

1.3

Order of Operations

What you should learn

GOAL 1 Use the order of operations to evaluate algebraic expressions.

GOAL 2 Use a calculator to evaluate **real-life** expressions, such as calculating sales tax in **Example 5**.

Why you should learn it

▼ To solve **real-life** problems, such as calculating the cost of admission for a family to a state fair in **Exs. 50 and 51**.



GOAL 1 USING THE ORDER OF OPERATIONS

Mathematicians have established an **order of operations** to evaluate an expression involving more than one operation. Start with operations within grouping symbols. Then evaluate powers. Then do multiplications and divisions from left to right. Finally, do additions and subtractions from left to right.

EXAMPLE 1 Evaluating Without Grouping Symbols

Evaluate the expression when $x = 4$.

a. $3x^2 + 1$

b. $32 \div x^2 - 1$

SOLUTION

$$\begin{aligned} \text{a. } 3x^2 + 1 &= 3 \cdot 4^2 + 1 \\ &= 3 \cdot 16 + 1 \\ &= 48 + 1 \\ &= 49 \end{aligned}$$

Substitute 4 for x .

Evaluate power.

Evaluate product.

Evaluate sum.

$$\begin{aligned} \text{b. } 32 \div x^2 - 1 &= 32 \div 4^2 - 1 \\ &= 32 \div 16 - 1 \\ &= 2 - 1 \\ &= 1 \end{aligned}$$

Substitute 4 for x .

Evaluate power.

Evaluate quotient.

Evaluate difference.

When you want to change the established order of operations for an expression, you *must* use parentheses or other grouping symbols.

ACTIVITY

Developing
Concepts

Investigating Grouping Symbols

Without grouping symbols, the expression $3 \cdot 4^2 + 8 \div 4$ has a value of 50. You can insert grouping symbols to produce a different value. For example:

$$3 \cdot (4^2 + 8) \div 4 = 3 \cdot (16 + 8) \div 4 = 3 \cdot 24 \div 4 = 72 \div 4 = 18$$

Insert grouping symbols in the expression $3 \cdot 4^2 + 8 \div 4$ to produce the indicated values.

1. 146

2. 54

3. 38

THE LEFT-TO-RIGHT RULE Operations that have the same priority, such as multiplication and division *or* addition and subtraction, are performed using the *left-to-right rule*, as shown in Example 2.

EXAMPLE 2 *Using the Left-to-Right Rule*

a. $24 - 8 - 6 = (24 - 8) - 6$ **Work from left to right.**
 $= 16 - 6$
 $= 10$

b. $15 \cdot 2 \div 6 = (15 \cdot 2) \div 6$ **Work from left to right.**
 $= 30 \div 6$
 $= 5$

c. $16 + 4 \div 2 - 3 = 16 + (4 \div 2) - 3$ **Divide first.**
 $= 16 + 2 - 3$
 $= (16 + 2) - 3$ **Work from left to right.**
 $= 18 - 3$
 $= 15$

STUDENT HELP

Study Tip

In part (c) of Example 2, you do not perform the operations from left to right because division has a higher priority than addition and subtraction.

ORDER OF OPERATIONS

1. First do operations that occur within grouping symbols.
2. Then evaluate powers.
3. Then do multiplications and divisions from left to right.
4. Finally, do additions and subtractions from left to right.

A fraction bar can act as a grouping symbol: $(1 + 2) \div (4 - 1) = \frac{1 + 2}{4 - 1}$

EXAMPLE 3 *Using a Fraction Bar*

$$\frac{7 \cdot 4}{8 + 7^2 - 1} = \frac{7 \cdot 4}{8 + 49 - 1}$$
 Evaluate power.

$$= \frac{28}{8 + 49 - 1}$$
 Simplify the numerator.

$$= \frac{28}{57 - 1}$$
 Work from left to right.

$$= \frac{28}{56}$$
 Subtract.

$$= \frac{1}{2}$$
 Simplify.

STUDENT HELP



HOMEWORK HELP

Visit our Web site www.mcdougallittell.com for extra examples.

Skills Review

For help with writing fractions in lowest terms, see pp. 781–783.

GOAL 2 EVALUATING EXPRESSIONS WITH A CALCULATOR

Many calculators use the established order of operations, but some do not. See if your calculator follows the established order of operations in Example 4. If your calculator does not follow the order of operations, you must enter the operations in the correct order.

EXAMPLE 4 Using a Calculator

 When you enter the following in your calculator, does the calculator display 6.1 or 0.6?

$$10.5 \text{ - } 6.3 \text{ } \div \text{ } 2.1 \text{ - } 1.4 \text{ ENTER}$$

SOLUTION

a. If your calculator uses order of operations, it will display 6.1.

$$\begin{aligned} 10.5 - 6.3 \div 2.1 - 1.4 &= 10.5 - (6.3 \div 2.1) - 1.4 \\ &= 10.5 - 3 - 1.4 \\ &= 6.1 \end{aligned}$$

b. If it displays 0.6, it performs the operations as they are entered.

$$\begin{aligned} [(10.5 - 6.3) \div 2.1] - 1.4 &= (4.2 \div 2.1) - 1.4 \\ &= 2 - 1.4 \\ &= 0.6 \end{aligned}$$



EXAMPLE 5 Calculating Sales Tax

 Suppose you live in a state that charges no sales tax on essentials, such as clothes or food, but charges 6% sales tax on nonessentials, such as CDs or games. You decide to buy a sweatshirt for \$24 and a video game for \$39. Your calculator follows the established order of operations. Which of the following keystroke sequences will show the correct amount you owe? Explain why.

a. $24 \text{ + } 39 \text{ + } 39 \text{ } \times \text{ } 0.06 \text{ ENTER}$

b. $39 \text{ + } 24 \text{ + } 24 \text{ } \times \text{ } 0.06 \text{ ENTER}$

c. $39 \text{ + } 24 \text{ + } (24 \text{ + } 39) \text{ } \times \text{ } 0.06 \text{ ENTER}$

SOLUTION

- a. \$65.34 is correct. Tax is added on the video game, but not the sweatshirt.
b. \$64.44 is wrong. Tax is added on the sweatshirt, but not the video game.
c. \$66.78 is wrong. Tax is added on both items.

GUIDED PRACTICE

Vocabulary Check ✓

Concept Check ✓

- Describe the order of operations agreed upon by mathematicians.
- If an expression without grouping symbols includes addition and an exponent, which operation should you do first?
- If an expression without grouping symbols includes multiplication and division, which operation should you do first?

Skill Check ✓

Evaluate the expression for the given value of the variable.

- $x^4 - 3$ when $x = 2$
- $a^3 + 10a$ when $a = 3$
- $\frac{22}{x} \div 2 + 16$ when $x = 11$
- $(x + 5) \div 4$ when $x = 9$
- $5 \cdot 6y$ when $y = 5$
- $\frac{16}{x} - 2$ when $x = 4$
- $\frac{16}{n} + 2^3 - 10$ when $n = 8$
- $b + 6 \div 4$ when $b = 1.5$
- ERROR ANALYSIS** Julio's calculator displayed 7 when he evaluated the following expression. Did his calculator use the established order of operations? If not, how can he use grouping symbols to find the correct value?

72 **+** 12 **÷** 4 **-** 14 **ENTER** 7

PRACTICE AND APPLICATIONS

STUDENT HELP

▶ **Extra Practice**
to help you master
skills is on p. 797.

EVALUATING VARIABLE EXPRESSIONS Evaluate the expression for the given value of the variable.

- $3 + 2x^3$ when $x = 2$
- $6 \cdot 2p^2$ when $p = 5$
- $13 + 3b$ when $b = 7$
- $\frac{x}{7} + 16$ when $x = 14$
- $\frac{4}{5} \div n + 13$ when $n = \frac{1}{5}$
- $y^4 \div 8$ when $y = 4$
- $t^5 - 10t$ when $t = 3$
- $3r^2 - 17$ when $r = 6$
- $27 - \frac{24}{b}$ when $b = 8$
- $\frac{9}{10} \cdot y - \frac{3}{10}$ when $y = \frac{1}{2}$

EVALUATING NUMERICAL EXPRESSIONS Evaluate the expression.

- $4 + 9 - 1$
- $5 + 8 \cdot 2 - 4$
- $10 \div (3 + 2) + 9$
- $3(2.7 \div 0.9) - 5$
- $\frac{1}{3}(9 \cdot 3) + 18$
- $3 \cdot 2 + \frac{5}{9}$
- $16 \div 8 \cdot 2^2$
- $7[(18 - 6) - 6]$
- $6(5 - 3)^2 + 3$
- $\frac{1}{2} \cdot 26 - 3^2$
- $6 \div 3 + 2 \cdot 7$
- $2 \cdot 3^2 \div 7$
- $[(7 - 4)^2 + 3] + 15$
- $[10 + (5^2 \cdot 2)] \div 6$
- $2.5 \cdot 0.5^2 \div 5$

STUDENT HELP

▶ HOMEWORK HELP

- Example 1:** Exs. 13–22
Example 2: Exs. 23–37
Example 3: Exs. 38–40,
42
Example 4: Exs. 43–46
Example 5: Ex. 41

EXPRESSIONS WITH FRACTION BARS Evaluate the expression.

38. $\frac{9 \cdot 2}{4 + 3^2 - 1}$

39. $\frac{13 - 4}{18 - 4^2 + 1}$

40. $\frac{5^3 \cdot 2}{1 + 6^2 - 8}$

41. **Writing** You decide to buy two rings from an outdoor vendor. One ring costs \$10.89. The other ring costs \$12.48. The sales tax is 8%. The vendor uses a calculator to obtain the price including sales tax for both rings and gets \$24.37. What mistake did the vendor make?

42. **LOGICAL REASONING** Which is correct? Explain.

A. $\frac{(9 - 7)^2 + 3}{5} = (9 - 7)^2 + 3 \div 5$

B. $\frac{(9 - 7)^2 + 3}{5} = [(9 - 7)^2 + 3] \div 5$

 **ORDER OF OPERATIONS** In Exercises 43–46, two calculators were used to evaluate the expression. They gave different results. Which calculator used the established order of operations? Rewrite the calculator steps with grouping symbols so that both calculators give the correct result.

43. 15  6  3  4  Calculator A: 12; Calculator B: 7

44. 15  9  3  7  Calculator A: 19; Calculator B: 9

45. 15  10  5  4  Calculator A: 21; Calculator B: 9

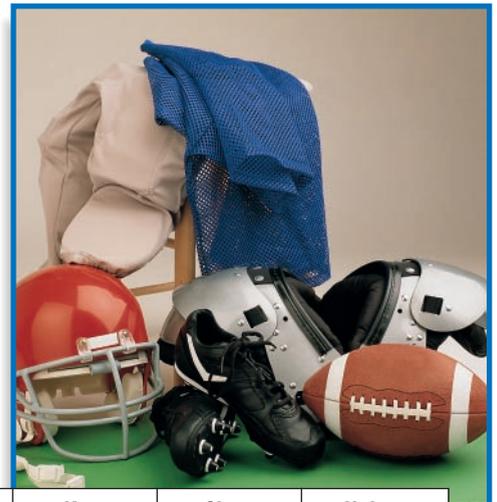
46. 4  3  6  2  Calculator A: 9; Calculator B: 15

47.  **HOTEL RATES** A hotel charges \$49.99 per room per night for adults and \$44.10 per room per night for senior citizens. The expression $2 \times \$49.99 + 3 \times \44.10 represents the total cost of five rooms for two adults and three senior citizens for an overnight stay. Where in the expression can you put grouping symbols to make sure it is evaluated correctly?

 **FOOTBALL UNIFORMS** In Exercises 48 and 49, use the following information.

The table shows the cost of parts of a professional football player's uniform. A sporting goods company offers a \$3200 discount for orders of 30 or more complete professional football player uniforms.

48. Write an expression that represents the cost for an order of 35 complete professional football player uniforms.
49. Evaluate the expression you wrote in Exercise 48.



Part of uniform	Jersey and pants	Shoulder pads	Lower body pads	Knee pads	Cleats	Helmet
Cost	\$230	\$300	\$50	\$25	\$100	\$200

ADMISSION PRICES In Exercises 50 and 51, use the table below. It shows the admission prices for the California State Fair in 1998. Suppose a family of 2 adults, 1 senior, and 3 children go to the State Fair. The children's ages are 13 years, 10 years, and 18 months.

50. Write an expression that represents the admission price for the family.

51. Evaluate the expression you wrote in Exercise 50.

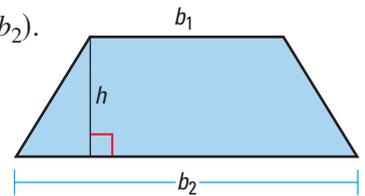
California State Fair Admission Prices	
Age	Admission price
General Admission (13–61 years of age)	\$7.00
Seniors (62 years and above)	\$5.00
Children (5–12 years)	\$4.00
Children (4 years and under)	Free

► Source: *Sacramento Bee*

STUDENT HELP

Study Tip
The expression $b_1 + b_2$ is read “ b sub 1 plus b sub 2”.

52. **GEOMETRY CONNECTION** The area A of a trapezoid with parallel bases of lengths b_1 and b_2 and height h is $A = \frac{1}{2}h(b_1 + b_2)$. Find the area of a trapezoid whose height is 2 meters and whose bases are 6 meters and 10 meters.



GEOMETRY CONNECTION In Exercises 53 and 54, use the following information. The surface area of a cylinder equals the lateral surface area ($2\pi r \cdot h$) plus the area of the two bases ($2 \cdot \pi r^2$).

53. Write the expression for the surface area of a cylinder.

54. Evaluate the expression when $h = 10.5$ centimeters and $r = 2.5$ centimeters. Use 3.14 as an approximation for π .

55. **MULTI-STEP PROBLEM** You are shopping for school supplies. A store is offering a 10% discount on binders and a 20% discount on packages of paper. You want to buy 5 binders originally marked \$2.50 each and 10 packages of paper originally marked \$1.30 each.

a. Write an expression that shows how much you will save after the discounts.

b. Evaluate the expression.

c. *Writing* If you have \$25 to spend on supplies, how much money will you have left over? Explain how you arrived at your answer.

★ Challenge

56. **CRITICAL THINKING** Without grouping symbols, the expression $2 \cdot 3^3 + 4$ has a value of 58. Insert grouping symbols in the expression $2 \cdot 3^3 + 4$ to produce the indicated values.

a. 62

b. 220

c. 4374

d. 279,936

57. Create a math puzzle like the one in Exercise 56 with an expression that produces different values when grouping symbols are inserted in different places.

Test Preparation

EXTRA CHALLENGE

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MIXED REVIEW

STUDENT HELP

Skills Review

For help with fractions, see pp. 781–783.

EVALUATING EXPRESSIONS Evaluate the expression for the given value of the variable. (Review 1.1)

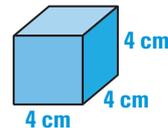
58. $8a$ when $a = 4$ 59. $\frac{24}{x}$ when $x = 3$ 60. $c + 15$ when $c = 12.5$
61. $\frac{4}{3} \cdot x$ when $x = \frac{1}{6}$ 62. $9d$ when $d = 0.5$ 63. $\frac{5}{16} - p$ when $p = \frac{3}{8}$

EVALUATING EXPONENTIAL EXPRESSIONS Evaluate the expression for the given value of the variable. (Review 1.2)

64. $(6w)^2$ when $w = 5$ 65. $4(t^3)$ when $t = 3$ 66. $9b^2$ when $b = 8$
67. $5x^2$ when $x = 16$ 68. $(7x)^3$ when $x = 2$ 69. $6y^5$ when $y = 4$
70.  **INTEREST EARNED** You deposit \$120 in a savings account that pays an annual interest rate of 2.5%. How much simple interest would you earn in 2 years? (Review 1.1)

71.  **DRIVING TIME** If you are driving at a constant speed of 68 miles per hour, how long will it take you to travel 170 miles? (Review 1.1)

72. **GEOMETRY CONNECTION** Find the volume of the cube shown at the right and find the area of one of its sides. (Review 1.2 for 1.4)



QUIZ 1

Self-Test for Lessons 1.1–1.3

Evaluate the expression when $x = 3$. (Lesson 1.1)

1. $6x$ 2. $\frac{36}{x}$ 3. $x + 29$ 4. $x - 2$
5. $x \div 3$ 6. $21 - x$ 7. $4.5x$ 8. $13.7 + x$

 **TRAVEL** Find the average speed. (Lesson 1.1)

9. 120 miles in 3 hours 10. 90 miles in 1.5 hours 11. 360 miles in 6 hours

Write the expression in exponential form. (Lesson 1.2)

12. six cubed 13. $4 \cdot 4 \cdot 4 \cdot 4 \cdot 4$ 14. $5y \cdot 5y \cdot 5y$
15. five squared 16. $3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot t$ 17. $7x \cdot 7x$

Evaluate the expression when $x = 6$. (Lesson 1.2)

18. x^5 19. $2x^3$ 20. $(2x)^3$ 21. $x^2 - 3$

Evaluate the expression. (Lesson 1.3)

22. $\frac{6 \cdot 3}{7 + (2^3 - 1)}$ 23. $\frac{2^5 - 12}{2(5^2 - 5)}$ 24. $\frac{(3^2 - 3)}{2 \cdot 9}$ 25. $\frac{2(17 + 2 \cdot 4)}{6^2 - 11}$

26.  **PACKING BOXES** A cubic packing box has dimensions of 1.2 feet on each side. What is the volume of the box? (Lesson 1.2)