11.1 Ratio and Proportion

What *you should learn*

GOAL 1 Solve proportions.

GOAL 2 Use proportions to solve **real-life** problems, such as making estimates about an archaeological dig in **Example 4**.

Why *you should learn it*

▼ To solve **real-life** problems such as creating a mural in **Exs. 41 and 42**.

REAL LIFE

GOAL 1 SOLVING PROPORTIONS

In Chapter 3 you solved problems involving ratios. An equation that states that two ratios are equal is a **proportion**.

$\frac{a}{b} = \frac{c}{d}$ *b* and *d* are nonzero.

This proportion is read as "a is to b as c is to d." When the ratios are written in this order, the numbers a and d are the **extremes** of the proportion and the numbers b and c are the **means** of the proportion.

PROPERTIES OF PROPORTIONS

RECIPROCAL PROPERTY

If two ratios are equal, their reciprocals, if they exist, are also equal.

If $\frac{a}{b} = \frac{c}{d}$, then $\frac{b}{a} = \frac{d}{c}$. **Example:** $\frac{2}{3} = \frac{4}{6} \Rightarrow \frac{3}{2} = \frac{6}{4}$

CROSS PRODUCT PROPERTY

The product of the extremes equals the product of the means.

If $\frac{a}{b} = \frac{c}{d}$, then $ad = bc$. **Example:** $\frac{2}{3} = \frac{4}{6} \Rightarrow 2 \cdot 6 = 3 \cdot 4$

When a proportion involves a single variable, finding the value of that variable is called **solving the proportion**.

EXAMPLE 1 *Using the Reciprocal Property*

Solve the proportion $\frac{3}{y} = \frac{5}{8}$.

SOLUTION

$\frac{3}{y} = \frac{5}{8}$ Write original proportion.

$\frac{y}{3} = \frac{8}{5}$ Use reciprocal property.

$y = 3 \cdot \frac{8}{5}$ Multiply each side by 3.

$y = \frac{24}{5}$ Simplify.

✓**CHECK** When you substitute to check, $\frac{3}{\frac{24}{5}}$ becomes $3 \cdot \frac{5}{24}$ which simplifies to $\frac{5}{8}$.

EXAMPLE 2 *Using the Cross Product Property*

Solve the proportion $\frac{x}{3} = \frac{12}{x}$.

SOLUTION

$$\frac{x}{3} = \frac{12}{x} \qquad \text{Write original proportion.}$$

$$x \cdot x = 3 \cdot 12 \qquad \text{Use cross product property.} \qquad \frac{x}{3} \times \frac{12}{x}$$

$$x^2 = 36 \qquad \text{Simplify.}$$

$$x = \pm 6 \qquad \text{Take square root of each side.}$$

▶ The solutions are $x = 6$ and $x = -6$. Check these in the original proportion.

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> **STUDENT HELP**
> **Study Tip**
> Remember to check your solution in the original proportion. Notice that Example 2 has two solutions so you need to check both of them.

Up to now, you have been checking solutions to make sure that you didn't make errors in the solution process. Even with no mistakes, it can happen that a trial solution does not satisfy the original equation. This type of solution is called **extraneous**. An extraneous solution should not be listed as an actual solution.

EXAMPLE 3 *Checking Solutions*

Solve the proportion $\frac{y^2 - 9}{y + 3} = \frac{y - 3}{2}$.

SOLUTION

$$\frac{y^2 - 9}{y + 3} = \frac{y - 3}{2} \qquad \text{Write original proportion.}$$

$$2(y^2 - 9) = (y + 3)(y - 3) \qquad \text{Use cross product property.}$$

$$2y^2 - 18 = y^2 - 9 \qquad \text{Use distributive property.}$$

$$y^2 = 9 \qquad \text{Isolate variable term.}$$

$$y = \pm 3 \qquad \text{Take square root of each side.}$$

> **STUDENT HELP**
> **Look Back**
> For help with solving an equation by finding square roots or by factoring, see pp. 505 and 613.

At this point, the solutions appear to be $y = 3$ and $y = -3$.

✓**CHECK** Check each solution by substituting it into the original proportion.

$y = 3$:

$$\frac{y^2 - 9}{y + 3} = \frac{y - 3}{2}$$

$$\frac{3^2 - 9}{3 + 3} \stackrel{?}{=} \frac{3 - 3}{2}$$

$$\frac{0}{6} \stackrel{?}{=} \frac{0}{2}$$

$$0 = 0$$

$y = -3$:

$$\frac{y^2 - 9}{y + 3} = \frac{y - 3}{2}$$

$$\frac{(-3)^2 - 9}{(-3) + 3} \stackrel{?}{=} \frac{(-3) - 3}{2}$$

$$\frac{0}{0} \neq \frac{-6}{2}$$

▶ You can conclude that $y = -3$ is extraneous because the check results in a false statement. The only solution is $y = 3$.

Focus on Applications

ARCHAEOLOGY In 1974, archaeologists discovered the tomb of Emperor Qin Shi Huang (259–210 B.C.) in China. Buried close to the tomb was an entire army of life-sized clay warriors.

INTERNET APPLICATION LINK
www.mcdougallittell.com

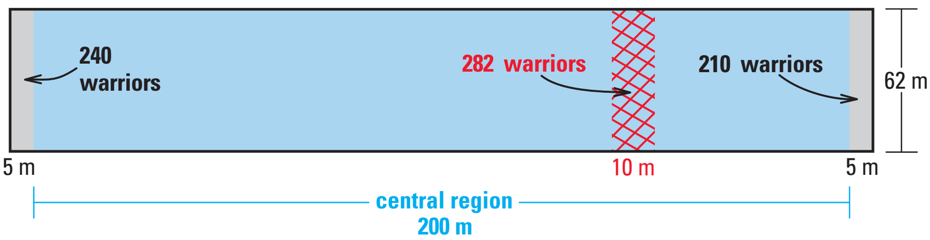
GOAL 2 Using Proportions in Real Life

When writing a proportion to model a situation, you can set up your proportion in more than one way.

EXAMPLE 4 *Writing and Using a Proportion*

Archaeology Archaeologists excavated three pits containing the clay army. To estimate the number of warriors in Pit 1 shown below, an archaeologist might excavate three sites. The sites at the ends together contain 450 warriors. The site in the central region contains 282 warriors. This 10-meter-wide site is thought to be representative of the 200-meter central region. Estimate the number of warriors in the central region. Then estimate the total number of warriors in Pit 1.

Not drawn to scale



Solution Let n represent the number of warriors in the 200-meter central region. You can find the value of n by solving a proportion.

$$\frac{\text{Number of warriors found}}{\text{Total number of warriors}} = \frac{\text{Number of meters excavated}}{\text{Total number of meters}}$$

$$\frac{282}{n} = \frac{10}{200}$$

▶ The solution is $n = 5640$, indicating that there are about 5640 warriors in the central region. With the 450 warriors at the ends, that makes a total of about 6090 warriors in Pit 1.

Real Life *Model Making*

EXAMPLE 5 *Writing and Using a Proportion*

You want to make a scale model of one of the clay horses found in the tomb. The clay horse is 1.5 meters tall and 2 meters long. Your scale model will be 18 inches long. How tall should it be?

Solution Let h represent the height of the model.

$$\frac{\text{Height of actual statue}}{\text{Length of actual statue}} = \frac{\text{Height of model}}{\text{Length of model}}$$

$$\frac{1.5}{2} = \frac{h}{18}$$

▶ The solution is $h = 13\frac{1}{2}$. Your scale model should be $13\frac{1}{2}$ inches tall.

Guided Practice

Vocabulary Check ✓

1. Write the extremes and the means of the proportion $\frac{3}{4} = \frac{9}{12}$.

Concept Check ✓

Write *yes* or *no* to tell whether the equation is a consequence of $\frac{a}{b} = \frac{c}{d}$.

2. $ac = bd$
3. $ba = dc$
4. $ad = bc$
5. $\frac{a}{b} = \frac{d}{c}$
6. $\frac{b}{a} = \frac{d}{c}$
7. $\frac{a}{d} = \frac{b}{c}$
8. Solve the proportion $\frac{4}{x+1} = \frac{7}{2}$ two ways—using the reciprocal property and using the cross product method. Which method do you prefer? Why?

Skill Check ✓

Solve the proportion. Check for extraneous solutions.

9. $\frac{x}{3} = \frac{2}{7}$
10. $\frac{6}{x} = \frac{5}{3}$
11. $\frac{2}{2x+1} = \frac{1}{5}$
12. $\frac{3}{x} = \frac{x+1}{4}$
13. $\frac{t-2}{t} = \frac{2}{t+3}$
14. $\frac{2u-3}{4u} = \frac{u-1}{u}$

MODEL MAKING **In Exercises 15 and 16, use Example 5. The proportion used in Example 5 compares height to length, but the situation could be described using other comparisons instead.**

15. Set up a different proportion to represent the situation in Example 5.
16. Solve the proportion you wrote in Exercise 15. Do you get the same solution as in Example 5?

Practice and Applications

STUDENT HELP

Extra Practice to help you master skills is on p. 807.

SOLVING PROPORTIONS **Solve the proportion. Check for extraneous solutions.**

17. $\frac{16}{4} = \frac{12}{x}$
18. $\frac{4}{2x} = \frac{7}{3}$
19. $\frac{5}{8} = \frac{c}{9}$
20. $\frac{x}{3} = \frac{2}{5}$
21. $\frac{5}{3c} = \frac{2}{3}$
22. $\frac{24}{5} = \frac{9}{y+2}$
23. $\frac{6}{3} = \frac{x+8}{-1}$
24. $\frac{r+4}{3} = \frac{r}{5}$
25. $\frac{w+4}{2w} = \frac{-5}{6}$
26. $\frac{5}{2y} = \frac{7}{y-3}$
27. $\frac{x+6}{3} = \frac{x-5}{2}$
28. $\frac{x-2}{4} = \frac{x+10}{10}$
29. $\frac{8}{x+2} = \frac{3}{x-1}$
30. $\frac{x-3}{18} = \frac{3}{x}$
31. $\frac{-2}{a-7} = \frac{a}{5}$
32. $\frac{u}{3} = \frac{1}{2u-1}$
33. $\frac{d}{d+4} = \frac{d-2}{d}$
34. $\frac{3x}{4x-1} = \frac{1}{x}$
35. $\frac{x-3}{x} = \frac{x}{x+6}$
36. $\frac{5}{m+1} = \frac{4m}{m}$
37. $\frac{2}{3t} = \frac{t-1}{t}$
38. $\frac{2}{6x+1} = \frac{2x}{1}$
39. $\frac{-2}{q} = \frac{q+1}{q^2}$
40. $\frac{6}{19n} = \frac{-2}{n^2+2}$

STUDENT HELP

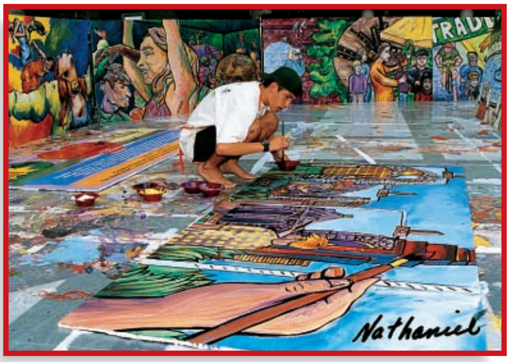
HOMEWORK HELP
Example 1: Exs. 17–40
Example 2: Exs. 17–40
Example 3: Exs. 17–40
Example 4: Exs. 41–43
Example 5: Exs. 41–43

STUDENT HELP

HOMEWORK HELP
Visit our Web site www.mcdougallittell.com for help with writing proportions to solve problems.

MURAL PROJECT **In Exercises 41–42, use the following information.**

The *Art is the Heart of the City* fence mural project in Charlotte, North Carolina, involved artists Cordelia Williams and Paul Rousso along with 22 students in grades 11 and 12. Students created drawings on paper. Slides of the drawings were made and projected to fit onto 4-foot-wide by 8-foot-long sheets of plywood used for the fence panels. Students traced and later painted the enlarged images.



41. If the paper used for the original drawings was 11 inches wide, how long did it need to be?

42. Suppose the length of the paintbrush on the panel shown is $2\frac{1}{2}$ feet. Use Exercise 41 to find the length of the paintbrush in the student's drawing.

43. **SCALE MODELS** A scale model uses a scale of $\frac{1}{16}$ inch to represent 1 foot. Explain how you can use a proportion and cross products to show that a scale of $\frac{1}{16}$ in. to 1 ft is the same as a scale of 1 in. to 192 in.

WHAT STUDENTS BUY **In Exercises 44–46, use the table. It shows the results of a survey in which 100 students were asked how they spent money last week.**

How 100 students spent money

Item	Number of students
Food	78
Clothes, accessories	20
Books, magazines, comics	15
Toys, stickers, games	14
Movie tickets	14
Arcade games	14
Gifts	13
Movie rentals	13
Music	12
Footwear	11
Grooming products	11

44. Estimate the number of students out of 500 that bought clothes or accessories in the last week.

45. Choose 3 items that were bought by different numbers of students. Based on the survey, how many students out of 20 would you predict to have bought each item?

46. **COLLECTING DATA** Choose one item from Exercise 45. Ask 20 students whether they bought that item. Compare the results with your prediction. Are the results what you expected? Explain.

FOCUS ON CAREERS

MODEL BUILDER
To make scale models, model builders must pay close attention to detail. They use proportions to find the correct sizes for objects they are modeling as in Exercise 43.

CAREER LINK
www.mcdougallittell.com

47. **SAMPLING FISH POPULATIONS** Researchers studying fish populations at Dryden Lake in New York caught, marked, and then released 232 Chain pickerel. Later a sample of 329 Chain pickerel were caught and examined. Of these, 16 were found to be marked. Use the proportion below to estimate the total Chain pickerel population in the lake.

$$\frac{\text{Marked pickerel in sample}}{\text{Total pickerel in sample}} = \frac{\text{Marked pickerel in lake}}{\text{Total pickerel in lake}}$$

Test Preparation

48. MULTI-STEP PROBLEM Base your answers on the 1997 population data shown in the table for people 25 years and older in the United States.

Total population (in thousands)	Number of people out of 100 with education level			
	Not a high school graduate	High school graduate	Some college, but less than 4 yr	At least 4 yr of college
170,581	17.9	33.8	24.5	23.8

DATA UPDATE of U.S. Bureau of the Census data at www.mcdougallittell.com

a. Out of 200 people aged 25 years or older, about how many would you expect to have just a high school education? to have at least 4 years of college?

b. Write and solve a proportion to estimate the number of people in the United States aged 25 years or older with at least 4 years of college.

c. Suppose a town has 20,000 residents aged 25 years or older. Write and solve a proportion to estimate the number of town residents 25 years or older who have completed at least 4 years of college.

d. *Writing* Suppose another town has 15,860 people aged 25 years or older and that 7581 of these people have completed at least 4 years of college. Explain how you can find out whether the number of college graduates in that town is typical for a town of that size.

★ **Challenge**

49. LOGICAL REASONING One way to prove that two proportions are equivalent is to apply the properties of equality to transform one of the proportions into the other proportion. Give a sequence of steps that transforms the proportion $\frac{a}{b} = \frac{c}{d}$ into $\frac{a}{c} = \frac{b}{d}$.

EXTRA CHALLENGE
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MIXED REVIEW

50. DECIMAL AND PERCENT FORM Copy and complete the table. If necessary, round to the nearest tenth of a percent. (Skills Review, pages 784–785)

Decimal	?	0.2	?	0.073	0.666…	?	?	2
Percent	78%	?	3%	?	?	176%	110%	?

FINDING SQUARE ROOTS **Find all square roots of the number or write *no square roots*. Check the results by squaring each root.** (Review 9.1)

51. 64 **52.** -9 **53.** 12 **54.** 169

55. -20 **56.** 50 **57.** $\frac{9}{25}$ **58.** 0.04

SIMPLIFYING RADICAL EXPRESSIONS **Simplify the radical expression.** (Review 9.2)

59. $\sqrt{18}$ **60.** $\sqrt{20}$ **61.** $\sqrt{80}$ **62.** $\sqrt{162}$

63. $9\sqrt{36}$ **64.** $\sqrt{\frac{11}{9}}$ **65.** $\frac{1}{2}\sqrt{28}$ **66.** $4\sqrt{\frac{5}{4}}$