

WHAT did you learn?

WHY did you learn it?

Evaluate and graph a square-root function. (12.1)

Investigate walking speeds of dinosaurs. (p. 711)

Add, subtract, multiply, and divide radical expressions. (12.2)

Compare speeds of pole-vaulters. (p. 720)

Solve a radical equation. (12.3)

Find how much centripetal force a person experiences on an amusement park ride. (p. 726)

Solve a quadratic equation by completing the square. (12.4)

Investigate a proof of the quadratic formula. (p. 732)

Choose a method for solving a quadratic equation. (12.4)

Develop efficient problem solving skills. (p. 732)

Use the Pythagorean theorem and its converse. (12.5)

Calculate the distance from home plate to second base. (p. 740)

Find the distance and midpoint between two points in a coordinate plane. (12.6)

Apply the distance and midpoint formulas to model real-life situations. (pp. 746, 747)

Find the trigonometric ratios *sine*, *cosine*, and *tangent*. (12.7)

Determine cloud height above the ground. (p. 756)

Use logical reasoning and proof. (12.8)

Prove that statements are true or false. (pp. 759, 760)

How does Chapter 12 fit into the BIGGER PICTURE of algebra?

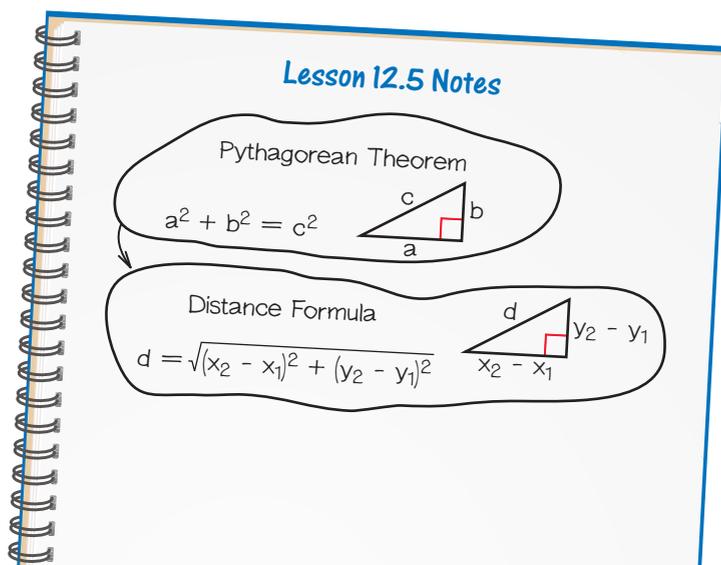
Radicals have many applications in geometry and in other fields. In this chapter you learned to add, subtract, multiply, and divide radical expressions and to solve radical equations.

The chapter also contains material that helps prepare you for other courses in mathematics. There are many applications involving geometry; an introduction to the trigonometric ratios sine, cosine, and tangent; and an introduction to some of the formal properties of algebra explored in Algebra 2.

STUDY STRATEGY

How did you use the notes in your notebook?

A map of ideas you made, using the **Study Strategy** on page 708, may look like this one.



VOCABULARY

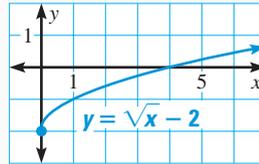
- square-root function, p. 709
- legs of a right triangle, p. 738
- midpoint between two points, p. 747
- postulates, or axioms, p. 758
- conjugates, p. 717
- hypothesis, p. 739
- midpoint formula, p. 747
- theorems, p. 759
- complete the square, p. 730
- conclusion, p. 739
- trigonometric ratio, p. 752
- conjecture, p. 759
- Pythagorean theorem, p. 738
- converse, p. 739
- sine, cosine, tangent, p. 752
- indirect proof, p. 760
- hypotenuse, p. 738
- distance formula, p. 745

12.1 FUNCTIONS INVOLVING SQUARE ROOTS

Examples on pp. 709–711

EXAMPLE To sketch the graph of $y = \sqrt{x} - 2$, note that the domain is the set of all nonnegative numbers. Then make a table of values, plot the points, and connect them with a smooth curve. The range is all numbers greater than or equal to -2 .

x	y
0	$y = \sqrt{0} - 2 = -2$
1	$y = \sqrt{1} - 2 = -1$
2	$y = \sqrt{2} - 2 \approx -0.6$
⋮	⋮
⋮	⋮
⋮	⋮



Identify the domain and the range of the function. Then graph the function.

1. $y = 11\sqrt{x}$
2. $y = 2\sqrt{x - 5}$
3. $y = \sqrt{x} + 3$

12.2 OPERATIONS WITH RADICAL EXPRESSIONS

Examples on pp. 716–718

EXAMPLE You can use radical operations to simplify radical expressions.

$$\begin{aligned}
 4\sqrt{20} - 3\sqrt{5} &= 4\sqrt{4 \cdot 5} - 3\sqrt{5} && \text{Perfect square factor} \\
 &= 4\sqrt{2 \cdot 2} \cdot \sqrt{5} - 3\sqrt{5} && \text{Product property} \\
 &= 8\sqrt{5} - 3\sqrt{5} && \text{Simplify.} \\
 &= 5\sqrt{5} && \text{Subtract like radicals.}
 \end{aligned}$$

Simplify the expression.

4. $\sqrt{5} + 2\sqrt{5} - \sqrt{3}$
5. $\sqrt{6} (2\sqrt{3} - 4\sqrt{2})$
6. $(3 - \sqrt{10})^2$
7. $(\sqrt{8} + \sqrt{3})^2$
8. $\frac{21}{\sqrt{3}}$
9. $\frac{8}{6 - \sqrt{4}}$

12.3

SOLVING RADICAL EQUATIONS

Examples on
pp. 722–724**EXAMPLE** Solve $\sqrt{3x - 2} = x$.

$$(\sqrt{3x - 2})^2 = x^2$$

$$3x - 2 = x^2$$

$$0 = x^2 - 3x + 2$$

$$0 = (x - 2)(x - 1)$$

$$x = 2 \text{ or } x = 1$$

Square both sides.

Simplify.

Write in standard form.

Factor.

Zero-product property

Solve the equation.

10. $2\sqrt{x} - 4 = 0$

11. $x = \sqrt{-4x - 4}$

12. $\sqrt{x - 3} + 2 = 8$

12.4

COMPLETING THE SQUARE

Examples on
pp. 730–733**EXAMPLE** Solve $x^2 - 6x - 1 = 6$ by completing the square.

$$x^2 - 6x = 7$$

Isolate the x^2 -term and the x -term.

$$x^2 - 6x + 9 = 7 + 9$$

Add $(-\frac{6}{2})^2$ to each side.

$$(x - 3)^2 = 16$$

Write left side as perfect square.

$$x - 3 = \pm 4$$

Find square root of each side.

$$x = 7 \text{ or } x = -1$$

Solve for x .

Solve the equation by completing the square.

13. $x^2 - 4x - 1 = 7$

14. $x^2 + 20x + 19 = 0$

15. $2x^2 - x - 4 = 10$

12.5

THE PYTHAGOREAN THEOREM AND ITS CONVERSE

Examples on
pp. 738–740**EXAMPLE** Given $a = 6$ and $c = 12$, find b .

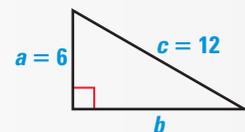
$$a^2 + b^2 = c^2$$

Write Pythagorean theorem.

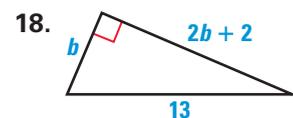
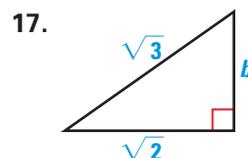
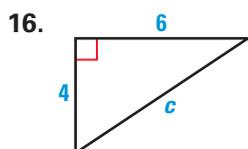
$$6^2 + b^2 = 12^2$$

Substitute for a and c .

$$b = 6\sqrt{3}$$

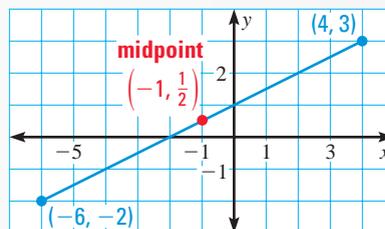
Solve for b .

Find each missing length.



EXAMPLE Find the distance d and the midpoint m between $(-6, -2)$ and $(4, 3)$.

$$\begin{aligned}d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\&= \sqrt{[4 - (-6)]^2 + [3 - (-2)]^2} \\&= \sqrt{125} \\&= 5\sqrt{5} \\m &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\&= \left(\frac{-6 + 4}{2}, \frac{-2 + 3}{2} \right) \\&= \left(-1, \frac{1}{2} \right)\end{aligned}$$



Find the distance and the midpoint between the two points.

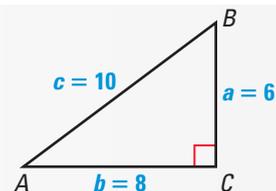
19. $(8, 5)$ and $(11, -4)$

20. $(-3, 6)$ and $(1, 7)$

21. $(-2, -2)$ and $(2, 8)$

EXAMPLES

$$\begin{aligned}\sin A &= \frac{\text{side opposite } \angle A}{\text{hypotenuse}} = \frac{a}{c} = \frac{6}{10} = 0.6 \\ \cos A &= \frac{\text{side adjacent to } \angle A}{\text{hypotenuse}} = \frac{b}{c} = \frac{8}{10} = 0.8 \\ \tan A &= \frac{\text{side opposite } \angle A}{\text{side adjacent to } \angle A} = \frac{a}{b} = \frac{6}{8} = 0.75\end{aligned}$$



22. Find the sine, the cosine, and the tangent of $\angle B$.

EXAMPLE Prove that for all numbers a and b , $(a + b) - b = a$.

$$\begin{aligned}(a + b) - b &= (a + b) + (-b) && \text{Definition of subtraction} \\ &= a + [b + (-b)] && \text{Associative property of addition} \\ &= a + 0 && \text{Inverse property of addition} \\ &= a && \text{Identity property of addition}\end{aligned}$$

23. Which basic axiom of algebra is represented by $\left(\frac{2}{3}\right)\left(\frac{4}{5}\right) = \left(\frac{4}{5}\right)\left(\frac{2}{3}\right)$?

24. Prove that $(c)(-b) = -cb$ for all real numbers c and b .

Identify the domain and the range of the function. Then sketch its graph.

1. $y = 12\sqrt{x}$

2. $y = \sqrt{2x + 7}$

3. $y = \sqrt{3x} - 3$

4. $y = \sqrt{x - 5}$

Simplify the expression.

5. $3\sqrt{2} - \sqrt{2}$

6. $(4 + \sqrt{7})(4 - \sqrt{7})$

7. $(4\sqrt{5} + 1)^2$

8. $\frac{8}{3 - \sqrt{5}}$

Solve the equation.

9. $\sqrt{y} + 6 = 10$

10. $\sqrt{2m + 3} - 6 = 4$

11. $n = \sqrt{9n - 18}$

12. $p = \sqrt{-3p + 18}$

Solve the equation by completing the square.

13. $x^2 - 6x = -5$

14. $x^2 - 2x = 2$

15. $x^2 + \frac{4}{5}x - 1 = 0$

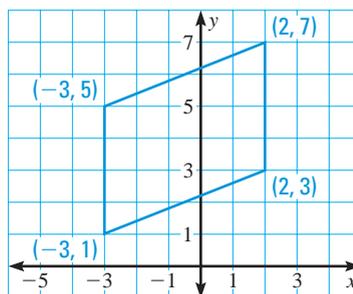
Determine whether the given lengths can be the sides of a right triangle.

16. 6, 18, 36

17. 9, 40, 41

18. 1.5, 3.6, 3.9

In Exercises 19 and 20, use the diagram.



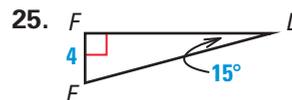
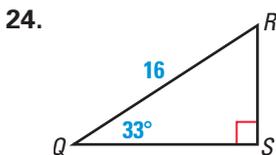
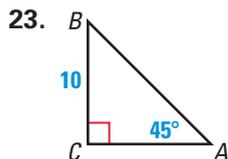
19. Find the perimeter of the parallelogram.

20. Find the coordinates of the midpoint of each side of the parallelogram.

21. Find the perimeter of the parallelogram whose vertices are the midpoints you found in Exercise 20.

22. Compare the perimeters you found in Exercises 19 and 21.

Find the missing lengths of the sides of the triangles. Round your answer to the nearest hundredth.



26. Prove that if a , b , and c are real numbers, and $a + c = b + c$, then $a = b$.

27. **PONY EXPRESS** Pony express stations were about 10 miles apart. The latitude-longitude coordinates of two former stations in western Nevada are (39.2 N, 119.0 W) and (39.2 N, 118.5 W). Find the coordinates of the station that was halfway between them.

28. **GATEWAY ARCH** You are standing 134 feet from the Gateway Arch in St. Louis, Missouri. You estimate that the angle between the ground and the line from where you stand to the top of the arch is 78° . Estimate the height of the Gateway Arch.