3.2

Solving Equations Using Multiplication and Division

What you should learn

GOAL 1 Solve linear equations using multiplication and division.

GOAL 2 Use multiplication and division equations to solve **real-life** and geometric problems as in **Example 4**.

Why you should learn it

▼ To model and solve **real-life** problems, such as finding how far away you are from a thunderstorm in **Exercise 54**.

GOAL 1 MULTIPLICATION AND DIVISION EQUATIONS

In this lesson you will study equations that can be solved by multiplying or dividing each side by the same nonzero number.

Remember, when you solve a linear equation your goal is to isolate the variable on one side of the equation. In the last lesson you used the fact that addition and subtraction are inverse operations to solve linear equations. In this lesson you will use the fact that multiplication and division are inverse operations.

TRANSFORMATIONS THAT PRODUCE EQUIVALENT EQUATIONS

- Multiply *each* side of the equation by the same nonzero number.
- Divide *each* side of the equation by the same nonzero number.

ORIGINAL EQUATION		EQUIVALENT EQUATION
$\frac{x}{2} = 3$	Multiply by 2. →	$x = 6$
$4x = 12$	Divide by 4. →	$x = 3$

EXAMPLE 1 *Dividing Each Side of an Equation*

Solve $-4x = 1$.

SOLUTION

On the left side of the equation, x is multiplied by -4. To isolate x, you need to undo the multiplication by applying the inverse operation of dividing by -4.

$-4x = 1$ — Write original equation.

$\frac{-4x}{-4} = \frac{1}{-4}$ — Divide each side by -4.

$x = -\frac{1}{4}$ — Simplify.

▶ The solution is $-\frac{1}{4}$. Check this in the original equation.

✓ **CHECK**

$-4x = 1$ — Write original equation.

$(-4)\left(-\frac{1}{4}\right) \stackrel{?}{=} 1$ — Substitute $-\frac{1}{4}$ for x.

$1 = 1$ — Solution is correct.

EXAMPLE 2 Multiplying Each Side of an Equation

Solve $\frac{x}{5} = -30$.

SOLUTION

On the left side of the equation, x is divided by 5. You can isolate x by multiplying each side by 5 to undo the division.

$$\frac{x}{5} = -30 \qquad \text{Write original equation.}$$

$$5\left(\frac{x}{5}\right) = 5(-30) \qquad \text{Multiply each side by 5.}$$

$$x = -150 \qquad \text{Simplify.}$$

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STUDENT HELP

Look Back For help with reciprocals, see page 108.

When you solve an equation with a fractional coefficient, such as $10 = -\frac{2}{3}m$, you can isolate the variable by multiplying by the reciprocal of the fraction.

EXAMPLE 3 Multiplying Each Side by a Reciprocal

Solve $10 = -\frac{2}{3}m$.

SOLUTION

$$10 = -\frac{2}{3}m \qquad \text{Write original equation.}$$

$$\left(-\frac{3}{2}\right)10 = \left(-\frac{3}{2}\right)\left(-\frac{2}{3}m\right) \qquad \text{Multiply each side by } -\frac{3}{2}.$$

$$-15 = m \qquad \text{Simplify.}$$

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The transformations used to isolate the variable in Lessons 3.1 and 3.2 are based on rules of algebra called **properties of equality**.

CONCEPT SUMMARY

PROPERTIES OF EQUALITY

ADDITION PROPERTY OF EQUALITY	If $a = b$, then $a + c = b + c$.
SUBTRACTION PROPERTY OF EQUALITY	If $a = b$, then $a - c = b - c$.
MULTIPLICATION PROPERTY OF EQUALITY	If $a = b$, then $ca = cb$.
DIVISION PROPERTY OF EQUALITY	If $a = b$ and $c \neq 0$, then $\frac{a}{c} = \frac{b}{c}$.

You have been using these properties to keep equations in balance as you solve them. For instance, in Example 2 you used the multiplication property of equality when you multiplied each side by 5.

FOCUS ON APPLICATIONS

REAL LIFE **MOVIE FRAMES** A motion picture camera takes a series of separate pictures as frames. These are projected in rapid sequence when the movie is shown.

GOAL 2 SOLVING REAL-LIFE PROBLEMS

EXAMPLE 4 *Modeling a Real-Life Problem*

RESTORING MOVIES A single picture on a roll of movie film is called a frame. Motion picture studios try to save some older films from decay by cleaning and restoring the film frame by frame. This process is very expensive and time-consuming.

a. The usual rate for taking and projecting professional movies is 24 frames per second. Find the total number of frames in a movie that is 90 minutes long.

b. If a worker can restore 8 frames per hour, how many hours of work are needed to restore all of the frames in a 90-minute movie?

SOLUTION

a. Let x = the total number of frames in the movie. To find the total number of seconds in the movie, multiply $90 \cdot 60$ because each minute is 60 seconds.

$$\frac{\text{Total number of frames in the movie}}{\text{Total number of seconds in the movie}} = \text{Number of frames per second}$$

$$\frac{x}{5400} = 24$$

▶ The solution is $x = 129{,}600$, so a 90-minute movie has 129,600 frames.

b. Let y = the number of hours of work and use the result from part (a).

$$\text{Number of frames restored per hour} \cdot \text{Number of hours of work} = \text{Total number of frames in the movie}$$

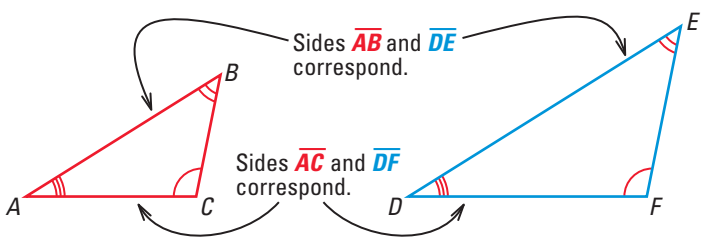
$$8 \cdot y = 129{,}600$$

▶ The solution is $y = 16{,}200$, so 16,200 hours of work are needed.

. . . . . .

You can model some real-life situations with an equation that sets two *ratios* equal. If a and b are two quantities measured in the *same* units, then the **ratio of a to b** is $\frac{a}{b}$.

Two triangles are **similar triangles** if they have equal corresponding angles. It can be shown that this is equivalent to the ratios of the lengths of corresponding sides being equal. Corresponding angles are marked with the same symbol.

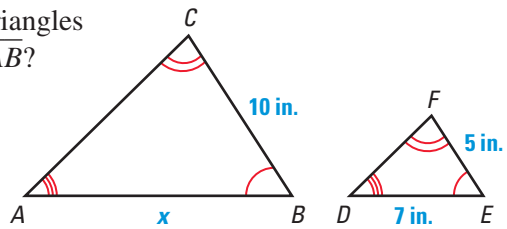


The ratio $\frac{\text{length of } \overline{AB}}{\text{length of } \overline{DE}}$ is equal to the ratio $\frac{\text{length of } \overline{AC}}{\text{length of } \overline{DF}}$.

STUDENT HELP **Skills Review** For help with ratios and rates, see p. 787.

EXAMPLE 5 Solving Problems with Similar Triangles

GEOMETRY CONNECTION The two triangles are similar. What is the length of side $\overline{AB}$?



SOLUTION

Set up equal ratios to find the unknown side length.

PROBLEM SOLVING STRATEGY

VERBAL MODEL

$$\frac{\text{Length of } \overline{AB}}{\text{Length of } \overline{DE}} = \frac{\text{Length of } \overline{BC}}{\text{Length of } \overline{EF}}$$

LABELS

Length of $\overline{AB}$ = x (inches)

Length of $\overline{DE}$ = 7 (inches)

Length of $\overline{BC}$ = 10 (inches)

Length of $\overline{EF}$ = 5 (inches)

ALGEBRAIC MODEL

$$\frac{x}{7} = \frac{10}{5}$$ Write algebraic model.

$$7\left(\frac{x}{7}\right) = 7\left(\frac{10}{5}\right)$$ Multiply each side by 7.

$$x = 14$$ Simplify.

▶ The length of $\overline{AB}$ is 14 inches.

GUIDED PRACTICE

Vocabulary Check ✓

1. Name two pairs of inverse operations.

Concept Check ✓

2. Describe six ways to transform an equation into an equivalent equation.
3. What is the first step you would use to solve each equation in Exercises 4–7?

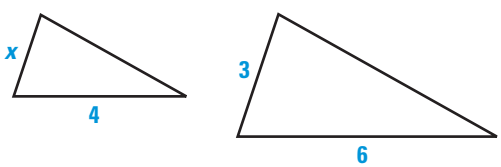
Skill Check ✓

Solve the equation.

4. $6x = 18$
5. $\frac{y}{4} = 8$
6. $\frac{r}{-5} = 20$
7. $\frac{5}{6}a = -10$
8. $-7b = -4$
9. $-3x = 5$
10. $-\frac{3}{8}t = -6$
11. $\frac{1}{7}x = \frac{5}{7}$

12. **CAR TRIPS** Write and solve an equation to find your average speed on a trip from St. Louis to Dallas. You drove 630 miles in $10\frac{1}{2}$ hours.

13. The two triangles are similar triangles. Write and solve an equation to find the unknown side length.



PRACTICE AND APPLICATIONS

STUDENT HELP

Extra Practice to help you master skills is on p. 799.

STATING INVERSES **State the inverse operation.**

14. Divide by 6.

15. Multiply by -2.

16. Divide by -4.

17. Multiply by $\frac{2}{3}$.

18. Multiply by $-\frac{9}{4}$.

19. Divide by $-\frac{4}{3}$.

EQUIVALENT EQUATIONS **Tell whether the equations are equivalent.**

20. $-4x = 44$ and $x = 11$

21. $21x = 7$ and $x = 3$

22. $\frac{x}{10} = -4$ and $x = -40$

23. $\frac{2}{3}x = 24$ and $x = 16$

SOLVING EQUATIONS **Solve the equation.**

24. $10x = 110$

25. $-21m = 42$

26. $18 = -2a$

27. $30b = 5$

28. $-4n = -24$

29. $288 = 16t$

30. $7r = -56$

31. $8x = 3$

32. $-10x = -9$

FOCUS ON APPLICATIONS

33. $\frac{y}{7} = 12$

34. $\frac{z}{2} = -5$

35. $\frac{1}{2}x = -20$

36. $\frac{1}{3}y = 82$

37. $\frac{m}{-4} = -\frac{3}{4}$

38. $0 = \frac{4}{5}d$

39. $-\frac{4}{5}x = 72$

40. $-\frac{1}{5}y = -6$

41. $\frac{t}{-2} = \frac{1}{2}$

42. $-\frac{2}{3}t = -16$

43. $\frac{3}{4}z = -5\frac{1}{2}$

44. $\frac{1}{3}y = 5\frac{2}{3}$

45. $\frac{3}{4}t = |-15|$

46. $-\frac{1}{2}b = -|-8|$

47. $-6y = -|27|$

REAL LIFE NEWSPAPERS Each week the average household reads five pounds of newspapers.
Source: Newspaper Association of America

48. **BUNDLING NEWSPAPERS** You are loading a large pile of newspapers onto a truck. You divide the pile into four equal-size bundles. You find that one bundle weighs 37 pounds. You want to find the weight x of the original pile. Which equation models the situation? Solve the correct equation.

A. $\frac{x}{4} = 37$ **B.** $4x = 37$

MODELING REAL-LIFE PROBLEMS **In Exercises 49–53, write and solve an equation to answer the question.**

49. Each household in the United States receives about 676 pieces of junk mail per year. About how many pieces does a household receive per week?

50. About one eighth of the population is left-handed. In what size school would you expect to find about 50 left-handed students?

51. You ate three of the eight pizza slices and you paid \$3.30 as your share of the cost. How much did the whole pizza cost?

52. It takes 45 peanuts to make one ounce of peanut butter. How many peanuts will be needed to make a 12-ounce jar of peanut butter?

53. A 10,000-square-foot pizza was created on October 11, 1987. This pie was eaten by about 30,000 people. On average, how much did each person eat?

STUDENT HELP

HOMEWORK HELP

Example 1: Exs. 14–47
Example 2: Exs. 14–47
Example 3: Exs. 14–47
Example 4: Exs. 48–54
Example 5: Exs. 58–59

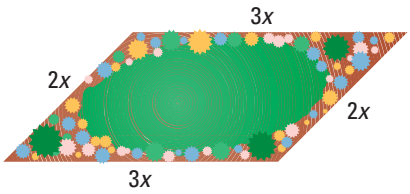
FOCUS ON APPLICATIONS

REAL LIFE THUNDERSTORMS You see a flash of lightning instantly since light travels so rapidly (186,000 miles/second). You hear the thunder later since sound takes about 5 seconds to travel a mile near the ground.

54. THUNDERSTORMS You can tell how many miles you are from a thunderstorm by counting the seconds between seeing the lightning and hearing the thunder, and then dividing by five. How many seconds would you count for a thunderstorm nine miles away?

55. CRITICAL THINKING Look back at Example 4. For each situation, write a different model than the one shown. The solutions should stay the same.

56. GARDENS A homeowner is installing a fence around the garden at the right. The garden has a perimeter of 216 feet. Write and solve an equation to find the garden's dimensions.



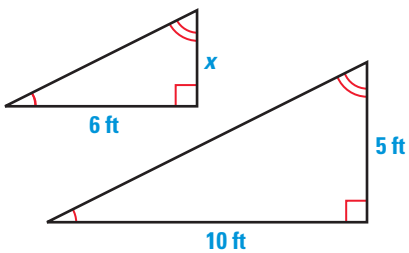
57. BALD EAGLES On page 129 you learned that bald eagles fly up to 30 miles per hour and dive at speeds up to 100 miles per hour. Using this information, write and solve an equation to answer each question.

a. What is the least amount of time that an eagle could take to fly 6 miles?

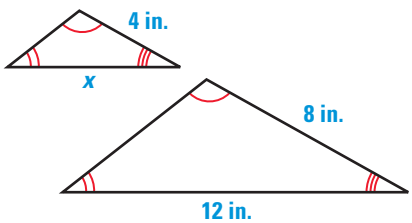
b. An eagle a mile above the water spots a fish. What is the shortest time it would take the eagle to dive for the fish? Express your answer in seconds.

GEOMETRY CONNECTION **In Exercises 58 and 59, the two triangles are similar. Write and solve an equation to find the length of the side marked *x*.**

58.



59.



COOKING **In Exercises 60 and 61, use the recipe shown below.**

60. You have only 3 cups of rice, so you decrease the recipe. To find the amount of each ingredient, you can write an equation that sets two ratios equal. For the rice you can use the ratio $\frac{3 \text{ cups}}{4 \text{ cups}} = \frac{3}{4}$ that compares the reduced amount to the original amount.

Choose and solve the equation you can use to find the number of teaspoons of soy sauce.

A. $\frac{3}{4} = \frac{2}{x}$ **B.** $\frac{3}{4} = \frac{x}{2}$

FRIED RICE

1 tablespoon vegetable oil
1/2 cup chopped bamboo shoots
1/2 cup chopped mushrooms
1/4 cup chopped scallions
4 cups cooked rice
1 cup cooked diced chicken
1/2 cup chicken broth
2 teaspoons soy sauce

STUDENT HELP

INTERNET HOMEWORK HELP Visit our Web site www.mcdougallittell.com for help with Exs. 60 and 61.

61. You have 5 cups of rice, so you increase the recipe. Write and solve an equation to find the amount of chicken.

Test Preparation

62. MULTIPLE CHOICE What is the first step you would use to solve $\frac{1}{4} = -7x$?

(A) Divide by 4. (B) Multiply by 4.

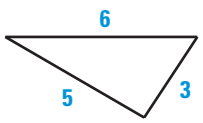
(C) Multiply by -7. (D) Divide by -7.

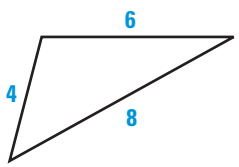
63. MULTIPLE CHOICE Solve $-\frac{5}{7}x = -2$.

(A) $\frac{14}{5}$ (B) $-\frac{14}{5}$ (C) $\frac{10}{7}$ (D) $\frac{7}{5}$

64. MULTIPLE CHOICE Which of the triangles below are similar triangles?

I. 

II. 

III. 

(A) I and II (B) I and III (C) II and III (D) None

Challenge

65. ATTENDANCE In 1997, the average home attendance at New York Yankees' baseball games was a little more than 150% of the average home attendance at Anaheim Angels' baseball games. The average home attendance at New York Yankees' baseball games was about 33,000. Write and solve an equation to estimate the average home attendance at Anaheim Angels' baseball games. ▶ Source: *ESPN 1998 Information Please® Sports Almanac*

EXTRA CHALLENGE
www.mcdougallittell.com

MIXED REVIEW

TRANSLATING VERBAL SENTENCES **Write the verbal sentence as an equation.** (Review 1.5 for 3.3)

66. The sum of 18 and five times a number is 108.

67. Twelve less than nine times a number is 60.

68. Five more than two thirds of a number is 11.

69. Eleven is two fifths of the quantity n decreased by thirteen.

SIMPLIFYING EXPRESSIONS **Simplify the expression.** (Review 2.6 for 3.3)

70. $15 - 8x + 12$ **71.** $4y - 9 + 3y$ **72.** $(x + 8)(-2) - 36$

73. $5(y + 3) + 7y$ **74.** $12x - (x - 2)(2)$ **75.** $-25y - 6(-y - 9)$

SOLVING EQUATIONS **Solve the equation.** (Review 3.1)

76. $4 + y = 12$ **77.** $t - 2 = 1$ **78.** $5 - (-t) = 14$

79. $x - 2 = 28$ **80.** $19 - x = 37$ **81.** $-9 - (-a) = -2$

82. PHOTOGRAPHY You take 24 pictures. Seven of the pictures cannot be developed because of bad lighting. Let x represent the number of pictures that can be developed successfully. Which of the following is a correct model for the situation? Solve the correct equation. (Review 3.1)

A. $x + 7 = 24$ **B.** $x - 7 = 24$