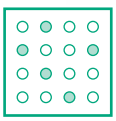
Appendix 1

Samples and Surveys

GOAL ***Recognize bias in samples and survey questions; analyze the results of surveys.***

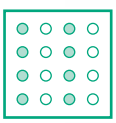
Surveys are often conducted to gather data about a *population*. A **population** is the entire group of people or objects that you want information about. When it is difficult to survey an entire population, a **sample**, or a part of the group, is surveyed.

TYPES OF SAMPLES



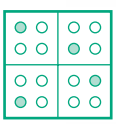
SIMPLE RANDOM SAMPLE

Members are chosen using a method that gives everyone an equally likely chance of being selected.



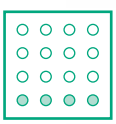
SYSTEMATIC SAMPLE

Members are chosen using a pattern, such as selecting every other person.



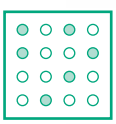
STRATIFIED SAMPLE

The population is first divided into groups. Then members are randomly chosen from each group.



CONVENIENCE SAMPLE

Members are chosen because they are easily accessible.



SELF-SELECTED SAMPLE

Members volunteer to participate.

EXAMPLE 1 *Classifying a Sample*

A teacher wants to select a group of 5 students. Classify the sample.

a. The teacher selects the first 5 students who raise their hands.

b. The teacher draws the names of 5 students from a hat.

SOLUTION

a. This is a self-selected sample because the students volunteer.

b. This is a simple random sample because each student has an equally likely chance of being selected.

In a **random sample**, each person or object has an equally likely chance of being selected. A random sample is most likely to produce a representative sample of a population. A sample that is not representative of a population is a **biased sample**. In a biased sample, one or more of the parts of a population are overrepresented or underrepresented. A non-random sample can result in a biased sample.

Simple random, systematic, and stratified samples are preferred because the resulting samples are usually random. The results of convenience and self-selected samples are more likely to be biased.

EXAMPLE 2 *Identifying Potentially Biased Samples*

Administrators at your school are considering adding lacrosse as a school-sponsored sport. They decide to survey students to see if there is enough interest in the sport. Decide whether the sampling method could result in a biased sample. Explain your reasoning.

a. Survey students that currently participate in sports at your school.

b. Survey every tenth student who enters the school in the morning.

SOLUTION

a. This method could result in a biased sample because students who already participate in sports are more likely to favor the addition of a new sport than other students in the school.

b. This method is not likely to result in a biased sample because a wide range of students will be surveyed.

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When conducting a survey, it is important that the survey questions are carefully written. If a question is poorly written, then the responses of the people surveyed may not accurately reflect their opinions or actions. These types of questions are called **biased questions**.

EXAMPLE 3 *Identifying Biased Questions*

Decide whether the survey question may be biased. Explain.

a. Do you agree with your school's attendance policy?

b. Due to the large number of accidents among drivers that are under the age of 18, do you support raising the minimum driving age to 18?

SOLUTION

a. This question may be biased. It assumes that the student being surveyed is familiar with the attendance policy.

b. This question may be biased. Respondents may think a "no" response means that they are in favor of drivers under 18 getting into accidents.

ANALYZING DATA To help you make conclusions about data collected from a survey, the data needs to be organized. Data displays are often used to organize and present data collected from a survey. However, the data should be accurately displayed so that it can be correctly interpreted.

EXAMPLE 4 *Making Conclusions About Data from a Survey*

You conduct a survey that asks 30 students the average number of minutes they spend talking on the phone each day. The information collected from the survey is shown at the right.

Minutes	Tally
0–14	𝍸 | | |
15–29	𝍸 𝍸 | |
30–44	𝍸 |
45–59	| | | |

a. Make a histogram of the data.

b. Make a conclusion about the data.

STUDENT HELP

Look Back
For help with making histograms, see p. 792.

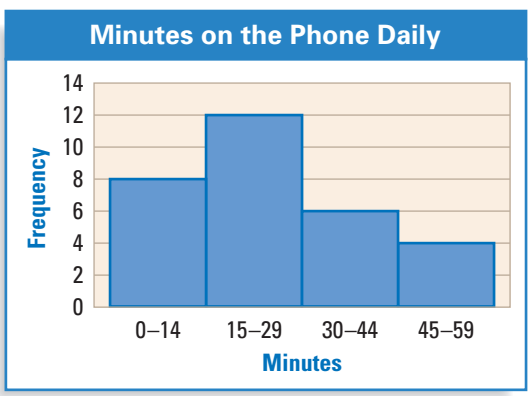
SOLUTION

a. Draw and label the horizontal and vertical axes.

List each interval from the table on the horizontal axis.

The greatest frequency is 12. So, start the vertical axis at 0 and end at 14, using increments of 2.

Draw a bar for each interval. The bars should have the same width.



b. The histogram shows that the interval with greatest frequency is the 15–29 minute interval.

In general, more than half of the students spend less than 30 minutes talking on the phone each day.

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ANALYZING SURVEYS The results of public opinion surveys often appear in newspapers and magazines. You should be critical of these surveys, especially if the results are meant to persuade you in some way. If you notice non-random sampling, biased questions, or poorly displayed data, then it is likely that the survey results are biased.

EXAMPLE 5 *Analyzing the Results of a Survey*

A survey asks shoppers whether they prefer to shop for clothes on-line or at a store. The results are shown at the right.

a. Tell why the results are biased.

b. Suggest a way to eliminate the bias.

SOLUTION

a. The results of the survey are biased because the sampling method is biased. The sample contains only on-line shoppers, so the group of people who prefer shopping for clothes at a store are underrepresented.

b. The survey should be conducted using a random sample. For example, conduct a telephone survey.

EXERCISES

In Exercises 1–4, classify the sample involving the following situation.
A fast-food restaurant decides to conduct a survey to find out if their customers are satisfied with the service at the restaurant.

1. Survey every twentieth customer.
2. Survey every tenth male customer and every tenth female customer.
3. Survey customers randomly chosen by the computer system.
4. Survey customers only when the restaurant is not busy.

In Exercises 5–8, classify the sample.

5. The staff at a town library want to know what new programs residents would be interested in. A computer generates a list of 100 randomly chosen town residents to be telephoned and surveyed.
6. The manager of a grocery store wants to know if shoppers are satisfied with the selection of items in the deli case. A deli customer is surveyed every 30 minutes.
7. The editor of a school newspaper wants to know how students feel about the extracurricular activities offered by the school. Twenty students from each class (freshman, sophomore, junior, and senior) are randomly chosen from the school directory and surveyed.
8. The manager of a hotel wants to know how guests rate the cleanliness of the rooms. Response cards are left in each room for guests to fill out during their stay.

Decide whether the sampling method could result in a random sample. Explain your reasoning.

9. A dentist wants to expand her office hours to better accommodate her patients. She decides to list her patients alphabetically and survey every tenth patient on the list about his or her time preferences.

10. The manager of a radio station wants to know the listeners' favorite songs. He decides to have listeners call the radio station and request their favorite songs.

Decide whether the sampling method could result in a biased sample. Explain your reasoning.

11. Administrators at your school want to know if more vegetarian items should be added to the lunch menu. They decide to survey every fifth student waiting in line to purchase lunch.

12. A Boston-based market research company wants to learn more about the television viewing habits of average teenagers. They decide to survey randomly chosen teenagers who live in Boston.

13. The editor of a snowboarding magazine wants to know the readers' favorite places to ride. The latest issue of the magazine includes a survey for readers to complete and return.

14. A restaurant critic wants to taste a variety of items at a restaurant. She randomly chooses two items from each of the following groups on the menu: appetizers, entrées, and desserts.

Decide whether the survey question may be biased. Explain.

15. Do you agree with the rules established for traveling on public transportation with a dog?

16. How often do you read the local newspaper?

17. Would you rather see an exciting laser show or visit a boring art museum?

18. The fewer trash cans that a community has, the more litter the community has. Should our community include more money in its budget for more trash cans and trash pick up?

In Exercises 19–21, use the following information.
Your class officers are planning a dance. They need to decide whether to have a disc jockey or a live band at the dance. They decide to survey a sample of students to determine their preference.

19. Describe a sampling method that can be used so that the sample will most likely be random.

20. Write a question to ask the members of the sample. The question should be carefully written so that it is not biased.

21. Which is more likely to give accurate results, 50 students randomly chosen from a class of 300, or 20 students randomly chosen from a class of 300? Explain your reasoning.

In Exercises 22 and 23, use the following information.
A sample of 100 students, 50 boys and 50 girls, are asked to name their favorite type of movie. The results are shown in the table below.

Group	Drama	Comedy	Action	Other
Boys	9	18	21	2
Girls	17	14	11	8

22. Make a double bar graph of the data.

23. Use your display from Exercise 22 to make a conclusion about the data.

In Exercises 24 and 25, use the following information.
A sample of 19 students enrolled in a swimming class are asked to hold their breath underwater for as long as possible. The results (in seconds) are listed below.

65, 29, 38, 50, 60, 43, 27, 48, 29, 79, 37, 45, 48, 32, 57, 35, 54, 53, 37

24. Make a histogram of the data using the following intervals: 20–29, 30–39, 40–49, 50–59, 60–69, 70–79.

25. Use your display from Exercise 24 to make a conclusion about the data.

In Exercises 26 and 27, tell why the results of the survey are biased. Then tell how to eliminate the bias.

26.

Sal's Pizza
is your favorite!

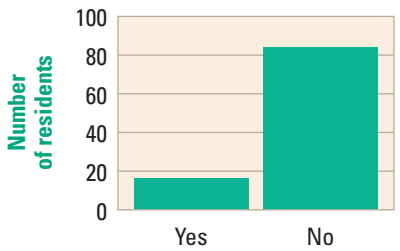
*Which pizza do you prefer?**

Sal's Pizza	72%
Pizza Primo	28%

* Based on a survey of 250 Sal's Pizza customers.

27.

Should Skateboarders be Allowed to Endanger the Safety of Pedestrians by Skateboarding on Sidewalks?*



*Based on a survey of 100 residents.

28. Find the results of a survey published in a magazine or newspaper. Do you think that the results of the survey are biased? Explain your reasoning.

In Exercises 29–32, you want to learn more about the music that students in your school listen to.

29. Describe how you would choose a sample of students to survey.

30. Write a survey that consists of 3 or 4 questions.

31. Give the survey to the sample and present your results.

32. Review your results. Would you change anything about the way the survey was designed or conducted? Explain.

Appendix 2

Probability of Compound Events

GOAL ***Find the probability of independent, dependent, mutually exclusive, and overlapping events.***

When you consider the probability of two events occurring, the events are called **compound events**. *Independent, dependent, mutually exclusive,* and *overlapping events* are examples of compound events.

Two events are **independent events** if the occurrence of one event does not affect the occurrence of the other. Two events are **dependent events** if the occurrence of one event *does* affect the occurrence of the other.

EXAMPLE 1 Deciding Whether Events Are Independent or Dependent

Decide whether the events are *independent* or *dependent*.

a. Each whole number from 1 through 10 is written on a piece of paper and placed in a hat. You randomly choose a piece of paper, do not put it back, then randomly choose another piece of paper.
Event *A*: Choose the 5 first.
Event *B*: Choose an odd number second.

b. You flip a coin and roll a number cube.
Event *A*: Get tails when flipping the coin.
Event *B*: Get a 2 when rolling the number cube.

SOLUTION

a. The events are dependent. Choosing the 5 and not putting it back affects the outcomes of choosing an odd number second.

b. The events are independent. Getting tails when flipping a coin does not affect the outcomes of rolling a number cube.

STUDENT HELP

Study Tip
In the formula for the probability of dependent events, $P(B \mid A)$ represents the probability that B will occur given that A has occurred.

PROBABILITIES OF INDEPENDENT AND DEPENDENT EVENTS

For two independent events A and B, the probability that both events occur is the product of the probabilities of the events.

$$P(A \text{ and } B) = P(A) \cdot P(B) \qquad \text{Events } A \text{ and } B \text{ are independent.}$$

For two dependent events A and B, the probability that both occur is the product of the probability of the first event and the conditional probability of the second event given the first event.

$$P(A \text{ and } B) = P(A) \cdot P(B \mid A) \qquad \text{Events } A \text{ and } B \text{ are dependent.}$$

EXAMPLE 2 Finding the Probability of Independent Events

Of the 35 CDs in your collection, 21 are rock and 14 are hip-hop. You randomly choose a CD and put it back. Then you randomly choose another CD. Find the probability of choosing a rock CD first and choosing a hip-hop CD second.

SOLUTION

Let event A be "choose a rock CD first" and let event B be "choose a hip-hop CD second." Since the first CD chosen is replaced before the second CD is chosen, events A and B are independent. The probability of A and B is shown below.

$$P(A \text{ and } B) = \frac{21}{35} \cdot \frac{14}{35} = \frac{6}{25} = 0.24$$

▶ The probability is 0.24.

EXAMPLE 3 Finding the Probability of Dependent Events

A drawer contains 9 white socks, 7 black socks, and 5 blue socks. You randomly choose one sock from the drawer. Then you randomly choose another sock without replacing the first. Find the probability of events A and B.

a. **Event *A***: Choose a white sock first.
Event *B*: Choose a black sock second.

b. **Event *A***: Choose a blue sock first.
Event *B*: Choose a blue sock second.

SOLUTION

a. First, find the probability of each event.

$$P(A) = \frac{9}{21}$$ Of the 21 socks, 9 are white.

$$P(B \mid A) = \frac{7}{20}$$ Of the remaining 20 socks, 7 are black.

Then multiply the probabilities.

$$P(A \text{ and } B) = \frac{9}{21} \cdot \frac{7}{20} = \frac{3}{20} = 0.15$$

▶ The probability is 0.15.

b. First, find the probability of each event.

$$P(A) = \frac{5}{21}$$ Of the 21 socks, 5 are blue.

$$P(B \mid A) = \frac{4}{20}$$ Of the remaining 20 socks, 4 are blue.

Then multiply the probabilities.

$$P(A \text{ and } B) = \frac{5}{21} \cdot \frac{4}{20} = \frac{1}{21} \approx 0.048$$

▶ The probability is about 0.048.

Mutually exclusive events are events that have no outcomes in common. Use the rule below to find the probability of mutually exclusive events.

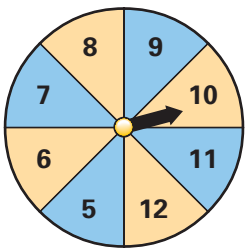
PROBABILITY OF MUTUALLY EXCLUSIVE EVENTS

For two mutually exclusive events A and B, the probability that either of the events occurs is the sum of the probabilities of the events.

$P(A \text{ or } B) = P(A) + P(B)$ Events A and B are mutually exclusive.

EXAMPLE 4 *Finding the Probability of Mutually Exclusive Events*

You are playing a game using the spinner at the right, which is divided into equal parts. On your spin, what is the probability that the spinner lands on an even number or a prime number?



SOLUTION

First, find the probabilities of the events. Let event A be getting an even number and let event B be getting a prime number.

$P(A) = \frac{4}{8}$ Of the 8 numbers, 4 are even.

$P(B) = \frac{3}{8}$ Of the 8 numbers, 3 are prime.

Then find the sum of the probabilities.

$$P(A \text{ or } B) = \frac{4}{8} + \frac{3}{8} = \frac{7}{8} = 0.875$$

▶ The probability that the spinner lands on an even number or a prime number is 0.875, or 87.5%.

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Overlapping events are events that have one or more outcomes in common. Use the rule below to find the probability of overlapping events.

PROBABILITY OF OVERLAPPING EVENTS

For two overlapping events A and B, the probability that either of the events occurs is the sum of the probabilities of the events, minus the probability that both events occur.

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ Events A and B are overlapping.

EXAMPLE 5 *Finding the Probability of Overlapping Events*

You roll a number cube. What is the probability that you get an odd number or a number greater than 3?

SOLUTION

Find the probabilities of the events. Let event A be getting an odd number and let event B be getting a number greater than 3.

$P(A) = \frac{3}{6}$ Of the 6 numbers, 3 are odd (1, 3, 5).

$P(B) = \frac{3}{6}$ Of the 6 numbers, 3 are greater than 3 (4, 5, 6).

$P(A \text{ and } B) = \frac{1}{6}$ Of the 6 numbers, 1 is odd and greater than 3 (5).

Find the probability of the events.

$$P(A \text{ or } B) = \frac{3}{6} + \frac{3}{6} - \frac{1}{6} = \frac{5}{6} \approx 0.83$$

▶ The probability that the number is odd or greater than 3 is about 0.83.

EXERCISES

Decide whether the events are *independent* or *dependent*.

1. You toss a coin two times.
   Event *A*: Get tails on the first toss.
   Event *B*: Get tails on the second toss.

2. A box contains blue and yellow paper clips. You randomly choose a paper clip, then choose another without replacing the first paper clip.
   Event *A*: The first paper clip is blue.
   Event *B*: The second paper clip is yellow.

3. A bag contains tulip bulbs and daffodil bulbs. You randomly choose a bulb, plant it, and randomly choose another bulb.
   Event *A*: The first bulb is a daffodil bulb.
   Event *B*: The second bulb is a daffodil bulb.

4. You randomly choose a card from a standard deck of 52 cards. You put the card back in the deck and then your friend randomly chooses a card.
   Event *A*: You choose the queen of diamonds.
   Event *B*: Your friend chooses the six of clubs.

5. Each week you and your sister assign chores by writing the chores on pieces of paper and randomly choose them from a hat.
   Event *A*: You choose "rake the leaves" the first week of October.
   Event *B*: You choose "rake the leaves" the second week of October.

Events *A* and *B* are independent. Find the indicated probability.

6. $P(A) = 0.5$
$P(B) = 0.8$
$P(A \text{ and } B) = \underline{?}$

7. $P(A) = 0.6$
$P(B) = \underline{?}$
$P(A \text{ and } B) = 0.24$

8. $P(A) = \underline{?}$
$P(B) = 0.5$
$P(A \text{ and } B) = 0.055$

Events *A* and *B* are dependent. Find the indicated probability.

9. $P(A) = 0.9$
$P(B \mid A) = 0.7$
$P(A \text{ and } B) = \underline{?}$

10. $P(A) = 0.2$
$P(B \mid A) = \underline{?}$
$P(A \text{ and } B) = 0.042$

11. $P(A) = \underline{?}$
$P(B \mid A) = 0.4$
$P(A \text{ and } B) = 0.34$

Decide whether the situation describes *independent* or *dependent* events. Then answer the question.

12. You plant a fruit tree. The probability that the tree will survive its first winter is 0.8. If it survives the first winter, the probability that the tree will bear fruit by its fourth year is 0.6. What is the probability that the tree will bear fruit by its fourth year?

13. You toss a coin and roll a number cube. What is the probability that you get tails and a 3?

14. A bucket of tennis balls contains 12 yellow, 5 orange, and 3 pink tennis balls. You randomly choose a ball, put it back, and randomly choose another ball. What is the probability that the first ball is yellow and the second ball is orange?

15. A bowl contains 22 green grapes and 28 purple grapes. You randomly choose a grape, eat it, and randomly choose another grape. What is the probability that both grapes are green?

In Exercises 16 and 17, use the following information.
The formula for finding the probability of independent events can be extended to three or more events. This is also true for dependent events.

16. You are assigned a 4 digit personal identification number (PIN) for use with your bank card. Each of the digits can be a whole number from 0 to 9, and the digits can repeat. What is the probability that your PIN is 1234?

17. In Exercise 16, suppose that the digits cannot repeat. What is the probability that your PIN is 1234?

In Exercises 18 and 19, read the first 100 words of a book. Count the number of words that satisfy each of the events below, which are independent.

Event *A*: The word starts with a vowel.
Event *B*: The word has more than four letters.
Event *C*: The word has one letter.

18. You randomly choose a word from the first 100 words. What is the probability that the word starts with a vowel and has more than four letters?

19. You randomly choose a word from the first 100 words. What is the probability that the word starts with a vowel and has one letter?

Events *A* and *B* are mutually exclusive events. Find *P*(*A* or *B*).

20. $P(A) = 0.2$
$P(B) = 0.12$

21. $P(A) = 0.07$
$P(B) = 0.33$

22. $P(A) = 62\%$
$P(B) = 15\%$

Events *A* and *B* are mutually exclusive events that involve rolling a number cube. Find *P*(*A* or *B*).

23. **Event *A***: Get a number greater than 4.
Event *B*: Get a number less than 4.

24. **Event *A***: Get a 2.
Event *B*: Get an odd number.

In Exercises 25–27, you randomly choose a letter from the word INDIANA. Find the probability of choosing the given letters.

25. I or N

26. N or D

27. A or N

28. A wallet contains six \$1 bills, one \$5 bill, and three \$10 bills. A bill is randomly chosen from the wallet. What is the probability that the bill is a \$1 bill or a \$5 bill?

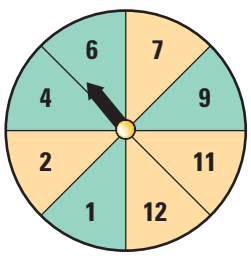
Events *A* and *B* are overlapping events. Find *P*(*A* or *B*).

29. $P(A) = 0.4$
$P(B) = 0.5$
$P(A \text{ and } B) = 0.2$

30. $P(A) = 0.5$
$P(B) = 0.5$
$P(A \text{ and } B) = 0.375$

31. $P(A) = 11\%$
$P(B) = 58\%$
$P(A \text{ and } B) = 23\%$

In Exercises 32–35, events *A* and *B* are overlapping events that involve spinning the spinner below. Find *P*(*A* or *B*).



32. **Event *A***: Lands on an odd number.
Event *B*: Lands on green.

33. **Event *A***: Lands on an even number.
Event *B*: Lands on yellow.

34. **Event *A***: Lands on green.
Event *B*: Lands on a number divisible by 3.

35. **Event *A***: Lands on a number greater than 4.
Event *B*: Lands on a number less than 11.

36. A standard deck of 52 playing cards contains 13 cards in each of the following four suits: hearts, diamonds, clubs, and spades. You randomly choose a card from the deck. What is the probability that the card is a 4 or a club?

Events *A* and *B* are *complementary*. Use the following information to find the indicated probability.

Two mutually exclusive events in which one or the other event must occur are called *complementary events*. The sum of the probabilities of two complementary events is 1. So, if A and B are complementary events, then $P(A) + P(B) = 1$.

37. $P(A) = \underline{?}$
$P(B) = 0.6$

38. $P(A) = 0.14$
$P(B) = \underline{?}$

39. $P(A) = \underline{?}$
$P(B) = 28\%$

Appendix 3

Estimation and Reasonableness

GOAL ***Use estimation to solve problems and to determine whether answers are reasonable.***

Some real-world problems require only an estimate for an answer instead of an exact answer. Before using estimation, you need to decide whether estimation is appropriate.

EXAMPLE 1 Deciding Whether Estimation is Appropriate

A music store is having a sale on CDs. All of the new releases have a sale price of $10.88, which includes tax. Erik wants to buy 3 CDs that are new releases. Decide whether an *estimate* or an *exact answer* is appropriate. Then solve.

a. How much money should Erik take with him to pay for the CDs?

b. What is the total cost of the CDs?

SOLUTION

a. An estimate is appropriate. To be sure that Erik has enough money, he should round up: $\$11 \cdot 3 = \$33$. So, Erik should bring $33.

b. An exact answer is appropriate.

$$\$10.88 \cdot 3 = \$32.64$$

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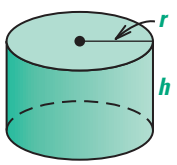
Estimation can often be used to check the reasonableness of an answer.

EXAMPLE 2 Using Estimation to Check Reasonableness

Determine whether the answers to the problem are reasonable.

Problem Use the formula $S = 2\pi r^2 + 2\pi rh$ to find the surface area S (in square meters) of the cylinder at the right.



$r = 6.1$ m
$h = 7.6$ m

Susan's answer: about 324 m^2

Len's answer: about 525 m^2

STUDENT HELP

Study Tip
The Greek letter π (pi) is often approximated by the number 3.14.

SOLUTION

To find an estimate of the surface area, round the radius, height, and pi to the nearest whole number: $S \approx 2(3)(6^2) + 2(3)(6)(8) = 504$ m^2.

The estimate of 504 square meters is not close to Susan's answer, but it is close to Len's answer. So, Susan's answer is not reasonable and Len's answer is reasonable.

ESTIMATION METHODS *Front-end estimation* can be used to estimate a sum when all the numbers have the same number of digits. When numbers being added have about the same value, *clustering* can be used to estimate their sum. Different types of estimation may yield different levels of accuracy.

EXAMPLE 3 Comparing Estimation Methods

You recently started working at a grocery store. The table below shows the amounts of your first 6 paychecks.

Week	1	2	3	4	5	6
Pay	$30.44	$31.62	$38.12	$40.01	$43.19	$46.88

a. Use front-end estimation to estimate your total pay for the 6 weeks.

b. Use clustering to estimate your total pay for the 6 weeks.

c. Find the actual total. Which method gives a closer estimate? Why?

SOLUTION

a. To use front-end estimation, add the values of the greatest digits to get a low estimate. Then use the remaining digits to adjust the sum and get a closer estimate.

1 Add the values of the tens' digits.

$$\begin{array}{r} 30.44 \\ 31.62 \\ 38.12 \\ 40.01 \\ 43.19 \\ +\ 46.88 \\ \hline 210 \end{array}$$

2 Approximate the sum of the remaining digits.

$$\begin{array}{rl} 30.44 & \searrow \\ 31.62 & \rightarrow \text{about } 10 \text{ more} \\ 38.12 & \nearrow \\ 40.01 & \searrow \\ 43.19 & \rightarrow \text{about } 10 \text{ more} \\ +\ 46.88 & \nearrow \\ \hline & \text{about } 20 \text{ more} \end{array}$$

3 Add the two sums.

$$\begin{array}{r} 210 \\ +\ 20 \\ \hline 230 \end{array}$$

▶ Your total pay for the first 6 weeks is about $230.

b. When a group of data are clustered, you can choose a convenient value near the center of the cluster to represent each data value in the group.

In the table, the amounts cluster around $40. An estimate of the sum is given by $6 \cdot 40 = 240$.

▶ Your total pay for the first 6 weeks is about $240.

c. The actual total is $230.26. The estimate obtained by front-end estimation is closer than the estimate obtained by clustering. The estimate obtained by clustering is greater than the estimate obtained by front-end estimation because the value chosen to represent the data in the cluster group is greater than the average data value in the cluster.

EXERCISES

Decide whether an *estimate* or an *exact answer* is appropriate.

1. the balance of a bank account on a bank statement
2. the number of hours spent doing homework during a week
3. the amount of money left as a tip for a waiter
4. a baseball player's batting average on a baseball card

Decide whether an *estimate* or an *exact answer* is appropriate. Then solve the problem.

5. You are helping prepare fundraising materials. It takes you 28 seconds to stuff an envelope and put an address label on the envelope. How many minutes should it take you to prepare 50 envelopes?
6. You have received the following scores on your science tests: 76, 82, 91, 87, 89. What is your average?

Determine whether the answer to the problem is reasonable. Explain your reasoning.

7. **Problem** Evaluate $5x - 5$ when $x = 6.1$. **Answer** 25.5
8. **Problem** Evaluate $3z^2 + z$ when $z = -1.9$. **Answer** -12.73
9. **Problem** Evaluate $2\pi r$ when $r = 4.8$ ft. **Answer** 9.6 ft
10. **Problem** Solve $9.85x = 66.98$. **Answer** $x = 6.8$
11. An airplane travels 1448 miles in 3 hours 20 minutes. A passenger calculates the average speed of the aircraft to be 580 miles per hour. Use an estimate to decide if the calculation is reasonable.

In Exercises 12–15, use the following information.
You are waiting in line to pay for several items at a grocery store. You want to be sure that you will have enough money for the items. The prices of the items are \$5.89, \$3.23, \$1.79, \$2.97, \$6.78, \$3.44, and \$4.88.

12. Estimate the total price by adding the values of the ones' digits, ignoring the decimal parts.
13. Is your estimate in Exercise 12 high or low? Is your estimate appropriate for the situation? Explain.
14. Estimate the total price by rounding each dollar amount up to the next whole number and adding the rounded amounts.
15. Is your estimate in Exercise 14 high or low? Is your estimate appropriate for the situation? Explain.
16. The weights (in pounds) of the starting defensive players for a football team are listed below. Use clustering to estimate the total weight of the starting defensive players.

216, 218, 220, 224, 230, 245, 251, 265, 268, 272, 278

In Exercises 17–19, use the following information.
You are mailing a box of items to a relative. The weights (in ounces) of the items are listed below.

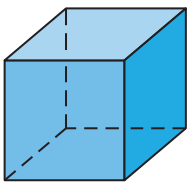
12.5, 13.2, 13.6, 20.8, 21.9, 22.7, 22.9

17. Use front-end estimation to estimate the total weight of the items in the box.

18. Use clustering to estimate the total weight of the items in the box.

19. Find the actual weight of the items in the box. Which method gives a closer estimate? Explain.

In Exercises 20 and 21, use the cube at the right.



20. Estimate the volume of the cube. (The volume V of a cube is given by the formula $V = s^3$, where s is the length of a side of the cube.)

21. Use a calculator to compare the actual volume of the cube to your estimate from Exercise 20. By what percent does your estimate differ from the actual volume?

In Exercises 22–25, use the following information.
Recall that if $a = b^2$, then $\sqrt{a} = b$. For example, because $9 = 3^2$, $\sqrt{9} = 3$.

22. Determine whether $\sqrt{21}$ is closer to 4 or 5. Explain your reasoning.

23. On a number line, between what two consecutive integers does $\sqrt{154}$ lie?

24. Find the value of $\sqrt{227}$ to the nearest whole number.

25. Estimate the value of $7.15 + 3.9\sqrt{89}$ by using whole number estimates.

In Exercises 26–28, use estimation to solve the problem.

26. The time it takes a pendulum to swing back and forth is called its *period*. The formula $T = 2\pi\sqrt{\frac{L}{384}}$ can be used to find the period T (in seconds) of a pendulum with a length of L inches. Estimate the period of a pendulum that is 90 inches long.

27. A sphere with radius r has a surface area S of 220 square centimeters. Use the formula $S = 4\pi r^2$ to estimate the length of the radius.

28. As Earth orbits the sun, the distance from Earth to the sun varies from about 9.14×10^7 to 9.45×10^7 miles. The speed of light is 186,282 miles per second. Estimate the time, in seconds, it takes for light from the sun to reach Earth when Earth is 9.1421555×10^7 miles from the sun.