1.4

Equations and Inequalities

GOAL ① CHECKING AND SOLVING EQUATIONS

An **equation** is formed when an equal sign ($=$) is placed between two expressions creating a left and right side of the equation. An equation that contains one or more variables is an **open sentence**. Here are some examples.

$$4 - b = 3 \qquad\qquad 3x + 1 = 7 \qquad\qquad a + 3 = 3 + a$$

When the variable in a single-variable equation is replaced by a number, the resulting statement can be true or false. If the statement is true, the number is a **solution of an equation**.

Substituting a number for a variable in an equation to see whether the resulting statement is true or false is called *checking* a possible solution.

EXAMPLE 1 *Substituting to Check Possible Solutions*

Check whether the numbers 2 and 3 are solutions of the equation $3x + 1 = 7$.

SOLUTION

To check the possible solutions, substitute them into the equation. If both sides of the equation have the same value, then the number is a solution.

x	$3x + 1 = 7$	Result	Conclusion
2	$3(2) + 1 \overset{?}{=} 7$	$7 = 7$	2 is a solution
3	$3(3) + 1 \overset{?}{=} 7$	$10 \neq 7$	3 is not a solution

is not equal to

▶ The number 2 is a solution of $3x + 1 = 7$. The number 3 is *not* a solution.

EXAMPLE 2 *Checking Possible Solutions*

Check whether the numbers 2, 3, and 4 are solutions of the equation $4x - 2 = 10$.

SOLUTION

x	$4x - 2 = 10$	Result	Conclusion
2	$4(2) - 2 \overset{?}{=} 10$	$6 \neq 10$	2 is not a solution
3	$4(3) - 2 \overset{?}{=} 10$	$10 = 10$	3 is a solution
4	$4(4) - 2 \overset{?}{=} 10$	$14 \neq 10$	4 is not a solution

▶ The number 3 is a solution, and 2 and 4 are *not* solutions of the equation.

Finding all the solutions of an equation is called **solving the equation**. Later in the book, you will study several ways to systematically solve equations.

Some equations are simple enough to solve in your head with mental math. For instance, to solve $x + 2 = 5$, ask yourself the question

"What number can be added to 2 to obtain 5?"

If you can see that the answer is 3, then you have solved the equation!

▶ ACTIVITY

Developing Concepts **Using Mental Math to Solve Equations**

1 Match the equation with the question that can be used to find a solution of the equation.

2 Then use mental math to solve the equation.

EQUATION	MENTAL MATH QUESTION
1. $x + 2 = 6$	**A.** 2 times what number gives 10?
2. $x - 3 = 4$	**B.** What number divided by 3 gives 1?
3. $2x = 10$	**C.** What number minus 3 gives 4?
4. $\frac{x}{3} = 1$	**D.** What number cubed gives 8?
5. $x^3 = 8$	**E.** What number plus 2 gives 6?

Using mental math to solve equations is something that you already do in everyday life. You probably don't think of it as solving equations, but the mental math process is the same.

Grocery Shopping

EXAMPLE 3 *Using Mental Math to Solve a Real-Life Equation*

You need to buy ingredients for nachos. In the supermarket you find that a bag of tortilla chips costs $2.99, beans cost $.99, cheese costs $3.99, two tomatoes cost $1.00, and olives cost $1.49. You have a ten-dollar bill. About how much more money do you need?

SOLUTION

You can ask the question: The total cost equals 10 plus what number of dollars? Let x represent any additional money you may need. Use rounding to estimate the total cost.

$$3 + 1 + 4 + 1 + 1.5 = 10 + x$$

$$10.5 = 10 + x$$

▶ Because the total cost of the ingredients is approximately 10.5 or $10.50, you can see that you need about $.50 more to purchase all the ingredients.

Another type of open sentence is an **inequality**. An inequality is formed when an inequality symbol, such as $<$, is placed between two expressions.

SYMBOL	MEANING
$<$	is less than
$\leq$	is less than or equal to
$>$	is greater than
$\geq$	is greater than or equal to

A **solution of an inequality** is a number that produces a true statement when it is substituted for the variable in the inequality.

EXAMPLE 4 *Checking Solutions of Inequalities*

Decide whether 4 is a solution of the inequality.

a. $2x - 1 < 8$ **b.** $x + 4 > 9$ **c.** $x - 3 \geq 1$

SOLUTION

INEQUALITY	SUBSTITUTION	RESULT	CONCLUSION
a. $2x - 1 < 8$	$2(4) - 1 \overset{?}{<} 8$	$7 < 8$	4 is a solution.
b. $x + 4 > 9$	$4 + 4 \overset{?}{>} 9$	$8 \not> 9$	4 is not a solution.
c. $x - 3 \geq 1$	$4 - 3 \overset{?}{\geq} 1$	$1 \geq 1$	4 is a solution.

EXAMPLE 5 *Checking Solutions in Real Life*

VETERINARIAN Your vet told you to restrict your cat's caloric intake to no more than 500 calories each day. Three times a day, you give your cat a serving of food containing x calories. Do the following values of x meet the vet's restriction?

a. 170 calories   **b.** 165 calories

SOLUTION

a. $3x \leq 500$ **Write original inequality.**

   $3(170) \leq 500$ **Substitute 170 for *x*.**

     $510 \not\leq 500$ **Simplify.**

▶ No. This is too many calories per serving.

b. $3x \leq 500$ **Write original inequality.**

   $3(165) \leq 500$ **Substitute 165 for *x*.**

     $495 \leq 500$ **Simplify.**

▶ Yes. If each serving has 165 calories, you will meet your goal.

GUIDED PRACTICE

Decide whether the following is an *expression*, an *equation*, or an *inequality*. Explain your decision.

1. $3x + 1 = 14$　　　　**2.** $7y - 6$　　　　**3.** $5(y^2 + 4) - 7$

4. $5x - 1 = 3 + x$　　**5.** $3x + 2 \le 8$　　**6.** $5x > 20$

7. Identify the left side and the right side of the equation $8 + 3x = 5x - 9$.

8. **ERROR ANALYSIS** Jan says her work shows that 6 is not a solution of $3x - 4 = 14$. What is a likely explanation for her error?

$$3 \cdot (6) - 4 = 6$$

9. Which question could be used to find the solution of the equation $5 - x = 1$?

   A. What number can 5 be subtracted from to get 1?

   B. What number can be subtracted from 5 to get 1?

   C. What number can 1 be subtracted from to get 5?

🌎 **ELECTIONS** **The number of votes received by the new student council president is represented by *x*. Match the sentence with the equation or inequality that represents it.**

   A. $x = 125$　　　**B.** $x < 125$　　　**C.** $x \ge 125$　　　**D.** $x \le 125$

10. She received no more than 125 votes.　**11.** She received at least 125 votes.

12. She received exactly 125 votes.　　　**13.** She received less than 125 votes.

PRACTICE AND APPLICATIONS

CHECKING SOLUTIONS OF EQUATIONS **Check whether the given number is a solution of the equation.**

14. $3b + 1 = 13; 4$　　**15.** $5 + x^2 = 17; 3$　　**16.** $4c + 2 = 2c + 8; 2$

17. $2y^3 + 3 = 5; 1$　　**18.** $5r - 10 = 11; 5$　　**19.** $4s - 4 = 30 - s; 7$

20. $6d - 5 = 20; 5$　　**21.** $\frac{x}{4} - 9 = 9; 36$　　**22.** $m + 4m = 60 - 2m; 10$

23. $10 + \frac{a}{7} = 12; 14$　**24.** $p^2 - 5 = 20; 6$　　**25.** $4h - h = \frac{12}{h}; 3$

MENTAL MATH **Write a question that could be used to solve the equation. Then use mental math to solve the equation.**

26. $x + 3 = 8$　　　**27.** $n + 6 = 11$　　　**28.** $p - 11 = 20$

29. $3y = 12$　　　　**30.** $\frac{x}{4} = 5$　　　　**31.** $4p = 36$

32. $4r - 1 = 11$　　**33.** $2t - 1 = 9$　　　**34.** $m^2 = 144$

35. $\frac{x}{7} = 3$　　　　**36.** $5q - 2 = 3$　　　**37.** $y^3 = 125$

CHECKING SOLUTIONS OF INEQUALITIES Check whether the given number is a solution of the inequality.

38. $n - 2 < 6$; 3

39. $5 + s > 8$; 4

40. $5 + 5x \geq 10$; 1

41. $4p - 1 \geq 8$; 2

42. $3r - 15 < 0$; 5

43. $11x \leq x - 7$; 9

44. $6 + y \leq 8$; 3

45. $29 - 4b > 5$; 7

46. $t^2 + 6 > 40$; 6

47. $a - 7 \geq 15$; 22

48. $6x - 16 < 20$; 7

49. $y^3 - 2 \leq 8$; 2

50. $r + 2r < 30$; 9

51. $a(3a + 2) > 50$; 4

52. $\dfrac{c + 5}{3} \leq 4$; 3

53. $\dfrac{25 - d}{d} \geq 4$; 5

54. $x^2 - 10 > 16$; 6

55. $n(21 - n) < 100$; 8

EQUATIONS AND INEQUALITIES Match the verbal sentence with its mathematical representation.

56. The sum of x and 16 is less than 32.

A. $\dfrac{x}{16} > 32$

57. The product of 16 and x is equal to 32.

B. $x^4 = 16$

58. The difference of x and 16 is 32.

C. $x + 16 < 32$

59. The quotient of x and 16 is greater than or equal to 32.

D. $16 + x \leq 32$

60. The product of 16 and x is less than 32.

E. $16x > 32$

61. The fourth power of x is 16.

F. $x - 16 = 32$

62. The sum of 16 and x is less than or equal to 32.

G. $16x = 32$

63. The quotient of x and 16 is greater than 32.

H. $\dfrac{x}{16} \geq 32$

64. The product of 16 and x is greater than 32.

I. $16x < 32$

65. 🌍 **COMPUTER CENTER** Your school is building a new computer center. Four hundred square feet of the center will be available for computer stations. Each station requires 20 square feet. You want to find how many computer stations can be placed in the new center. You write the equation $20x = 400$ to model the situation. What do 20, x, and 400 represent? Solve the equation. Check your solution.

66. 🌍 **PLAYING A COMPUTER GAME** You are playing a new computer game. For every eight screens you complete, you receive a bonus. You want to know how many bonuses you will receive after completing 96 screens. You write the equation $8x = 96$ to model the situation. What do 8, x, and 96 represent? Solve the equation. Check your solution.

67. 🌍 **BUYING GAS** You are taking a trip by automobile with the family of a friend. You have $65 to help pay for gas. It costs $15 to fill the tank. Can you completely fill the gas tank four times? You use the inequality $15x \leq 65$ to model the situation. What do 15, x, and 65 represent?

68. 🌍 **BUYING A VIOLIN** You are budgeting money to buy a violin and bow that cost $250 including tax. If you save $5 per week, will you have enough money in a year? You write the inequality $5n \geq 250$ to model the situation. What do 5, n, and 250 represent? Solve the inequality.

🌐 **APPLYING FORMULAS** In Exercises 69–76, match the problem with the formula needed to solve the problem. Then use the Guess, Check, and Revise strategy or another problem-solving strategy to solve the problem.

Area of a rectangle	$A = lw$	Distance	$d = rt$
Simple interest	$I = Prt$	Volume of a cube	$V = s^3$
Temperature	$C = \frac{5}{9}(F - 32)$	Surface area of a cube	$S = 6s^2$

69. What is the average speed of a runner who completes a 10,000-meter race in 25 minutes?

70. How long must $1000 be invested at an annual interest rate of 3% to earn $300 in simple interest?

71. A car travels 60 miles per hour for a distance of 300 miles. How long did the trip take?

72. Carpeting costs $20 per square yard. You carpet a room that has a width of 15 feet for $800. What is the length of the room in feet?

73. You measure the temperature of a substance in a chemistry lab at 32°F. What is the temperature in degrees Celsius?

74. A piece of cheese cut in the shape of a cube has a volume of 27 cubic inches. What is the length of each edge of the piece of cheese?

75. You want to construct a patio of 80 square feet with a length of 10 feet. What is the width of the patio?

76. A cubic storage box is made with 96 square feet of wood. What is the length of each edge?

77. 🌐 **AIRCRAFT DESIGN** In the diagram, let x represent the length of a passenger jet and let y represent the jet's wingspan.

a. What does the equation $1.212y = x$ say about the relationship between the length and the wingspan of the passenger jet?

b. Is the passenger jet longer than it is wide or wider than it is long? Explain your reasoning.

199 ft 11 in.

242 ft 4 in.
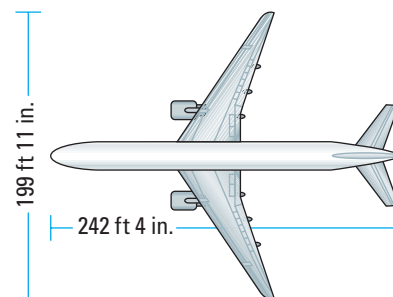
78. 🌐 **MACH NUMBERS** The *Mach number* of an aircraft is the ratio of its maximum speed to the speed of sound. Copy and complete the table. Use the equation $v = 660m$, where v is the speed (in miles per hour) of the aircraft and m is the Mach number, to find the speed of each aircraft.

Airplane type	X-15A-2	Supersonic transport	Commercial jet
Mach number, m	6.7	2.2	0.9
Speed, v	?	?	?
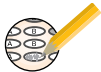
79. MULTIPLE CHOICE You tune and restore pianos. As a piano tuner you charge $75 per tuning. The expenses for your piano restoration business are $2600 per month. Which of the following inequalities could you use to find the number of pianos p you must tune per month in order to at least meet your business expenses?

    (A) $75p \leq 2600$ **(B)** $75p \geq 2600$ **(C)** $\dfrac{75}{p} \geq 2600$ **(D)** $75p = 2600$

80. MULTIPLE CHOICE The width of a soccer field cannot be greater than 100 yards. The area cannot be greater than 13,000 square yards. Which of the following would you use to find the possible lengths of a soccer field?

    (A) $100x \geq 13,000$ **(B)** $100x \leq 13,000$

    (C) $100 + x \leq 13,000$ **(D)** $100x = 130,000$

81. MULTIPLE CHOICE For which inequality is $x = 238$ a solution?

    (A) $250 \geq x + 12$ **(B)** $250 < x + 12$ **(C)** $250 > x + 12$

★ Challenge

🌐 **BUSINESS** **You plan to start your own greeting card business. Your startup cost of buying a computer and color printer is $1400. You also want to run an ad for $50 a week for 4 weeks. You plan to sell each card for $1.79. How many cards must you sell to equal or exceed your initial costs?**

82. Write an inequality that models the situation. Solve the inequality to find the minimum number of cards you must sell.

83. *Writing* Try raising the price per card. How does the price increase change the number of cards you must sell to equal or exceed your initial costs? What are some factors you need to consider in choosing a price?

MIXED REVIEW

EVALUATING EXPRESSIONS Evaluate the expression for the given value of the variable. **(Review 1.1)**

84. $1.2n$ when $n = 4.8$ **85.** $\dfrac{1}{12} + x$ when $x = \dfrac{1}{6}$

86. $b - 12$ when $b = 43$ **87.** $\dfrac{4}{5} \cdot y$ when $y = \dfrac{1}{5}$

EXPONENTIAL FORM Write the expression in exponential form. **(Review 1.2)**

88. three cubed   **89.** y squared   **90.** $6 \cdot 6 \cdot 6 \cdot 6 \cdot 6$

91. $c \cdot c \cdot c \cdot c$ **92.** five to the fourth power   **93.** $5y \cdot 5y \cdot 5y \cdot 5y$

94. three squared   **95.** $9a \cdot 9a \cdot 9a \cdot 9a \cdot 9a \cdot 9a$ **96.** seven squared

97. one to the third power   **98.** $6 \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x$ **99.** $t \cdot t \cdot t \cdot t \cdot t \cdot t$

100. 🌐 **WATER TEMPERATURE** The temperature of the water in a swimming pool is 78° Fahrenheit. What is the temperature of the water in degrees Celsius? Use the formula $C = \dfrac{5}{9}(F - 32)$, where F is the Fahrenheit temperature and C is the Celsius temperature. **(Review 1.3 for 1.5)**