.

a. $(x - 3)(x + 2) = 0$ **b.** $(x + 1)(x + 3) = 0$

c. $(x - 2)(x - 2) = 0$ **d.** $(x - 1)(x + 2) = 0$

3. Write a general rule for solving an equation that is in factored form.

In this activity, you may have noticed that the product of two factors is zero *only* when at least one of the factors is zero. This is because the real numbers satisfy the **zero-product property**.

ZERO-PRODUCT PROPERTY

Let a and b be real numbers. If $ab = 0$, then $a = 0$ or $b = 0$.

Examples 1, 2, and 3 show how to use this property to solve factored equations.

EXAMPLE 1 Using the Zero-Product Property

Solve the equation $(x - 2)(x + 3) = 0$.

SOLUTION Use the zero-product property: either $x - 2 = 0$ or $x + 3 = 0$.

$(x - 2)(x + 3) = 0$ Write original equation.

$x - 2 = 0$ Set first factor equal to 0.

$x = 2$ Solve for x.

$x + 3 = 0$ Set second factor equal to 0.

$x = -3$ Solve for x.

▶ The solutions are 2 and -3. Check these in the original equation.

EXAMPLE 2 Solving a Repeated-Factor Equation

Solve $(x + 5)^2 = 0$.

SOLUTION

This equation has a *repeated* factor. To solve the equation you need to set only $x + 5$ equal to zero.

$(x + 5)^2 = 0$ Write original equation.

$x + 5 = 0$ Set repeated factor equal to 0.

$x = -5$ Solve for x.

▶ The solution is -5. Check this in the original equation.

EXAMPLE 3 Solving a Factored Cubic Equation

Solve $(2x + 1)(3x - 2)(x - 1) = 0$.

SOLUTION

$(2x + 1)(3x - 2)(x - 1) = 0$ Write original equation.

$2x + 1 = 0$ Set first factor equal to 0.

$x = -\frac{1}{2}$ Solve for x.

$3x - 2 = 0$ Set second factor equal to 0.

$x = \frac{2}{3}$ Solve for x.

$x - 1 = 0$ Set third factor equal to 0.

$x = 1$ Solve for x.

▶ The solutions are $-\frac{1}{2}$, $\frac{2}{3}$, and 1. Check these in the original equation.

STUDENT HELP

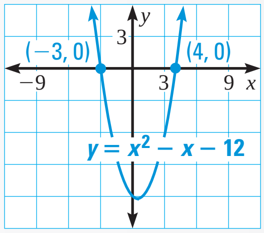
Study Tip
A polynomial equation in one variable can have as many solutions as it has linear factors that include the variable. As you saw in Example 2, an equation may have fewer solutions than factors if a factor is repeated.

GOAL 2 RELATING FACTORS AND X-INTERCEPTS

CONCEPT SUMMARY — FACTORS, SOLUTIONS, AND X-INTERCEPTS

For any quadratic polynomial $ax^2 + bx + c$, if one of the following statements is true, then all three statements are true.

- $(x - p)$ is a factor of the quadratic expression $ax^2 + bx + c$.
  Example: $(x - 4)$ and $(x + 3)$ are factors of $x^2 - x - 12$.
- $x = p$ is a solution of the quadratic equation $ax^2 + bx + c = 0$.
  Example: $x = 4$ and $x = -3$ are solutions of $x^2 - x - 12 = 0$.
- p is an x-intercept of the graph of the function $y = ax^2 + bx + c$.
  Example: 4 and -3 are x-intercepts of $y = x^2 - x - 12$.



EXAMPLE 4 *Relating x-Intercepts and Factors*

STUDENT HELP

Study Tip
The statements above about factors, solutions, and x-intercepts are *logically equivalent* because each statement implies the truth of the other statements.

To sketch the graph of $y = (x - 3)(x + 2)$:

First solve $(x - 3)(x + 2) = 0$ to find the x-intercepts: 3 and -2.

Then find the coordinates of the vertex.

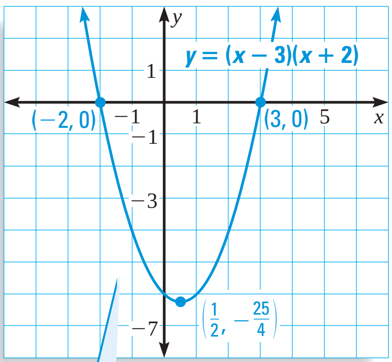
- The x-coordinate of the vertex is the average of the x-intercepts.

$$x = \frac{3 + (-2)}{2} = \frac{1}{2}$$

- Substitute to find the y-coordinate.

$$y = \left(\frac{1}{2} - 3\right)\left(\frac{1}{2} + 2\right) = -\frac{25}{4}$$

- The coordinates of the vertex are $\left(\frac{1}{2}, -\frac{25}{4}\right)$.



Sketch the graph by using the x-intercepts and vertex coordinates.

EXAMPLE 5 *Using a Quadratic Model*

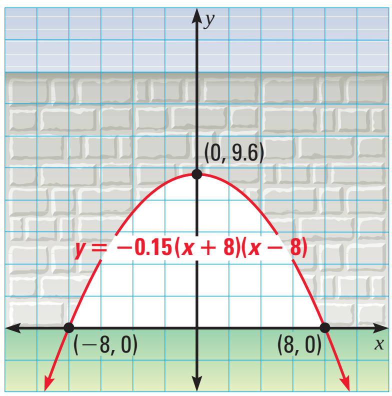
REAL LIFE
Archways

An arch is modeled by the equation $y = -0.15(x - 8)(x + 8)$, with x and y measured in feet. How wide is the arch at the base? How high is the arch?

SOLUTION Sketch a graph of the model.

- The x-intercepts are $x = 8$ and $x = -8$.
- The vertex is at $(0, 9.6)$.

▶ The arch is $8 - (-8)$, or 16 feet wide at the base and 9.6 feet high.



GUIDED PRACTICE

Vocabulary Check ✓

1. What is the zero-product property?

Concept Check ✓

2. Are 1 and -4 the solutions of $(x + 1)(x - 4) = 0$? Explain.
3. Are -5, 2, and 3 the solutions of $3(x - 2)(x + 5) = 0$? Explain.

Tell whether the statement about $x^2 + 12x + 32$ is *true* or *false*.

4. $x^2 + 12x + 32 = (x + 4)(x + 8)$
5. $x = -4$ and $x = -8$ are solutions of $x^2 + 12x + 32 = 0$.
6. $(x + 4)$ and $(x + 8)$ are factors of $x^2 + 12x + 32$.
7. The x-intercepts of the graph of $y = x^2 + 12x + 32$ are -4 and -8.

Skill Check ✓

Does the graph of the function have x-intercepts of 4 and -5?

8. $y = 2(x + 4)(x - 5)$
9. $y = 4(x - 4)(x - 5)$
10. $y = -(x - 4)(x + 5)$
11. $y = 3(x + 5)(x - 4)$

In Exercises 12–17, use the zero-product property to solve the equation.

12. $(b + 1)(b + 3) = 0$
13. $(t - 3)(t - 5) = 0$
14. $(x - 7)^2 = 0$
15. $(y + 9)(y - 2) = 0$
16. $(3d + 6)(2d + 5) = 0$
17. $\left(2w + \frac{1}{2}\right)^2 = 0$
18. Sketch the graph of $y = (x - 2)(x + 5)$. Label the vertex and the x-intercepts.

PRACTICE AND APPLICATIONS

STUDENT HELP

Extra Practice to help you master skills is on p. 806.

ZERO-PRODUCT PROPERTY **Use the zero-product property to solve the equation.**

19. $(x + 4)(x + 1) = 0$
20. $(y + 3)^2 = 0$
21. $(t + 8)(t - 6) = 0$
22. $(w - 17)^2 = 0$
23. $(b - 9)(b + 8) = 0$
24. $(d + 7)^2 = 0$
25. $(y - 2)(y + 1) = 0$
26. $(z + 2)(z + 3) = 0$
27. $(v - 7)(v - 5) = 0$
28. $\left(t + \frac{1}{2}\right)(t - 4) = 0$
29. $4(c + 9)^2 = 0$
30. $(u - 3)\left(u - \frac{2}{3}\right) = 0$
31. $(y - 5.6)^2 = 0$
32. $(a - 40)(a + 12) = 0$
33. $7(b - 5)^3 = 0$

STUDENT HELP

HOMEWORK HELP
Example 1: Exs. 19–43
Example 2: Exs. 19–43
Example 3: Exs. 38–43
Example 4: Exs. 44–52
Example 5: Exs. 53–58

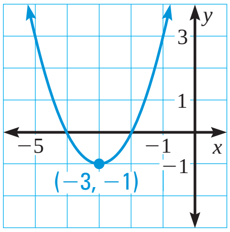
SOLVING FACTORED EQUATIONS **Solve the equation.**

34. $(4x - 8)(7x + 21) = 0$
35. $(2d + 8)(3d + 12) = 0$
36. $5(3m + 9)(5m - 15) = 0$
37. $8(9n + 27)(6n - 9) = 0$
38. $(6b - 18)(2b + 2)(2b + 2) = 0$
39. $(4y - 5)(2y - 6)(3y - 4) = 0$
40. $(x + 44)(3x - 2)^2 = 0$
41. $(5x - 9.5)^2(3x + 6.3) = 0$
42. $\left(\frac{1}{2}x + 2\right)\left(\frac{2}{3}x + 6\right)\left(\frac{1}{6}x - 1\right) = 0$
43. $\left(2n - \frac{1}{4}\right)\left(5n + \frac{3}{10}\right)\left(3n - \frac{2}{3}\right) = 0$

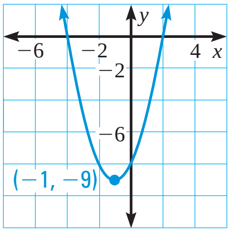
MATCHING FUNCTIONS AND GRAPHS **Match the function with its graph.**

A. $y = (x + 2)(x - 4)$ **B.** $y = (x - 2)(x + 4)$ **C.** $y = (x + 4)(x + 2)$

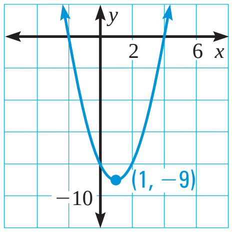
44.



45.



46.



SKETCHING GRAPHS **Find the x-intercepts and the vertex of the graph of the function. Then sketch the graph of the function.**

47. $y = (x - 4)(x + 2)$ **48.** $y = (x + 5)(x + 3)$ **49.** $y = (-x - 2)(x + 2)$

50. $y = (-x - 1)(x + 7)$ **51.** $y = (x - 2)(x - 6)$ **52.** $y = (-x + 4)(x + 3)$

STUDENT HELP

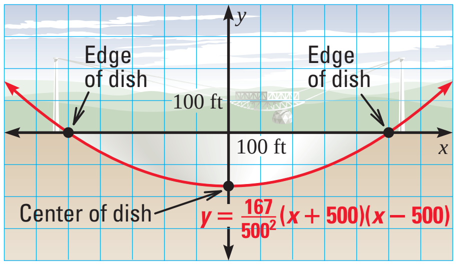
INTERNET **HOMEWORK HELP** Visit our Web site www.mcdougallittell.com for help with Exs. 53–58.

RADIO TELESCOPE **In Exercises 53 and 54, use the cross section of the radio telescope dish shown below.**

The cross section of the telescope's dish can be modeled by the polynomial function

$$y = \frac{167}{500^2}(x + 500)(x - 500)$$

where x and y are measured in feet, and the center of the dish is where $x = 0$.



53. Explain how to use the algebraic model to find the width of the dish.

54. Use the model to find the coordinates of the center of the dish.

GATEWAY ARCH **In Exercises 55 and 56, use the following information.** The Gateway Arch in St. Louis, Missouri, has the shape of a *catenary* (a U-shaped curve similar to a parabola). It can be approximated by the following model, where x and y are measured in feet. ▶ Source: National Park Service

Gateway Arch model: $y = -\frac{7}{1000}(x + 300)(x - 300)$

55. According to the model, how far apart are the legs of the arch?

56. How high is the arch?

FOCUS ON APPLICATIONS

REAL LIFE **THE BARRINGER METEOR CRATER** was formed about 49,000 years ago when a nickel and iron meteorite struck the desert at about 25,000 miles per hour.

BARRINGER METEOR CRATER **In Exercises 57 and 58, use the following equation, where x and y are measured in meters, to model a cross section of the Barringer Meteor Crater, near Winslow, Arizona.**
▶ Source: Jet Propulsion Laboratory

Barringer Meteor Crater model: $y = \frac{1}{1800}(x - 600)(x + 600)$

57. Assuming the lip of the crater is at $y = 0$, how wide is the crater?

58. What is the depth of the crater?

Test Preparation

59. MULTI-STEP PROBLEM You sell hot dogs for \$1.00 each at your concession stand at a baseball park and have about 200 customers. You want to increase the price of a hot dog. You estimate that you will lose three sales for every \$.10 increase. The following equation models your hot dog sales revenue R, where n is the number of \$.10 increases.

Concession stand revenue model: $R = (1 + 0.1n)(200 - 3n)$

a. To find your revenue from hot dog sales, you multiply the price of each hot dog sold by the number of hot dogs sold. In the formula above, what does $1 + 0.1n$ represent? What does $200 - 3n$ represent?

b. How many times would you have to raise the price by \$.10 to reduce your revenue to zero? Make a graph to help find your answer.

c. Decide how high you should raise the price to make the most money. Explain how you got your answer.

★ **Challenge**

60. *Writing* Write your own multi-step problem about selling a product like the one in Exercise 59. Include a model that shows the relationship between price and number of items sold. Explain what each factor in the model represents.

MIXED REVIEW

DECIMAL FORM **Rewrite in decimal form.** (Review 8.4)

61. 2.1×10^5 **62.** 4.443×10^{-2} **63.** 8.57×10^8 **64.** 1.25×10^6

65. 3.71×10^{-3} **66.** 9.96×10^6 **67.** 7.22×10^{-4} **68.** 8.17×10^7

MULTIPLYING EXPRESSIONS **Find the product.** (Review 10.2 for 10.5)

69. $(x - 2)(x - 7)$ **70.** $(x + 8)(x - 8)$ **71.** $(x - 4)(x + 5)$

72. $(x + 6)(x - 7)$ **73.** $\left(x + \frac{2}{3}\right)\left(x - \frac{1}{3}\right)$ **74.** $(x - 3)\left(x - \frac{1}{6}\right)$

75. $(2x + 7)(3x - 1)$ **76.** $(5x - 1)(5x + 2)$ **77.** $(3x + 1)(8x - 3)$

78. $(2x - 4)\left(\frac{1}{4}x - 2\right)$ **79.** $(x + 10)(x + 10)$ **80.** $(3x + 5)\left(\frac{2}{3}x - 3\right)$

EXPONENTIAL MODELS **Tell whether the situation can be represented by a model of *exponential growth* or *exponential decay*. Then write a model that represents the situation.** (Review 8.5, 8.6)

81. COMPUTER PRICES From 1996 to 2000, the average price of a computer company's least expensive home computer system decreased by 16% per year.

82. MUSIC SALES From 1995 to 1999, the number of CDs a band sold increased by 23% per year.

83. COOKING CLUB From 1996 to 2000, the number of members in the cooking club decreased by 3% per year.

84. INTERNET SERVICE From 1993 to 1998, the total revenues for a company that provides Internet service increased by about 137% per year.