1.5 A Problem Solving Plan Using Models

What you should learn

GOAL 1 Translate verbal phrases into algebraic expressions.

GOAL 2 Use a verbal model to write an algebraic equation or inequality to solve a **real-life** problem, such as making a decision about an airplane's speed in **Example 3**.

Why you should learn it

▼ To solve **real-life** problems such as finding out how many plates of dim sum were ordered for lunch in **Example 2**.

GOAL 1 TRANSLATING VERBAL PHRASES

To translate verbal phrases into algebra, look for words that indicate operations.

Operation	Verbal Phrase	Expression
Addition	The *sum* of six and a number	$6 + x$
	Eight *more than* a number	$y + 8$
	A number *plus* five	$n + 5$
	A number *increased* by seven	$x + 7$
Subtraction	The *difference* of five and a number	$5 - y$
	Four *less than* a number	$x - 4$
	Seven *minus* a number	$7 - n$
	A number *decreased* by nine	$n - 9$
Multiplication	The *product* of nine and a number	$9x$
	Ten *times* a number	$10n$
	A number *multiplied* by three	$3y$
Division	The *quotient* of a number and four	$\frac{n}{4}$
	Seven *divided* by a number	$\frac{7}{x}$

Order is important for subtraction and division, but *not* for addition and multiplication. "Four less than a number" is written as $x - 4$, *not* $4 - x$. On the other hand, "the sum of six and a number" can be written as $6 + x$ or $x + 6$.

EXAMPLE 1 *Translating Verbal Phrases into Algebra*

Translate the phrase into an algebraic expression.

SOLUTION

a. Three more than the quantity five times a number n

$5n + 3$ **Think: 3 more than what?**

b. Two less than the sum of six and a number m

$(6 + m) - 2$ **Think: 2 less than what?**

c. A number x decreased by the sum of 10 and the square of a number y

$x - (10 + y^2)$ **Think: x decreased by what?**

GOAL 2 USING A VERBAL MODEL

STUDENT HELP

Study Tip

When you translate verbal expressions, the words "the quantity" tell you what to group. In Example 1, you write $5n + 3$, not $(5 + 3)n$.

In English there is a difference between a phrase and a sentence. Verbal phrases translate into mathematical expressions and verbal sentences translate into equations or inequalities.

Phrase	The sum of six and a number	**Expression**	$6 + x$
Sentence	The sum of six and a number is twelve.	**Equation**	$6 + x = 12$
Sentence	Seven times a number is less than fifty.	**Inequality**	$7x < 50$

Sentences that translate into equations have words that tell how one quantity relates to another. In the first sentence, the word "is" says that one quantity is equal to another. In the second sentence, the words "is less than" indicate an inequality.

Writing algebraic expressions, equations or inequalities that represent real-life situations is called **modeling**. The expression, equation or inequality is a **mathematical model** of the real-life situation. When you write a mathematical model, we suggest that you use three steps.

WRITE A VERBAL MODEL. → **ASSIGN LABELS.** → **WRITE AN ALGEBRAIC MODEL.**

REAL LIFE *Dining Out*

EXAMPLE 2 *Writing an Algebraic Model*

You and three friends are having a dim sum lunch at a Chinese restaurant that charges \$2 per plate. You order lots of plates of wontons, egg rolls, and dumplings. The waiter gives you a bill for \$25.20, which includes tax of \$1.20. Use mental math to solve the equation for how many plates your group ordered.

SOLUTION

Be sure that you understand the problem situation before you begin. For example, notice that the tax is added after the cost of the plates of dim sum is figured.

PROBLEM SOLVING STRATEGY

VERBAL MODEL

$$\boxed{\text{Cost per plate}} \cdot \boxed{\text{Number of plates}} = \boxed{\text{Bill}} - \boxed{\text{Tax}}$$

LABELS

Cost per plate = **2** (dollars)

Number of plates = p (plates)

Amount of bill = **25.20** (dollars)

Tax = **1.20** (dollars)

ALGEBRAIC MODEL

$$2p = 25.20 - 1.20$$
$$2p = 24.00$$
$$p = 12$$

▶ Your group ordered 12 plates of food costing \$24.00.

FOCUS ON CAREERS

REAL LIFE **JET PILOT** Pilots select a route, an altitude, and a speed that will provide the fastest and safest flight.

INTERNET CAREER LINK
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A PROBLEM SOLVING PLAN USING MODELS

VERBAL MODEL	Ask yourself what you need to know to solve the problem. Then write a verbal model that will give you what you need to know.
LABELS	Assign labels to each part of your verbal model.
ALGEBRAIC MODEL	Use the labels to write an algebraic model based on your verbal model.
SOLVE	Solve the algebraic model and answer the original question.
CHECK	Check that your answer is reasonable.

Verbal and algebraic modeling can be used as part of a general problem solving plan that includes solving and checking to see that an answer is reasonable.

EXAMPLE 3 *Using a Verbal Model*

JET PILOT A jet pilot is flying from Los Angeles, CA to Chicago, IL at a speed of 500 miles per hour. When the plane is 600 miles from Chicago, an air traffic controller tells the pilot that it will be 2 hours before the plane can get clearance to land. The pilot knows that at its present altitude the speed of the jet must be greater than 322 miles per hour or the jet will stall.

a. At what speed would the jet have to fly to arrive in Chicago in 2 hours?

b. Is it reasonable for the pilot to fly directly to Chicago at the reduced speed from part (a) or must the pilot take some other action?

SOLUTION

a. You can use the formula (rate)(time) = (distance) to write a verbal model.

PROBLEM SOLVING STRATEGY

VERBAL MODEL

$$\text{Speed of jet} \cdot \text{Time} = \text{Distance to travel}$$

LABELS

Speed of jet = x (miles per hour)

Time = 2 (hours)

Distance to travel = 600 (miles)

ALGEBRAIC MODEL

$2x = 600$ Write algebraic model.

$x = 300$ Solve with mental math.

▶ To arrive in 2 hours, the pilot would have to slow the jet down to a speed of 300 miles per hour.

b. It is not reasonable for the pilot to fly at 300 miles per hour, because the jet will stall. The pilot should take some other action, such as circling in a holding pattern, to use some of the time.

GUIDED PRACTICE

Vocabulary Check ✓

In Exercises 1 and 2, consider the verbal phrase: *the difference of 7 and a number n.*

1. What operation does the word *difference* indicate?
2. Translate the verbal phrase into an algebraic expression.

Concept Check ✓

3. Is order important in the expression in Exercise 2?
4. Describe how to use a verbal model to solve a problem.

Skill Check ✓

Match the verbal phrase with its corresponding algebraic expression.

5. Eleven decreased by the quantity four times a number x — **A.** $4x - 11$
6. Four increased by the quantity eleven times a number x — **B.** $4(x - 11)$
7. Four times the quantity of a number x minus eleven — **C.** $11 - 4x$
8. Four times a number x decreased by eleven — **D.** $11x + 4$

Write the verbal sentence as an equation or an inequality.

9. A number x increased by ten is 24.
10. The product of seven and a number y is 42.
11. Twenty divided by a number n is less than or equal to two.
12. Ten more than a number x is greater than fourteen.

PRACTICE AND APPLICATIONS

STUDENT HELP

Extra Practice to help you master skills is on p. 797.

TRANSLATING PHRASES **Write the verbal phrase as an algebraic expression. Use *x* for the variable in your expression.**

13. Nine more than a number
14. One half multiplied by a number
15. Three more than half of a number
16. A number increased by seven
17. Quotient of a number and two tenths
18. Product of four and a number
19. Two cubed divided by a number
20. Difference of ten and a number
21. Five squared minus a number
22. Twenty-nine decreased by a number

TRANSLATING VERBAL SENTENCES **Write the verbal sentence as an equation or an inequality.**

23. Nine is greater than three times a number s.
24. Twenty-five is the quotient of a number y and 3.5.
25. The product of 14 and a number x is one.
26. Nine less than the product of ten and a number d is eleven.
27. Three times the quantity two less than a number x is ten.
28. Five decreased by eight is four times y.

STUDENT HELP

HOMEWORK HELP

Example 1: Exs. 13–38
Example 2: Exs. 39–48
Example 3: Exs. 49–57

TRANSLATING VERBAL SENTENCES **In Exercises 29–38, write the verbal sentence as an equation, or an inequality.**

29. Twenty-three less than the difference of thirty-eight and a number n is less than eight.

30. A number t increased by the sum of seven and the square of another number s is ten.

31. Five less than the difference of twenty and a number x is greater than or equal to ten.

32. Fourteen plus the product of twelve and a number y is less than or equal to fifty.

33. Nine plus the quotient of a number b and ten is greater than or equal to eleven.

34. Seventy divided by the product of seven and a number p is equal to one.

35. A number q is equal to or greater than one hundred.

36. A number x squared plus forty-four is equal to the number x to the fourth power times three.

37. The quotient of thirty-five and a number t is less than or equal to seven.

38. Fifty multiplied by the quantity twenty divided by a number n is greater than or equal to two hundred fifty.

ALGEBRAIC MODELING **In Exercises 39–48, write an equation or an inequality to model the real-life situation.**

39. Ben's hourly wage b at his after school job is \$1.50 less than Eileen's hourly wage e.

40. The distance s to school is $\frac{1}{5}$ mile more than the distance c to the Community Center swimming pool.

41. The length c of the Colorado River is three times the length r of the Connecticut River, plus 229 miles.

42. Pi (π) is the quotient of the circumference C and the diameter d of a circle.

43. The volume V of a cube with a side length s is less than or equal to thirty minus three.

44. The product of \$25 and the number m of club memberships is greater than or equal to \$500.

45. The perimeter P of a square is equal to four times the difference of a number s and two.

46. The simple interest earned on a principal of three hundred dollars at an annual interest rate of x percent is less than or equal to seventy-two dollars.

47. The area A of a trapezoid is equal to one half times the sum of seven and nine, times a number h plus seven.

48. The square of the length c of the hypotenuse of a right triangle is equal to four squared plus three squared.

FUNDRAISING **In Exercises 49–53, use the following information.**
The science club is selling magazine subscriptions at $15 each. The club wants to raise $315 for science equipment.

49. Write a verbal model that relates the number of subscriptions, the cost of each subscription, and the amount of money the club needs to raise.

50. Assign labels and write an algebraic model based on your verbal model.

51. Use mental math to solve the equation.

52. How many subscriptions does the science club need to sell to raise $315?

53. Check to see if your answer is reasonable.

STUDENT HELP
INTERNET HOMEWORK HELP
Visit our Web site www.mcdougallittell.com for help with problem solving in Exs. 54–57.

LAW ENFORCEMENT **In Exercises 54–57, use the following information.**
Jeff lives in a state in which speeders are fined $20 for each mile per hour (mi/h) over the speed limit. Jeff was given a ticket for $260 for speeding on a road where the speed limit is 45 miles per hour. Jeff wants to know how fast he was driving.

$$\boxed{\text{Fine per mi/h over speed limit}} \cdot \boxed{\text{Miles per hour over speed limit}} = \boxed{\text{Amount of ticket}}$$

54. Assign labels to the three parts of the verbal model.

55. Use the labels to translate the verbal model into an algebraic model.

56. Use mental math to solve the equation. What does the solution represent?

57. Perform unit analysis to check that the equation is set up correctly.

58. **CLASS ELECTION** You are running for class president. At 2:30 on election day you have 95 votes and your opponent has 120 votes. Forty-five more students will be voting. Let x represent the number of students (of the 45) who vote for you.

a. Write an inequality that shows the values of x that will allow you to win the election.

b. What is the smallest value of x that is a solution of the inequality?

FOCUS ON APPLICATIONS

REAL LIFE MOUNTAIN BIKES
More than 50% of all bicycles sold in the United States are mountain bikes.
INTERNET DATA UPDATE of Bicycle Product Suppliers Association data at www.mcdougallittell.com

MOUNTAIN BIKES **In Exercises 59 and 60, use the following information.**
You are shopping for a mountain bike. A store sells two different models. The model that has steel wheel rims costs $220. The model with aluminum wheel rims costs $480. You have a summer job for 12 weeks. You save $20 per week, which would allow you to buy the model with the steel wheel rims. You want to know how much more money you would have to save each week to be able to buy the model with the aluminum wheel rims.

59. Write a verbal model and an algebraic model for how much more money you would have to save each week.

60. Use mental math to solve the equation. What does the solution represent?

61. **BASKETBALL** The girls' basketball team scored 544 points in 17 games last year. This year the coach has set a goal for the team to score at least 5 more points per game. If 18 games are scheduled for this year, write an inequality that represents the total number of points the team must equal or exceed to meet their season goal.

Test Preparation

62. MULTIPLE CHOICE Translate the phrase "a number decreased by the quotient of three and four."

Ⓐ $n - \frac{3}{4}$ Ⓑ $\frac{3}{4} - n$ Ⓒ $\frac{n-3}{4}$ Ⓓ $\frac{3}{4-n}$

63. MULTIPLE CHOICE Give the correct algebraic translation of "Howard's hourly wage h is \$2 greater than Marla's hourly wage m."

Ⓐ $h < m + 2$ Ⓑ $h = m + 2$ Ⓒ $m = h + 2$ Ⓓ $h > m + 2$

64. MULTIPLE CHOICE A jet is flying nonstop from Baltimore, Maryland, to Jacksonville, Florida, at a speed r of 500 miles per hour. The distance d between the two cities is about 680 miles. Which equation models the number of hours t the flight will take?

Ⓐ $t = \frac{680}{500}$ Ⓑ $t = 680(500)$ Ⓒ $680 = \frac{t}{500}$ Ⓓ $t = \frac{500}{680}$

Challenge

SCHOOL DANCE **In Exercises 65 and 66, use the following information.** You are in charge of the music for a school dance. The school's budget allows only \$300 for music, which is enough to hire a disc jockey for 4 hours. You would rather hire a live band, but the band charges \$135 per hour. Your school does not allow students to be charged an admission fee. To raise the money for a live band, you obtain permission for a voluntary contribution of \$1.25 per person.

EXTRA CHALLENGE
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65. How much extra money do you need to raise?

66. How many students must contribute \$1.25 to cover the cost of a live band?

MIXED REVIEW

COMPARING DECIMALS **Compare using <, =, or >.** (Skills Review, p. 779)

67. 0.3 ? 0.30 **68.** 21.1 ? 20.99 **69.** 6.7 ? 6.079

70. 5.68 ? 5.678 **71.** 0.333 ? 0.3333 **72.** 18.45 ? 18.5

73. EATING HABITS The table gives the number of servings per day of fruits and vegetables consumed by adults in California. Create a bar graph of the data. (Skills Review, pp. 792–794)

Servings of Fruits and Vegetables Eaten by California Adults					
Year	1989	1991	1993	1995	1997
Servings per day	3.8	3.9	3.7	4.1	3.8

▶ Source: California Department of Health Services

74. PLANT GROWTH Kudzu is a vine native to Japan. Kudzu can grow a foot per day during the summer months. Write an expression that shows how much a 20-foot kudzu vine can grow during August. (Review 1.1)

EVALUATING VARIABLE EXPRESSIONS **Evaluate the expression.** (Review 1.3)

75. $4 + 3x$ when $x = 2$ **76.** $y \div 8$ when $y = 32$

77. $5 \cdot 2p^2$ when $p = 6$ **78.** $t^4 - t$ when $t = 7$

FOCUS ON APPLICATIONS

REAL LIFE **PLANT GROWTH** Kudzu was introduced to the United States in 1876. Today, kudzu covers over 7 million acres of the southern United States.

QUIZ 2

Self-Test for Lessons 1.4 and 1.5

Evaluate the expression. (Lesson 1.3)

1. $12 \div (7 - 3)^2 + 2$

2. $32 - 5 \cdot (2 + 1) + 4$

3. $x^2 + 4 - x$ when $x = 6$

4. $y \div 3 + 2$ when $y = 30$

5. $\frac{r}{s} \cdot 7$ when $r = 30$ and $s = 5$

6. $5x^2 - y$ when $x = 4$ and $y = 26$

Check whether the given number is a solution of the equation or inequality. (Lesson 1.4)

7. $2x + 6 = 18; 9$

8. $13 - 3x = 7; 2$

9. $4y + 7 = 5 + 5y; 2$

10. $2s + s = 4s; 6$

11. $3x - 4 > 0; 2$

12. $8 - 2y > 4; 3$

13. **PIZZA PARTY** You and three friends bought a pizza. You paid \$2.65 for your share $\left(\frac{1}{4}\text{ of the pizza}\right)$. Write an equation that models the situation. What was the total cost of the pizza? (Lesson 1.5)

MATH & History — Problem Solving

APPLICATION LINK
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THEN

IN THE NINTH CENTURY, mathematician al-Khwarizmi studied at the House of Wisdom in Baghdad. He developed and published important concepts in algebra. In the 1100s, al-Khwarizmi's text was translated from Arabic into Latin, making his ideas available to Western scholars. The step-by-step problem solving techniques invented by al-Khwarizmi are called *algorithms*—a Latinized form of al-Khwarizmi.

NOW

TODAY, computer programmers use algorithms to write new programs. Algorithms are also an important part of every algebra course. In this chapter you learned the following algorithm.

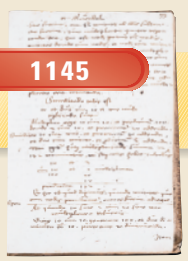
Write a verbal model. → **Assign labels.** → **Write an algebraic model.**

1. Write a word problem and use the algorithm shown to solve the problem.
2. Give two more examples of step-by-step methods you have learned in your study of mathematics. Show a worked-out problem using each method.

825

Scholars study at the House of Wisdom.

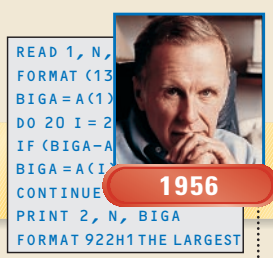
al-Khwarizmi's work is translated into Latin.



1145

1840

Ada Byron Lovelace writes the first computer program.



1956

John Backus develops the programming language Fortran.