11.6 Adding and Subtracting Rational Expressions

What *you should learn*

GOAL 1 Add and subtract rational expressions that have like denominators.

GOAL 2 Add and subtract rational expressions that have unlike denominators.

Why *you should learn it*

▼ To model **real-life** problems, such as planning your route for a car trip in **Example 5**.

REAL LIFE

GOAL 1 FRACTIONS WITH LIKE DENOMINATORS

To add or subtract rational expressions with *like* denominators, combine their numerators, and write the result over the common denominator.

ADDING OR SUBTRACTING WITH LIKE DENOMINATORS

Let a, b, and c be polynomials, with $c \neq 0$.

TO ADD, add the numerators. $\quad \frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$

TO SUBTRACT, subtract the numerators. $\quad \frac{a}{c} - \frac{b}{c} = \frac{a-b}{c}$

EXAMPLE 1 *Adding and Subtracting Expressions*

a. $\frac{5}{x} + \frac{x-5}{x} = \frac{5+x-5}{x}$ **Add numerators.**

$= \frac{x}{x}$ **Simplify.**

$= 1$

b. $\frac{7}{2m-3} - \frac{3m}{2m-3} = \frac{7-3m}{2m-3}$ **Subtract numerators.**

EXAMPLE 2 *Simplifying After Subtracting*

Simplify $\frac{4x}{3x^2-x-2} - \frac{x-2}{3x^2-x-2}$.

SOLUTION

$\frac{4x}{3x^2-x-2} - \frac{x-2}{3x^2-x-2} = \frac{4x-(x-2)}{3x^2-x-2}$ **Subtract.**

$= \frac{3x+2}{3x^2-x-2}$ **Simplify.**

$= \frac{3x+2}{(3x+2)(x-1)}$ **Factor.**

$= \frac{\cancel{3x+2}}{\cancel{(3x+2)}(x-1)}$ **Divide out common factor.**

$= \frac{1}{x-1}$ **Simplified form**

STUDENT HELP

Skills Review
For help with adding fractions with unlike denominators, see pp. 781–783.

GOAL 2 FRACTIONS WITH UNLIKE DENOMINATORS

To add or subtract rational expressions with *unlike* denominators, you must first rewrite the expressions so that they have *like* denominators. The like denominator that you usually use is the least common multiple of the original denominators. It is called the **least common denominator** or LCD.

EXAMPLE 3 *Adding with Unlike Denominators*

Simplify $\frac{7}{6x} + \frac{5}{8x^2}$.

SOLUTION

To find the least common denominator, first completely factor the denominators. You get $6x = 2 \cdot 3 \cdot x$ and $8x^2 = 2^3 \cdot x^2$. The LCD contains the highest power of each factor that appears in either denominator, so the LCD is $2^3 \cdot 3 \cdot x^2$, or $24x^2$.

$$\frac{7}{6x} + \frac{5}{8x^2} = \frac{7 \cdot 4x}{6x \cdot 4x} + \frac{5 \cdot 3}{8x^2 \cdot 3} \qquad \text{Rewrite fractions using LCD.}$$

$$= \frac{28x}{24x^2} + \frac{15}{24x^2} \qquad \text{Simplify numerators and denominators.}$$

$$= \frac{28x + 15}{24x^2} \qquad \text{Add fractions.}$$

STUDENT HELP

Study Tip
You can always find a common denominator by multiplying the two denominators, but it is often easier to use the LCD. For instance, using $24x^2$ rather than $48x^3$ in Example 3 makes the expressions in the numerators simpler.

EXAMPLE 4 *Subtracting with Unlike Denominators*

Simplify $\frac{x+2}{x-1} - \frac{12}{x+6}$.

SOLUTION

Neither denominator can be factored. The least common denominator is the product $(x - 1)(x + 6)$ because it must contain both of these factors.

$$\frac{x+2}{x-1} - \frac{12}{x+6} = \frac{(x+2)(x+6)}{(x-1)(x+6)} - \frac{12(x-1)}{(x-1)(x+6)} \qquad \text{Rewrite fractions using LCD.}$$

$$= \frac{x^2 + 8x + 12}{(x-1)(x+6)} - \frac{12x - 12}{(x-1)(x+6)} \qquad \text{Simplify numerators. Leave denominators in factored form.}$$

$$= \frac{x^2 + 8x + 12 - (12x - 12)}{(x-1)(x+6)} \qquad \text{Subtract fractions.}$$

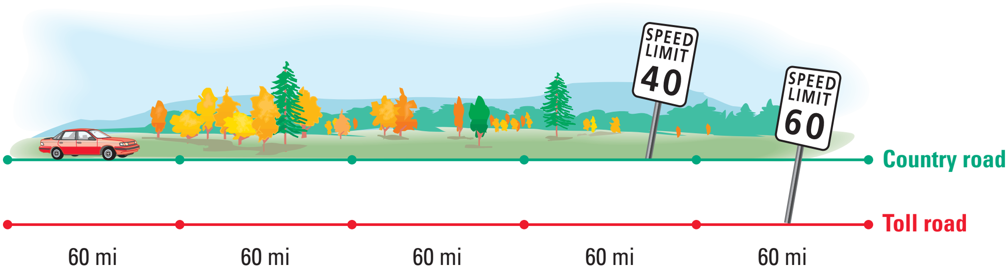
$$= \frac{x^2 + 8x + 12 - 12x + 12}{(x-1)(x+6)} \qquad \text{Use distributive property.}$$

$$= \frac{x^2 - 4x + 24}{(x-1)(x+6)} \qquad \text{Combine like terms.}$$

REAL LIFE
Travel Planning

EXAMPLE 5 *Writing and Using a Model*

You are planning a 300-mile car trip. You can make the trip using a combination of two roads: a toll road on which you can average 60 miles per hour and a scenic country road on which you can average 40 miles per hour. To avoid tolls and to enjoy the scenery, you want to spend as much time as possible on the country road. However, you do not want your total trip time to be too long.



Let x represent the number of miles you travel on the country road.

a. Write an expression for the total time.

b. Evaluate the expression at 60-mile intervals.

SOLUTION

PROBLEM SOLVING STRATEGY

a. VERBAL MODEL

$$\boxed{\text{Total time}} = \frac{\text{Distance on country road}}{\text{Speed on country road}} + \frac{\text{Distance on toll road}}{\text{Speed on toll road}}$$

LABELS

Total time $= T$ (hours)

Distance on country road $= x$ (miles)

Speed on country road $= 40$ (miles per hour)

Distance on toll road $= 300 - x$ (miles)

Speed on toll road $= 60$ (miles per hour)

ALGEBRAIC MODEL

$$T = \frac{x}{40} + \frac{300 - x}{60}$$ Write algebraic model.

$$= \frac{3x}{120} + \frac{2(300 - x)}{120}$$ Rewrite fractions using LCD.

$$= \frac{3x + 600 - 2x}{120}$$ Add fractions.

$$= \frac{x + 600}{120}$$ Combine like terms.

b. Substitute x-values of 0, 60, 120, 180, 240, and 300 into the expression.

Distance (country) x	0	60	120	180	240	300
Total time T	5.0	5.5	6.0	6.5	7.0	7.5

The table can help you decide how many miles to drive on the country road.

GUIDED PRACTICE

Vocabulary Check ✓

1. Explain what is meant by the *least common denominator* of two rational expressions.

Concept Check ✓

2. **ERROR ANALYSIS** **Explain the error in the following problem.**

$$\frac{3}{x-1} - \frac{2}{x} = \frac{3x}{x(x-1)} - \frac{2(x-1)}{x(x-1)}$$

$$= \frac{3x}{x(x-1)} - \frac{2x-2}{x(x-1)}$$

$$= \frac{3x-2x-2}{x(x-1)} = \frac{x-2}{x(x-1)}$$

3. You can use $x - 3$ as the LCD when finding the sum $\frac{5}{x-3} + \frac{2}{3-x}$. What number can you multiply the numerator and the denominator of the second fraction by to get an equivalent fraction with $x - 3$ as the new denominator?

Skill Check ✓

Simplify the expression.

4. $\frac{1}{3x} + \frac{5}{3x}$

5. $\frac{5x}{x+4} + \frac{20}{4+x}$

6. $\frac{x}{x^2-9} - \frac{3x+1}{x^2-9}$

7. $\frac{3}{10x} - \frac{1}{4x^2}$

8. $\frac{x+6}{x+1} - \frac{4}{2x+3}$

9. $\frac{x-2}{2x-10} + \frac{x+3}{x-5}$

PRACTICE AND APPLICATIONS

STUDENT HELP

Extra Practice to help you master skills is on p. 807.

ADDING AND SUBTRACTING **Simplify the expression.**

10. $\frac{7}{2x} + \frac{x+2}{2x}$

11. $\frac{4}{x+1} + \frac{2x-2}{x+1}$

12. $\frac{7x}{x^3} - \frac{6x}{x^3}$

13. $\frac{2}{3x-1} - \frac{5x}{3x-1}$

14. $\frac{3x}{4x+1} + \frac{5x}{4x+1}$

15. $\frac{-8}{3x^2} + \frac{11}{3x^2}$

16. $\frac{9}{4x} + \frac{7}{-5x}$

17. $\frac{11}{6x} + \frac{2}{13x}$

18. $\frac{9}{5x} - \frac{2}{x^2}$

19. $\frac{4}{x+4} - \frac{7}{x-2}$

20. $\frac{3}{x+3} + \frac{4x}{2x+6}$

21. $\frac{9}{x^2-3x} + \frac{3}{x-3}$

22. $\frac{2x}{x-1} - \frac{7x}{x+4}$

23. $\frac{x}{x-10} + \frac{x+4}{x+6}$

24. $\frac{4x}{5x-2} - \frac{2x}{5x+1}$

25. $\frac{x+8}{3x-1} + \frac{x+3}{x+1}$

26. $\frac{3x+10}{7x-4} - \frac{x}{4x+3}$

27. $\frac{2x+1}{3x-1} - \frac{x+4}{x-2}$

28. $\frac{x}{x^2+5x-24} + \frac{8}{x^2+5x-24}$

29. $\frac{x^2+1}{x^2-4} + \frac{5x}{x^2-4} - \frac{2x+11}{x^2-4}$

30. $\frac{x^2-9}{x+3} + \frac{x^2+9}{x-3}$

31. $\frac{2}{x+1} + \frac{3}{x-2} + \frac{3}{x+4}$

STUDENT HELP

HOMEWORK HELP

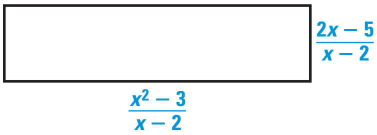
Example 1: Exs. 10–31
Example 2: Exs. 10–31
Example 3: Exs. 16–31
Example 4: Exs. 18–31
Example 5: Exs. 36–38

FOCUS ON APPLICATIONS

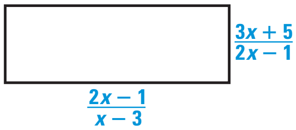
TRAVEL BY BOAT Many cities with harbors offer a variety of types of water transportation. These include regular commuter service, sight-seeing tours, and recreational trips.

GEOMETRY CONNECTION **In Exercises 32 and 33, find an expression for the perimeter of the rectangle.**

32.



33.



LOGICAL REASONING **In Exercises 34 and 35, use the expression $\frac{2x-5}{x-2}$ and the table feature of a graphing calculator or spreadsheet software.**

34. Construct a table that shows the value of the numerator, the value of the denominator, and the value of the entire rational expression when the value of x is 10, 100, 1000, 10,000, 100,000, and 1,000,000.

35. Use the table from Exercise 34. As x gets large, what happens to the values of the numerator? of the denominator? of the entire rational expression? Why do you think these results occur?

TRAVEL BY BOAT **In Exercises 36–38, use the following information.**
A boat moves through still water at x kilometers (km) per hour. It travels 24 km upstream against a current of 2 km per hour and then returns with the current. The rate upstream is $x - 2$ because the boat moves against the current, and the rate downstream is $x + 2$ because the boat moves with the current.

36. Write an expression for the total time for the round trip.

37. Write your answer to Exercise 36 as a single rational expression.

38. Use your answer to Exercise 37 to find how long the round trip will take if the boat travels 10 kilometers per hour through still water.

MOWING LAWNS **In Exercises 39–41, use the following information.**
You are choosing a business partner for a student lawn-care business you are starting. It takes you an average of 35 minutes to mow a lawn, so your rate is 1 lawn in 35 minutes or $\frac{1}{35}$ of a lawn per minute. Let x represent the average time (in minutes) it takes a possible partner to mow a lawn.

39. Write an expression for the partner's rate (that is, the part of a lawn the partner can mow in 1 minute). Then write an expression for the combined rate of you and your partner (the part of a lawn that you both can mow in 1 minute if you work together).

40. Write your answer to Exercise 39 as a single rational expression.

41. The table shows the mowing times of possible partners.

Possible mowing partner	A	B	C
Mowing time (in minutes)	40	45	55

a. Use your expression from Exercise 40 to find the combined rate of you with each possible partner.

b. Which partner(s) can you choose if you want a combined rate of $\frac{1}{20}$ of a lawn per minute or faster?

STUDENT HELP
INTERNET HOMEWORK HELP Visit our Web site www.mcdougallittell.com for help with Exs. 39–41.

COMBINING OPERATIONS **Simplify the expression.**

42. $\dfrac{2x}{x+5} - \dfrac{3x+2}{x+5} - \dfrac{4}{x+5}$

43. $\left(\dfrac{3x^2}{56}\right)\left(\dfrac{3}{x} + \dfrac{5}{x}\right)$

44. $\left(\dfrac{3x-5}{x} + \dfrac{1}{x}\right) \div \left(\dfrac{x}{6x-8}\right)$

45. $\dfrac{x-2}{x+6} \div \dfrac{x+8}{4x-24} \cdot \dfrac{x-8}{x-2}$

EXTENSION: TRINOMIAL DENOMINATORS **When you add rational expressions, you may need to factor a trinomial to find the LCD. Study the sample below. Then simplify the expressions in Exercises 46–49.**

Sample: $\dfrac{2x}{x^2-1} + \dfrac{3}{x^2+x-2} = \dfrac{2x}{(x+1)(x-1)} + \dfrac{3}{(x-1)(x+2)}$

The LCD is $(x+1)(x-1)(x+2)$.

Note: If you just used $(x^2-1)(x^2+x-2)$ as the common denominator, the factor $(x-1)$ would be included twice.

46. $\dfrac{2}{x-3} + \dfrac{x}{x^2+3x-18}$

47. $\dfrac{2}{x^2-4} + \dfrac{3}{x^2+x-6}$

48. $\dfrac{7x+2}{16-x^2} + \dfrac{7}{x-4}$

49. $\dfrac{5x-1}{2x^2-7x-15} - \dfrac{-3x+4}{2x^2+5x+3}$

Test Preparation

50. MULTIPLE CHOICE Find the LCD of $\dfrac{-2}{x+9}$ and $\dfrac{5x}{x^2+9x}$.

(A) x^2+9 (B) $x(x+9)$ (C) $x(x-9)$ (D) $(x+9)(x^2+9x)$

51. MULTIPLE CHOICE Simplify the expression $\dfrac{x}{x-1} - \dfrac{1}{2x+1}$.

(A) $\dfrac{x-1}{(x-1)(2x+1)}$ (B) $-\dfrac{x}{x-1}$

(C) $\dfrac{2x^2+1}{(x-1)(2x+1)}$ (D) $\dfrac{2x^2-1}{(x-1)(2x+1)}$

52. MULTIPLE CHOICE You are making a 350-mile car trip. You decide to drive a little faster to save time. Choose an expression for the time saved if the car's average speed s is increased by 5 miles per hour.

(A) $\dfrac{350}{s+5}$ (B) $\dfrac{s+5}{350} - \dfrac{s}{350}$ (C) $\dfrac{350}{s} - \dfrac{350}{s+5}$ (D) $350(s+5) - 350s$

★ **Challenge**

AVERAGE SPEED **In Exercises 53–55, you will write and simplify a general expression for the average speed traveled when making a round trip. Let *d* represent the one-way distance. Let *x* represent the speed while traveling there and let *y* represent the speed while traveling back.**

53. Write an expression for the total time for the round trip. Use addition to write your answer as a single rational expression.

54. To find the average speed on the trip, you need to divide the total distance by the total time. Use your answer from Exercise 53 to write an expression for the average speed. Then simplify that expression as far as possible.

55. What do you notice about the variables in the final answer? If your distance is doubled what happens to the average speed?

EXTRA CHALLENGE
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MIXED REVIEW

SIMPLIFYING EXPRESSIONS **Simplify the expression.** (Review 8.3 for 11.7)

56. $\frac{5}{10x}$ **57.** $\frac{4m^2}{6m}$ **58.** $\frac{16x^4}{32x^8}$ **59.** $\frac{42x^4y^3}{6x^3y^9}$

STANDARD FORM **Write the equation in standard form.** (Lesson 9.5 for 11.7)

60. $6x^2 = 5x - 7$ **61.** $9 - 6x = 2x^2$ **62.** $-4 + 3y^2 = y$

CHOOSING MODELS FROM DATA **Make a scatter plot of the data. Then tell whether a *linear, exponential,* or *quadratic* model fits the data.** (Review 9.8)

63. $(-1, 16), (0, 4), (1, -2), (2, -2), (3, 4), (5, 34)$

64. $(-5, 6), (-4, 3), (-2, -3), (-1, -6), (0, -9), (1, -12)$

65. **GAME SHOW** A contestant on a television game show must guess the price of a trip within \$1000 of the actual price in order to win. The actual price of the trip is \$8500. Write an absolute-value inequality that shows the range of possible guesses that will win the trip. (Review 6.4)

QUIZ 2

Self-Test for Lessons 11.4–11.6

Simplify the expression if possible. (Lesson 11.4)

1. $\frac{15x^2}{10x}$ **2.** $\frac{5x}{11x + x^2}$ **3.** $\frac{3 - x}{x^2 - 5x + 6}$ **4.** $\frac{x^2 - 7x + 12}{x^2 + 3x + 18}$

Simplify the expression. (Lessons 11.5 and 11.6)

5. $\frac{5x^2}{2x} \cdot \frac{14x^2}{10x}$

6. $\frac{5}{10 + 4x} \cdot (20 + 8x)$

7. $\frac{3x + 12}{4x} \div \frac{x + 4}{2x}$

8. $\frac{5x^2 - 30x + 45}{x + 2} \div (5x - 15)$

9. $\frac{x}{x^2 - 2x - 35} + \frac{5}{x^2 - 2x - 35}$

10. $\frac{4x - 1}{3x^2 + 8x + 5} - \frac{x - 6}{3x^2 + 8x + 5}$

CANOEING **In Exercises 11–13, use the following information.** You are on a canoe trip. You can paddle your canoe at a rate of x miles per hour in still water. The stream is flowing at a rate of 2 miles per hour, so your rate of travel downstream (with the current) is $x + 2$ miles per hour. You travel 15 miles downstream and 15 miles back upstream. (Lesson 11.6)

11. Write an expression for the travel time downstream and an expression for the travel time upstream.

12. Write and simplify an expression for the total travel time.

13. Find the total travel time if your rate of paddling in still water is about 4 miles per hour.