

Rates, Ratios, and Percents

GOAL 1 USING RATES AND RATIOS

In this lesson you will apply rates, ratios, and percents and use your equation solving skills to solve problems. If a and b are two quantities measured in different units, then the **rate of a per b** is $\frac{a}{b}$. Rates are often expressed as *unit rates*. A **unit rate** is a rate per one given unit, such as 60 miles per 1 gallon.

EXAMPLE 1 Interpreting Large Numbers

SPENDING How can you relate this information to individual spending?

Estimated Spending for Clothes and Food in the United States in 1996		
Clothing & Shoes	Restaurants	Food at home
\$183 billion	\$190 billion	\$300 billion

 **DATA UPDATE** of U.S. Bureau of Labor Statistics at www.mcdougallittell.com

SOLUTION One solution is to find the average rate of spending *per person* so that people can compare themselves to the average. You can do this by dividing by the total population, which was about 266 million in 1996.

Clothing and shoes: about \$688 per person

Restaurants: about \$714 per person

Food at home: about \$1128 per person

EXAMPLE 2 Using Collected Data

MILEAGE You have recorded your car mileage and gasoline use for 2 months. Estimate the number of miles you can drive on a full 18-gallon tank of gasoline.

Number of miles	290	242	196	237	184
Number of gallons	12.1	9.8	8.2	9.5	7.8

SOLUTION You drove a total of 1149 miles and used 47.4 gallons of gasoline, so your average rate was $1149 \div 47.4$, or about 24.2 miles per gallon. Let x represent the number of miles you can drive on 18 gallons. To estimate x , you can solve the following equation.

$$\frac{\text{Number of miles} \rightarrow x}{\text{Number of gallons} \rightarrow 18} = 24.2 \leftarrow \text{Average rate in miles per gallon}$$

▶ The solution is $x = 435.6$, so your estimate is 436 miles on an 18-gallon tank.

What you should learn

GOAL 1 Use rates and ratios to model and solve **real-life** problems, as in **Example 3**.

GOAL 2 Use percents to model and solve **real-life** problems, as in **Example 6**.

Why you should learn it

▼ To solve **real-life** problems such as interpreting the results of a survey on outdoor recreation in **Exs. 40–42**.





EXAMPLE 3 Applying Unit Analysis

- a. You are visiting Mexico and you want to exchange \$180 for pesos. The rate of currency exchange is 9.990 pesos per United States dollar. You want to find out how many pesos you will receive.

UNIT ANALYSIS You can use unit analysis to write an equation.

$$\cancel{\text{dollars}} \cdot \frac{\cancel{\text{pesos}}}{\cancel{\text{dollar}}} = \text{pesos}$$

$$D \cdot \frac{9.990}{1} = P \quad \text{Write equation.}$$

$$180 \cdot \frac{9.990}{1} = P \quad \text{Substitute 180 for } D \text{ dollars.}$$

$$1798.2 = P \quad \text{Simplify.}$$

▶ You will receive 1798 pesos.

- b. When you leave Mexico, you are surprised to find that you have exactly 180 pesos left. You want to find out how many United States dollars you will receive. The rate of currency exchange is now 9.987 pesos per dollar.

UNIT ANALYSIS $\cancel{\text{pesos}} \cdot \frac{\cancel{\text{dollar}}}{\cancel{\text{pesos}}} = \text{dollars}$

$$P \cdot \frac{1}{9.987} = D \quad \text{Write equation.}$$

$$180 \cdot \frac{1}{9.987} = D \quad \text{Substitute 180 for } P \text{ pesos.}$$

▶ After simplifying, you find that $D \approx 18.02343046$. You will receive \$18.

EXAMPLE 4 Using Ratios to Write an Equation

TULIPS You tested a sample of 50 tulip bulbs from a shipment of 4500 bulbs for resistance to a type of mold. You found that 32 of the bulbs had a natural resistance, and 18 did not. Use these results to estimate the number of bulbs in the shipment that will have a natural resistance to the mold.

SOLUTION

One way to answer the question is to write a ratio. Let x represent the number of resistant bulbs in the shipment.

$$\frac{\text{Resistant bulbs in sample}}{\text{Total bulbs in sample}} = \frac{\text{Resistant bulbs in shipment}}{\text{Total bulbs in shipment}}$$

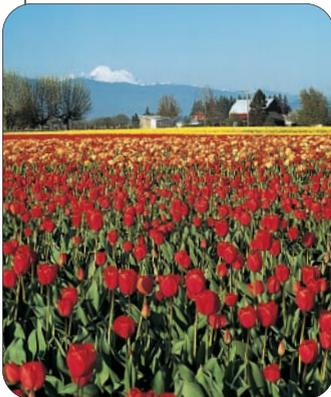
$$\frac{32}{50} = \frac{x}{4500} \quad \text{Write equation.}$$

$$4500 \cdot \frac{32}{50} = x \quad \text{Multiply each side by 4500.}$$

$$2880 = x \quad \text{Simplify.}$$

▶ About 2880 of the 4500 bulbs in the shipment will be resistant to the mold.

FOCUS ON APPLICATIONS



TULIPS About 25% of tulip bulbs sold in the United States are grown in the United States. The Skagit Valley area, shown above, is the largest U.S. producer of tulips.

GOAL 2 USING PERCENTS



EXAMPLE 5 Finding Percents

What percent of 15–17-year-olds said that they are dating?

A Survey of Teen Dating		
Age Group	Number Surveyed	Number Who Are Dating
12–14-year-olds	1074	216
15–17-year-olds	992	482
18–19-year-olds	307	205

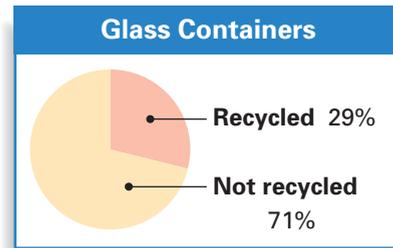
► Source: Simmons Market Research Bureau

SOLUTION To find the percent, divide the number who are dating by the number surveyed and then write the result as a percent.

$\frac{482}{992} \approx 0.485887097$, so about 49% of 15–17-year-olds said that they are dating.

EXAMPLE 6 Using a Graph

GLASS RECYCLING The circle graph shows the percent of glass containers recycled in the United States in a recent year. About 3.2 million tons of glass containers were recycled. Estimate the total weight of glass containers used that year.



DATA UPDATE of U.S. Environmental Protection Agency data at www.mcdougallittell.com

SOLUTION

VERBAL MODEL

$$\begin{array}{|c|} \hline \text{Percent of glass containers recycled (in decimal form)} \\ \hline \end{array} \cdot \begin{array}{|c|} \hline \text{Total weight of glass containers used} \\ \hline \end{array} = \begin{array}{|c|} \hline \text{Weight of glass containers recycled} \\ \hline \end{array}$$

LABELS

$$\begin{array}{l} \text{Percent of glass containers recycled} = 0.29 \quad (\text{no units}) \\ \text{Total weight of glass containers used} = x \quad (\text{tons}) \\ \text{Weight of glass containers recycled} = 3,200,000 \quad (\text{tons}) \end{array}$$

ALGEBRAIC MODEL

$$0.29 \cdot x = 3,200,000$$

Write algebraic model.

$$x = \frac{3,200,000}{0.29}$$

Divide each side by 0.29.

$$x \approx 11,034,482.76$$

Simplify.

► About 11.0 million tons of glass containers were used that year.

FOCUS ON APPLICATIONS



GLASS RECYCLING When one glass bottle is recycled, the energy saved could light a 100-watt light bulb for four hours.

GUIDED PRACTICE

Vocabulary Check ✓

Concept Check ✓

1. Explain the difference between a rate and a ratio.
2. Which model would you use to change 14 yards to feet? Explain.
A. $14 \text{ yards} \cdot \frac{1 \text{ yard}}{3 \text{ feet}}$ B. $14 \text{ yards} \cdot \frac{3 \text{ feet}}{1 \text{ yard}}$
3.  **SURVEY** You took a survey of your class and found that 18 out of the 31 students have a pet at home. Explain how you can use your results to make a prediction for the 1746 students in your school.
4. Look back at Example 1. Find the per person rate of spending for a month for each of the given categories.

Skill Check ✓

EXCHANGE RATES Convert the currency using the given exchange rate.

5. Convert 200 euros to United States dollars. (1 euro is 1.066 dollar.)
6. Convert 340 U.S. dollars to South African rand. (1 rand is 0.607 dollar.)

In Exercises 7–9, write and solve an equation to find the unknown number.

7. 45% of 280 = $\underline{\quad}$ 8. 7.5% of 340 = $\underline{\quad}$ 9. 20% of $\underline{\quad}$ = 15

10.  **TIPPING** What percent was the server's tip if the customer left \$1.75 for a \$12.50 meal?

PRACTICE AND APPLICATIONS

STUDENT HELP

▶ **Extra Practice**
to help you master
skills is on p. 799.

UNIT RATES Find the unit rate.

11. \$2 for 5 cans of dog food
12. \$121.50 for working 18 hours
13. \$1.39 for $1\frac{1}{2}$ quarts of juice
14. 6 ounces for 2.5 servings

FINDING SPEEDS In Exercises 15–18, find the average speed.

15. Fly 1200 miles in 4 hours
16. Hike 52 miles in 3 days
17. Swim 2 miles in 40 minutes
18. Drive 69 kilometers in $\frac{3}{4}$ hour

19. *Writing* A store sells a box of 5 frozen yogurt bars for \$1.20. The store also sells a box containing 7 of the same frozen yogurt bars for \$1.59. Which is the better buy? Explain how you decided.

IN ONE YEAR In Exercises 20 and 21, find the average rate as indicated.

20. The United States postal service receives 38 million changes of address per year. Find the rate per day. Round to the nearest thousand.
21. 1,100,000,000 miles are driven to move the mail in the United States per year. Find the rate per week. Round to the nearest million miles.
22.  **EQUATION SOLVING** Give a calculator keystroke sequence that you could use to find the value of x in Example 4.

STUDENT HELP

▶ HOMEWORK HELP

Example 1: Exs. 11–21

Example 2: Exs. 11–21,
27–29

Example 3: Exs. 23–26,
30–31

Example 4: Ex. 32

Example 5: Exs. 33–37

Example 6: Exs. 38–42

MEASUREMENT In Exercises 23–26, convert the measure. Round your answer to the nearest tenth.

23. 12 fluid ounces to cups (1 cup = 8 fluid ounces)
24. 21 inches to centimeters (1 inch = 2.54 centimeters)
25. 56 miles to kilometers (1 mile = 1.609 kilometers)
26. length of a football field (100 yards, not including the end zones) to meters (1 yard = 0.9144 meter)
27.  **CAR MILEAGE** A car uses fuel at a rate of 15 miles per gallon. Estimate how many miles the car can travel on 20 gallons of fuel.

 **BOOKS** A library has 14,588 books which fill its 313 equal-size shelves.

28. What is the average number of books per shelf?
29. The library plans to install 50 new shelves of this size. Write and solve an equation to estimate how many more books the library will be able to hold.

 **EXCHANGE RATES** In Exercises 30 and 31, use 114.520 yen per United States dollar as the rate of currency exchange. You are visiting Japan and have taken \$325 to spend on your trip.

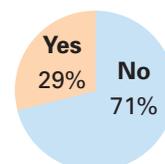
30. If you exchange the entire amount, how many yen will you receive?
31. You have 605 yen left after your trip. How many dollars will you get back?
32.  **CALLING FROM THE AIR** You are conducting a survey on the use of airplane phones. You survey 320 adults and find that 288 of them never made a phone call from an airplane. If you surveyed 3500 adults, how many of them would you predict *have* made a phone call from an airplane? Explain.

FINDING PERCENTS Find the percent. Round to the nearest whole percent.

33. \$2.25 tip on a cab fare of \$14
34. Tax of \$.68 on an item priced at \$11.29
35. 292 people in favor out of 450 people surveyed

 **MOON ROCKS** In Exercises 36–38, round to the nearest tenth. A total of 382 kilograms of lunar samples (rocks, dust, and so on) were collected during the six Apollo moon landings between 1969 and 1972.

36. The largest rock collected weighs 11.7 kilograms. This single rock is what percent of the total weight of the samples?
37. The 110.5 kilograms of lunar samples collected by the Apollo 17 astronauts represent what percent of the total weight of the samples?
38. About 7.5% of the lunar samples (by weight) have been analyzed and then returned for storage in the Return Sample Vault at NASA's Johnson Space Center. What is the combined weight of the samples in this vault?
39.  **CONSUMER ACTION** Americans ages 18 years and older were asked, "Have you ever stopped buying a product specifically because the manufacturer of that product pollutes the environment?" Of those surveyed, 719 said no. About how many people took part in the survey?



► Source: Wirthlin Worldwide

STUDENT HELP

 **HOMEWORK HELP**
Visit our Web site
www.mcdougallittell.com
for help with Exs. 36–38.

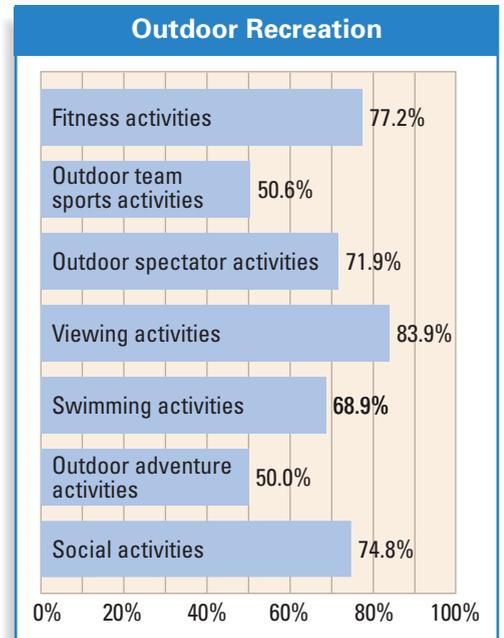
 **APPLICATION LINK**
Visit our Web site
www.mcdougallittell.com
for more information
about moon rocks.



In the survey, Basketball was the most popular team sport.

INTERPRETING DATA In Exercises 40–42, use the graph. It shows the percent of 16-to-24-year-olds participating in the top seven categories of outdoor recreation.

40. Viewing activities include sightseeing, visiting a nature center, and so on. Estimate how many students participate in viewing activities in a college with 1400 students.
41. Estimate the survey size if a survey of 16-to-24-year-olds found that 270 of them participate in fitness activities.
42. Write your own question based on the survey data. Then find the answer.



► Source: National Survey on Recreation and the Environment

43. **CRITICAL THINKING** Compare Example 2 and Example 4 in Goal 1. How are the methods shown alike? How are they different?

Test Preparation

QUANTITATIVE COMPARISON In Exercises 44–46, choose the statement below that is true about the given values of x .

- (A) The value of x in column A is greater.
- (B) The value of x in column B is greater.
- (C) The two values of x are equal.
- (D) The relationship cannot be determined from the given information.

	Column A	Column B
44.	The solution of the equation $160\% \text{ of } 400 = x$	The solution of the equation $16\% \text{ of } 4000 = x$
45.	The solution of the equation $\frac{x}{3} = \frac{7}{8}$	The solution of the equation $\frac{x}{8} = \frac{7}{3}$
46.	The solution of the equation $15\% \text{ of } x = 30$	The solution of the equation $30\% \text{ of } x = 15$

★ Challenge

LITERATURE CONNECTION Use this information in Exercises 47 and 48.

In *Citizen of the Galaxy* by Robert Heinlein, Thorby learns that he owns “an interest in a company . . . through a chain of six companies—18% of 31% of 43% of 19% of 44% of 27%, a share so microscopic that he lost track.” Let x represent the total worth of the company in which Thorby owns a share.

47. Write and simplify an expression to show Thorby’s share of the company.
48. Suppose Thorby’s share of the company is worth 1000 credits. Write and solve an equation to find the total worth of the company.

EXTRA CHALLENGE

► www.mcdougallittell.com

MIXED REVIEW

WRITING INEQUALITIES Graph the numbers on a number line. Then write two inequalities that compare the two numbers. (Review 2.1)

49. 4 and -3 50. -2.4 and 3.2 51. -0.2 and -0.21
52. -1.8 and -2 53. $\frac{3}{4}$ and $-\frac{5}{6}$ 54. $-1\frac{1}{3}$ and -1.75

SOLVING EQUATIONS Solve the equation. Round the result to the nearest hundredth. Check the rounded solution. (Review 3.6)

55. $-11a - 16 = 5$ 56. $2(4 - 3x) = -x$
57. $2.62x - 4.03 = 7.65 - 5.34x$ 58. $2.6(4.2y - 6.5) = -7.1y - 2.8$

59.  **POPULATION PROJECTIONS** The table shows the projected number (in millions) of people 85 years and older in the United States for different years. Make a line graph of the data. (Review 1.6 for 4.1)

Year	2000	2010	2020	2030	2040	2050
Number of people 85 years and older (in millions)	4.1	5.0	5.0	5.8	8.3	9.6

► Source: *Statistical Abstract of the United States, 1997*

QUIZ 3

Self-Test for Lessons 3.7 and 3.8

Write y as a function of x and evaluate when x is -2 , 0 , and 2 . (Lesson 3.7)

1. $3x + 2y = 12$ 2. $\frac{y}{3} - 4 = x$ 3. $\frac{1}{2}(2x + 10y) = 8$

Solve the given formula for the specified variable. (Lesson 3.7)

4. Solve $V = lwh$ for w . 5. Solve $A = \frac{1}{2}bh$ for h . 6. Solve $P = C + rC$ for r .

Convert the given currency or measure. (Lesson 3.8)

7. 125 Swiss francs to United States dollars (1 Swiss franc is 0.746 dollar.)
8. 50.8 centimeters to inches (1 inch = 2.54 centimeters)

In Exercises 9–11, write and solve an equation to find the answer. (Lesson 3.8)

9.  **CAR MILEAGE** A car uses fuel at a rate of 33 miles per gallon. Estimate how many miles the car will travel on 12 gallons of fuel.
10.  **POPULATION** A school's population increased by 42 students. This was a 3% increase. About how many students attended before the increase?
11.  **SURVEY** Out of 525 people surveyed, 210 answered yes. Estimate how many people would answer yes if 900 people were surveyed.
12. Use the information in Exercise 11. What percent of the 525 people surveyed answered yes? (Lesson 3.8)