12.2 Operations with Radical Expressions

What you should learn

GOAL 1 Add, subtract, multiply, and divide radical expressions.

GOAL 2 Use radical expressions in **real-life** situations, as in finding the speed of a pole-vaulter in **Ex. 58**.

Why you should learn it

▼ To solve **real-life** problems such as finding distances to the horizon from a schooner in **Example 5**.

REAL LIFE

GOAL 1 USING RADICAL OPERATIONS

You can use the distributive property to simplify sums and differences of radical expressions when the expressions have the same radicand.

Sum: $\sqrt{2} + 3\sqrt{2} = (1 + 3)\sqrt{2} = 4\sqrt{2}$

Difference: $\sqrt{2} - 3\sqrt{2} = (1 - 3)\sqrt{2} = -2\sqrt{2}$

In part (b) of Example 1, the first step is to remove a perfect square factor from under the radical sign, as you learned on page 512.

EXAMPLE 1 *Adding and Subtracting Radicals*

a. $2\sqrt{2} + \sqrt{5} - 6\sqrt{2} = -4\sqrt{2} + \sqrt{5}$ — Subtract like radicals.

b. $4\sqrt{3} - \sqrt{27} = 4\sqrt{3} - \sqrt{9 \cdot 3}$ — Perfect square factor

$= 4\sqrt{3} - \sqrt{9} \cdot \sqrt{3}$ — Use product property.

$= 4\sqrt{3} - 3\sqrt{3}$ — Simplify.

$= \sqrt{3}$ — Subtract like radicals.

EXAMPLE 2 *Multiplying Radicals*

a. $\sqrt{2} \cdot \sqrt{8} = \sqrt{16}$ — Use product property.

$= 4$ — Simplify.

b. $\sqrt{2}(5 - \sqrt{3}) = 5\sqrt{2} - \sqrt{2} \cdot \sqrt{3}$ — Use distributive property.

$= 5\sqrt{2} - \sqrt{6}$ — Use product property.

c. $(1 + \sqrt{5})^2 = 1^2 + 2\sqrt{5} + (\sqrt{5})^2$ — Use square of a binomial pattern.

$= 1 + 2\sqrt{5} + 5$ — Evaluate powers.

$= 6 + 2\sqrt{5}$ — Simplify.

d. $(a - \sqrt{b})(a + \sqrt{b}) = a^2 - (\sqrt{b})^2$ — Use sum and difference pattern.

$= a^2 - b$ — Simplify.

The expressions $(a + \sqrt{b})$ and $(a - \sqrt{b})$ are **conjugates**. As you can see in part (d) of Example 2, when a and b are integers, the product $(a + \sqrt{b})(a - \sqrt{b})$ does not involve radicals. Here are some other examples of conjugates.

EXPRESSION	CONJUGATE	PRODUCT
$4 + \sqrt{7}$	$4 - \sqrt{7}$	$4^2 - (\sqrt{7})^2 = 16 - 7 = 9$
$\sqrt{3} - c$	$\sqrt{3} + c$	$(\sqrt{3})^2 - c^2 = 3 - c^2$
$p + \sqrt{q}$	$p - \sqrt{q}$	$p^2 - (\sqrt{q})^2 = p^2 - q$

A simplified fraction does not have a radical in the denominator. To simplify some radical expressions with radicals in the denominator, you can use conjugates. For others, you may be able to write an equivalent expression with a perfect square under the radical sign in the denominator.

STUDENT HELP

INTERNET

HOMEWORK HELP Visit our Web site www.mcdougallittell.com for extra examples.

EXAMPLE 3 *Simplifying Radicals*

a. $\dfrac{3}{\sqrt{5}} = \dfrac{3}{\sqrt{5}} \cdot \dfrac{\sqrt{5}}{\sqrt{5}}$ Multiply numerator and denominator by $\sqrt{5}$.

$= \dfrac{3\sqrt{5}}{\sqrt{5} \cdot \sqrt{5}}$ Multiply fractions.

$= \dfrac{3\sqrt{5}}{5}$ Simplify.

b. $\dfrac{1}{c - \sqrt{d}} = \dfrac{1}{c - \sqrt{d}} \cdot \dfrac{c + \sqrt{d}}{c + \sqrt{d}}$ Multiply numerator and denominator by the conjugate.

$= \dfrac{c + \sqrt{d}}{(c - \sqrt{d})(c + \sqrt{d})}$ Multiply fractions.

$= \dfrac{c + \sqrt{d}}{c^2 - d}$ Simplify.

EXAMPLE 4 *Checking Quadratic Formula Solutions*

Check that $2 + \sqrt{3}$ is a solution of $x^2 - 4x + 1 = 0$.

SOLUTION

You can check the solution by substituting into the equation.

$x^2 - 4x + 1 = 0$ Write original equation.

$(2 + \sqrt{3})^2 - 4(2 + \sqrt{3}) + 1 \stackrel{?}{=} 0$ Substitute $2 + \sqrt{3}$ for x.

$4 + 4\sqrt{3} + 3 - 8 - 4\sqrt{3} + 1 \stackrel{?}{=} 0$ Multiply.

$0 = 0$ Solution is correct.

GOAL 2 USING RADICALS AS REAL-LIFE MODELS

REAL LIFE
Sailing

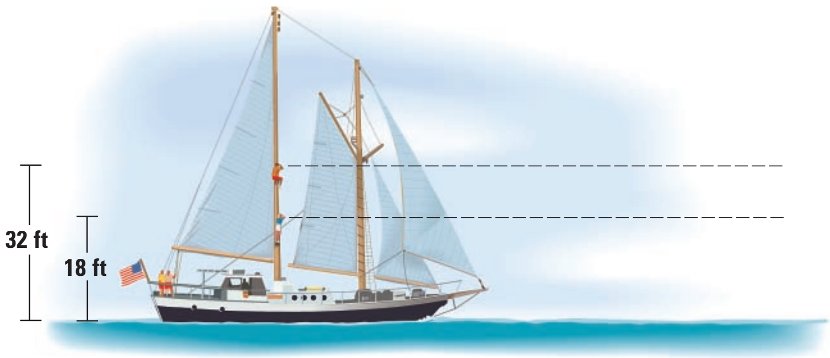
EXAMPLE 5 *Using a Radical Model*

You and a friend are spending the summer working on a schooner. The distance d (in miles) you can see to the horizon can be modeled by the following equation where h is your eye-level height (in feet) above the water.

Distance to horizon: $d = \sqrt{\frac{3h}{2}}$

a. Your eye-level height is 32 feet and your friend's eye-level height is 18 feet. Write an expression that shows how much farther you can see than your friend. Simplify the expression.

b. To the nearest tenth of a mile, how much farther can you see?



SOLUTION

a. Subtract the distance your friend can see from the distance you can see.

PROBLEM SOLVING STRATEGY

VERBAL MODEL

$\boxed{\text{Difference in distances}} = \boxed{\text{Your distance}} - \boxed{\text{Your friend's distance}}$

LABELS

Difference in distances $= D$ (miles)

Your distance $= \sqrt{\frac{3(32)}{2}}$ (miles)

Your friend's distance $= \sqrt{\frac{3(18)}{2}}$ (miles)

ALGEBRAIC MODEL

$$D = \sqrt{\frac{3(32)}{2}} - \sqrt{\frac{3(18)}{2}}$$

$$= \sqrt{48} - \sqrt{27}$$

$$= \sqrt{16 \cdot 3} - \sqrt{9 \cdot 3}$$

$$= 4\sqrt{3} - 3\sqrt{3}$$

$$= \sqrt{3}$$

▶ You can see $\sqrt{3}$ miles farther than your friend.

b. Use a calculator or the Square Root Table on p. 811 to find that $\sqrt{3} \approx 1.7$.

▶ You can see about 1.7 miles farther than your friend.

GUIDED PRACTICE

Vocabulary Check ✓

1. Complete the following sentence: Two radical expressions are _?_ if they have the same radicand.

2. Write a radical expression and its conjugate.

Concept Check ✓

3. Explain how to simplify $\frac{\sqrt{3}}{\sqrt{3}-1}$.

Skill Check ✓

Simplify the expression.

4. $4\sqrt{5} + 5\sqrt{5}$
5. $3\sqrt{7} - 2\sqrt{7}$
6. $3\sqrt{6} + \sqrt{24}$
7. $\sqrt{3} \cdot \sqrt{8}$
8. $(2 + \sqrt{3})^2$
9. $\sqrt{3}(5\sqrt{3} - 2\sqrt{6})$
10. $\frac{4}{\sqrt{13}}$
11. $\frac{3}{8 - \sqrt{10}}$
12. $\frac{6}{\sqrt{10}}$

Show whether the expression is a solution of the equation.

13. $6g^2 - 156 = 0; \sqrt{26}$
14. $x^2 - 48 = 0; -4\sqrt{3}$
15. $x^2 - 12x + 5 = 0; 6 + \sqrt{31}$
16. $x^2 - 8x + 8 = 0; 4 + 2\sqrt{2}$

17. **SAILING** In Example 5, suppose your eye-level height is 24 feet and your friend's is 12 feet. Write an expression that shows how much farther you can see than your friend. Simplify the expression.

PRACTICE AND APPLICATIONS

STUDENT HELP

Extra Practice to help you master skills is on p. 808.

ADDING AND SUBTRACTING RADICALS **Simplify the expression.**

18. $5\sqrt{7} + 2\sqrt{7}$
19. $\sqrt{3} + 5\sqrt{3}$
20. $11\sqrt{3} - 12\sqrt{3}$
21. $2\sqrt{6} - \sqrt{6}$
22. $\sqrt{32} + \sqrt{2}$
23. $\sqrt{75} + \sqrt{3}$
24. $\sqrt{80} - \sqrt{45}$
25. $\sqrt{72} - \sqrt{18}$
26. $\sqrt{147} - 7\sqrt{3}$
27. $4\sqrt{5} + \sqrt{125} + \sqrt{45}$
28. $3\sqrt{11} + \sqrt{176} + \sqrt{11}$
29. $\sqrt{24} - \sqrt{96} + \sqrt{6}$
30. $\sqrt{243} - \sqrt{75} + \sqrt{300}$

MULTIPLYING AND DIVIDING RADICALS **Simplify the expression.**

31. $\sqrt{3} \cdot \sqrt{12}$
32. $\sqrt{5} \cdot \sqrt{8}$
33. $(1 + \sqrt{13})(1 - \sqrt{13})$
34. $\sqrt{3}(5\sqrt{2} + \sqrt{3})$
35. $\sqrt{6}(7\sqrt{3} + 6)$
36. $(\sqrt{6} + 5)^2$
37. $(\sqrt{a} - b)^2$
38. $(\sqrt{c} + d)(3 + \sqrt{5})$
39. $(2\sqrt{3} - 5)^2$
40. $\frac{5}{\sqrt{7}}$
41. $\frac{2}{\sqrt{2}}$
42. $\frac{9}{5 - \sqrt{7}}$
43. $\frac{3}{\sqrt{48}}$
44. $\frac{6}{10 + \sqrt{2}}$
45. $\frac{\sqrt{3}}{\sqrt{3} - 1}$
46. $\frac{14}{60 - \sqrt{578}}$
47. $\frac{12}{7 - \sqrt{3}}$
48. $\frac{4 + \sqrt{3}}{a - \sqrt{b}}$

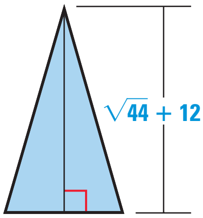
STUDENT HELP

HOMEWORK HELP
Example 1: Exs. 18–30
Example 2: Exs. 31–39
Example 3: Exs. 40–48
Example 4: Exs. 52–57
Example 5: Exs. 58, 59

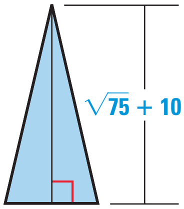
GEOMETRY CONNECTION **Find the area. (See Table of Formulas on page 813.)**

49.

50.



51.



CHECKING SOLUTIONS **In Exercises 52–57, solve the quadratic equation. Check the solutions.**

52. $x^2 + 10x + 13 = 0$

53. $x^2 - 4x - 6 = 0$

54. $x^2 - 4x - 15 = 0$

55. $4x^2 - 2x - 1 = 0$

56. $x^2 - 6x - 1 = 0$

57. $a^2 - 6a - 13 = 0$

58. **POLE-VAULTING** A pole-vaulter's approach velocity v (in feet per second) and height reached h (in feet) are related by the following equation.

Pole-vaulter model: $v = 8\sqrt{h}$

If you are a pole-vaulter and reach a height of 20 feet and your opponent reaches a height of 16 feet, approximately how much faster were you running than your opponent? Round your answer to the nearest hundredth.

59. **BIOLOGY** Many birds drop clams or other shellfish in order to break the shell and get the food inside. The time t (in seconds) it takes for an object such as a clam to fall a certain distance d (in feet) is given by the equation

$$t = \frac{\sqrt{d}}{4}.$$

A gull drops a clam from a height of 50 feet. A second gull drops a clam from a height of 32 feet. Find the difference in the times that it takes for the clams to reach the ground. Round your answer to the nearest hundredth.

FALLING OBJECTS **In Exercises 60–62, use the following information.** The average speed of an object S (in feet per second) that is dropped a certain distance d (in feet) is given by the following equation.

Falling object model: $S = \dfrac{d}{\frac{\sqrt{d}}{4}}$

60. Rewrite the equation with the right-hand side in simplest form.

61. Use either equation to find the average speed of an object that is dropped from a height of 400 feet.

62. **CHECKING GRAPHICALLY** Graph both the falling object model and your equation from Exercise 60 on the same screen to check that you simplified correctly.

Test Preparation

63. **MULTIPLE CHOICE** Simplify $\sqrt{5}(6 + \sqrt{5})^2$.

Ⓐ $41 + 2\sqrt{5}$ Ⓑ $53\sqrt{5}$ Ⓒ $41\sqrt{5} + 60$ Ⓓ $101\sqrt{5}$

64. **MULTIPLE CHOICE** Which of the following is the difference $\sqrt{3} - 5\sqrt{9}$?

Ⓐ $\sqrt{3} - 3$ Ⓑ $\sqrt{3} - 15$ Ⓒ $-4\sqrt{3}$ Ⓓ $\sqrt{3} - 45$

65. **MULTIPLE CHOICE** Simplify $\frac{3}{5 - \sqrt{5}}$.

Ⓐ $\frac{15 + 3\sqrt{5}}{20}$ Ⓑ $\frac{15 + \sqrt{5}}{20}$ Ⓒ $\frac{15 + \sqrt{15}}{20}$ Ⓓ $\frac{15 - 3\sqrt{5}}{20}$

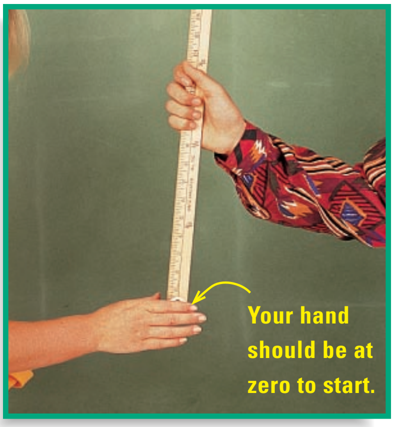
★ Challenge

SCIENCE CONNECTION **In Exercises 66–68, use the following information.**

Find a partner to help you with the following experiment. Take a ruler and drop it from a fixed height. Once the ruler is released, your partner should catch it as quickly as possible. Measure how far the ruler falls before your partner catches it. The reaction time can be calculated by the equation

$$t = \sqrt{\frac{d}{192}}$$

where t is the reaction time (in seconds) and d is the distance (in inches) the ruler falls.



66. Show how to arrive at this equation from the equation $t = \frac{\sqrt{d}}{4}$ in Exercise 59, where d is in feet.

67. Suppose for three trials, the ruler falls 4 inches, 1 inch, and 3 inches. Calculate the average reaction time for the three trials.

68. Perform the experiment several times, recording the distance the ruler falls each time. Organize the results in a table. Calculate the average reaction time for your first five trials. Compare results with your classmates.

EXTRA CHALLENGE
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MIXED REVIEW

PERCENT PROBLEMS **Solve the percent problem.** (Review 11.2)

69. What is 30% of 160? **70.** 105 is what percent of 240?

SKETCHING GRAPHS **Sketch the graph of the function.** (Review 11.8)

71. $y = \frac{1}{x - 6} - 1$ **72.** $y = \frac{1}{x - 5} + 2$ **73.** $y = \frac{2}{x - 6} + 9$

FINDING DOMAINS AND RANGES **Find the domain and the range of the function.** (Review 12.1 for 12.3)

74. $f(x) = \sqrt{x} - 3$ **75.** $f(x) = \sqrt{x - 8}$ **76.** $f(x) = \sqrt{\frac{1}{2}x^2}$

77. $f(x) = \sqrt{x} + 4$ **78.** $f(x) = 6x$ **79.** $f(x) = \sqrt{x + 3}$