11.7 Dividing Polynomials

What you should learn

GOAL 1 Divide a polynomial by a monomial or by a binomial factor.

GOAL 2 Use polynomial long division.

Why you should learn it

▼ To provide alternative forms of rational expressions to model **real-life** situations, as in the sports equipment problem in **Exs. 50–54**.

REAL LIFE

GOAL 1 DIVIDING A POLYNOMIAL

To divide a polynomial by a *monomial*, divide each term by the monomial, keeping the same signs between terms. Simplify each fraction.

ACTIVITY Developing Concepts

Investigating Polynomial Division

1. With a partner, discuss how to get from the original division problem to the simplified expression.

$$(2x^2 - 3) \div (6x) = \frac{?}{6x} = \frac{2x^2}{?} - \frac{?}{?} = \frac{x}{3} - \frac{1}{2x}$$

2. Explain how you could use multiplication to check that $(2x^2 - 3) \div (6x) = \frac{x}{3} - \frac{1}{2x}$. Carry out the check.

EXAMPLE 1 *Dividing a Polynomial by a Monomial*

Divide $12x^2 - 20x + 8$ by $4x$.

SOLUTION

$$\frac{12x^2 - 20x + 8}{4x} = \frac{12x^2}{4x} - \frac{20x}{4x} + \frac{8}{4x}$$ Divide each term of numerator by $4x$.

$$= \frac{3x(4x)}{4x} - \frac{5(4x)}{4x} + \frac{2(4)}{4x}$$ Find common factors.

$$= \frac{3x\cancel{(4x)}}{\cancel{4x}} - \frac{5\cancel{(4x)}}{\cancel{4x}} + \frac{2\cancel{(4)}}{\cancel{4}x}$$ Divide out common factors.

$$= 3x - 5 + \frac{2}{x}$$ Simplified form

To divide a polynomial by a *binomial,* first look to see whether the numerator and the denominator have a common factor. If they do, you can divide out the common factor.

$$\frac{x^2 - 2x - 3}{x - 3} = \frac{(x + 1)\cancel{(x - 3)}}{\cancel{x - 3}}$$

$$= x + 1$$

GOAL 2 USING LONG DIVISION

You can use **polynomial long division** to divide polynomials that do not have common factors. First review the process for long division in arithmetic.

EXAMPLE 2 *The Long Division Algorithm*

Use long division to divide 658 by 28.

SOLUTION

$$\begin{array}{r} 23 \\ 28\overline{)658} \\ \underline{56} \\ 98 \\ \underline{84} \\ 14 \end{array}$$

1. Think: $\frac{65}{28} \approx 2$
2. Subtract 2×28 from 65.
3. Bring down the 8. Think: $\frac{98}{28} \approx 3$
4. Subtract 3×28 from 98.
5. Remainder is 14.

Dividend, Quotient, Remainder, Divisor

$$\frac{658}{28} = 23 + \frac{14}{28} = 23 + \frac{1}{2} = 23\tfrac{1}{2}$$

> **STUDENT HELP**
>
> **Study Tip**
> To check division, you can multiply the answer by the divisor OR you can multiply the quotient by the divisor and add the remainder. You should get the dividend in either way. In Example 2, $28\left(23\frac{1}{2}\right) = 658$ and $(28)(23) + 14 = 658$.

EXAMPLE 3 *Polynomial Long Division*

Divide $x^2 + 2x + 4$ by $x - 1$.

SOLUTION

$$\begin{array}{r} x + 3 \\ x - 1\overline{)x^2 + 2x + 4} \\ \underline{x^2 - x} \\ 3x + 4 \\ \underline{3x - 3} \\ 7 \end{array}$$

To subtract $(-x)$ from $2x$, you add x.

1. Think: $\frac{x^2}{x} = x$
2. Subtract $x(x - 1)$.
3. Bring down $+ 4$. Think: $\frac{3x}{x} = 3$
4. Subtract $3(x - 1)$.
5. Remainder is 7.

Quotient, Dividend, Divisor, Remainder

$$\frac{x^2 + 2x + 4}{x - 1} = x + 3 + \frac{7}{x - 1}$$

▶ The answer is $x + 3 + \frac{7}{x - 1}$.

When dividing polynomials, remember that the polynomials need to be written in standard form. That is, the terms should be written in descending order according to their degrees.

If a term is missing in the standard form of the dividend, insert a term with a coefficient of 0 to hold the place.

EXAMPLE 4 *Adding a Place-holding Zero*

Divide $6x^2 - 11$ by $x + 2$.

SOLUTION

First insert a place-holding term $0x$ in the dividend: $6x^2 - 11 = 6x^2 + 0x - 11$.

$$
\begin{array}{r l}
 & \textbf{1. Think: } \frac{6x^2}{x} = 6x \\
\color{red}{6x - 12} & \\
x + 2\overline{)6x^2 + 0x - 11} & \\
\underline{6x^2 + 12x} & \textbf{2. Subtract } (6x)(x + 2). \\
-12x - 11 & \textbf{3. Bring down } -11\textbf{. Think: } \frac{-12x}{x} = -12 \\
\underline{-12x - 24} & \textbf{4. Subtract } -12(x + 2). \\
13 & \textbf{5. Remainder is 13.}
\end{array}
$$

▶ The answer is $6x - 12 + \frac{13}{x + 2}$.

EXAMPLE 5 *Rewriting in Standard Form*

Divide $m^2 - 16m - 21$ by $4 + 2m$.

SOLUTION

First write the divisor $4 + 2m$ in standard form as $2m + 4$.

$$
\begin{array}{r l}
 & \textbf{1. Think: } \frac{m^2}{2m} = \frac{1}{2}m \\
\color{red}{\frac{1}{2}m - 9} & \\
2m + 4\overline{)m^2 - 16m - 21} & \\
\underline{m^2 + 2m} & \textbf{2. Subtract } \frac{1}{2}m(2m + 4). \\
-18m - 21 & \textbf{3. Bring down } -21\textbf{. Think: } \frac{-18m}{2m} = -9 \\
\underline{-18m - 36} & \textbf{4. Subtract } -9(2m + 4). \\
15 & \textbf{5. Remainder is 15.}
\end{array}
$$

▶ The answer is $\frac{1}{2}m - 9 + \frac{15}{2m + 4}$.

STUDENT HELP

INTERNET **HOMEWORK HELP** Visit our Web site www.mcdougallittell.com for extra examples.

GUIDED PRACTICE

Vocabulary Check ✓

1. How is polynomial long division like long division with whole numbers? How is it different?

Concept Check ✓

2. For which of the divisions in Exercises 9–14 do you not need to use long division? Explain why not.

3. For the student's long division work shown at the right, complete the answer.

$$\frac{x^2-5}{x-1} = \text{Quotient} + \frac{\text{Remainder}}{\text{Divisor}}$$

$$= \boxed{?} + \frac{\boxed{?}}{\boxed{?}}$$

$$\begin{array}{r} x + 1 \\ x - 1\overline{)x^2 + 0x - 5} \\ \underline{x^2 - x\phantom{{}-5}} \\ x - 5 \\ \underline{x - 1} \\ -4 \end{array}$$

4. Explain how to check the answer to the division in Exercise 3.

Skill Check ✓

Set up the long division problem, but do not perform the division.

5. Divide $y^2 + 8$ by $y + 2$.
6. Divide $-x^2 - 4x + 21$ by $-x + 3$.
7. Divide $8y^2 - 2y$ by $3y + 5$.
8. Divide $72 - 18x + x^2$ by $x - 6$.

Divide.

9. Divide 856 by 29.
10. Divide $18x^2 + 45x - 36$ by $9x$.
11. Divide $x^2 - 8x + 15$ by $x - 3$.
12. Divide $y^2 + 6y + 2$ by $y + 3$.
13. Divide $10b^3 - 8b^2 - 5b$ by $-2b$.
14. Divide $2x^2 - x + 4$ by $3x - 6$.

PRACTICE AND APPLICATIONS

STUDENT HELP

Extra Practice to help you master skills is on p. 807.

DIVIDING BY A MONOMIAL **Divide.**

15. Divide $8x + 13$ by 2.
16. Divide $16y - 9$ by 4.
17. Divide $9c^2 + 3c$ by c.
18. Divide $9m^3 + 4m^2 - 8m$ by m.
19. Divide $-2x^2 - 12x$ by $-2x$.
20. Divide $7p^3 + 18p^2$ by p^2.
21. Divide $9a^2 - 54a - 36$ by $3a$.
22. Divide $16y^3 - 36y^2 - 64$ by $-4y^2$.

STUDENT HELP

HOMEWORK HELP

Example 1: Exs. 15–22
Example 2: Exs. 23–26
Example 3: Exs. 23–46
Example 4: Exs. 27–46
Example 5: Exs. 27–46

CHECKING QUOTIENTS AND REMAINDERS **Match the polynomial division problem with the correct answer.**

23. $(5x^2 + 2x + 3) \div (x + 2)$
24. $(12x - 5) \div (2x - 3)$
25. $(10x^2 - 7x - 12) \div (2x - 3)$
26. $(2x^2 + 5x + 2) \div (x + 2)$

A. $6 + \frac{13}{2x - 3}$
B. $5x + 4$
C. $2x + 1$
D. $5x - 8 + \frac{19}{x + 2}$

DIVIDING POLYNOMIALS **Divide.**

27. Divide $a^2 - 3a + 2$ by $a - 1$.

28. Divide $y^2 + 5y + 7$ by $y + 2$.

29. Divide $2b^2 - 3b - 4$ by $b - 2$.

30. Divide $3p^2 + 10p + 3$ by $p + 3$.

31. Divide $5g^2 + 14g - 2$ by $g + 3$.

32. Divide $c^2 - 25$ by $c - 5$.

33. Divide $x^2 - 3x - 59$ by $x - 9$.

34. Divide $d^2 + 15d + 45$ by $d + 5$.

35. Divide $-x^2 - 6x - 16$ by $x + 2$.

36. Divide $-x^2 + 9x - 12$ by $-x - 2$.

37. Divide $b^2 - 7b + 4$ by $b + 3$.

38. Divide $5 - 7m + 3m^2$ by $m - 3$.

39. Divide $x^2 + 9$ by $-x - 4$.

40. Divide $4x^2 + 12x - 10$ by $x - 2$.

41. Divide $-5m^2 + 2$ by $m - 1$.

42. Divide $4 + 11q + 6q^2$ by $2q + 1$.

43. Divide $4 - s^2$ by $s + 5$.

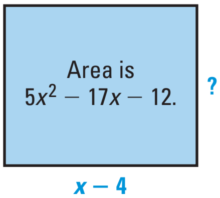
44. Divide $16a^2 - 25$ by $3 + 4a$.

45. Divide $c^2 - 7c + 21$ by $2c - 6$.

46. Divide $b^2 - 7b - 12$ by $4b + 4$.

GEOMETRY CONNECTION **The area and one dimension of the rectangle are shown. Find the missing dimension.**

47.



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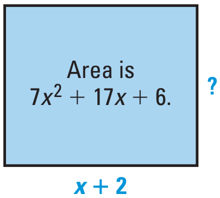

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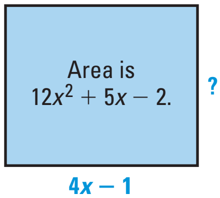


EXERCISE AND OTHER SPORTS EQUIPMENT **In Exercises 50–54, use the models below that approximate spending in the United States from 1988 to 1997. Let *t* represent the number of years since 1988.**

Dollars spent on exercise equipment (in millions): $E = 200t + 1400$

Total dollars spent on sports equipment (in millions): $S = 900t + 9900$

50. Write a rational model for the ratio of the money spent on exercise equipment to the total money spent on sports equipment. Simplify the model by dividing out the greatest common factor.

51. Use long division to write the model from Exercise 50 in another form.

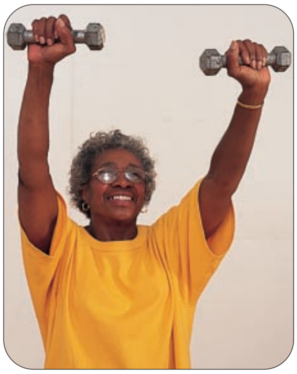
52. Copy and complete the table. Use the rewritten model from Exercise 51. Round to the nearest thousandth.

Year t	0	1	2	3	4	5	6	7	8	9
Ratio of E to S	?	?	?	?	?	?	?	?	?	?

53. Use the table. Was the ratio increasing or decreasing from 1988 to 1997?

54. **CRITICAL THINKING** Look back at the model you found in Exercise 51. How can you tell from the model if the ratio is increasing or if it is decreasing?

FOCUS ON APPLICATIONS



REAL LIFE **EXERCISE EQUIPMENT**
In 1997 Americans spent about 3 billion dollars on exercise equipment.
INTERNET DATA UPDATE of National Sporting Goods Association data at mcdougallittell.com

Test Preparation

55. MULTI-STEP PROBLEM Suppose you are 14 years old and your brother is 4 years old.

a. In t years, your age will be $14 + t$. What will your brother's age be?

b. Write the ratio of your age in t years to your brother's age in t years. Then use long division to rewrite this ratio.

c. Use the rewritten ratio to find the ratio of your ages now, in 5 years, in 10 years, in 25 years, in 50 years, and in 80 years.

d. Use your answers to part (c). Is the ratio of your ages getting smaller or larger as time goes by? What value do the ratios approach?

e. *Writing* Look at the original form of the ratio and the rewritten form of the ratio. Which form of the ratio makes it easier for you to recognize the trend that you described in part (d)? Explain your choice.

★ Challenge

AREA MODELS In Exercises 56–60, use the diagrams.

Diagram 1

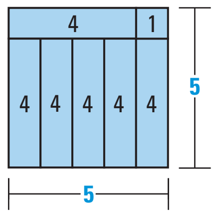
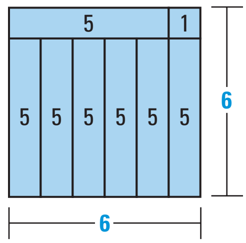


Diagram 2



56. Diagram 1 models $5^2 \div 4$. Write the quotient and the remainder.

57. What division problem does Diagram 2 model? Write the quotient and the remainder.

58. Draw diagrams to model $4^2 \div 3$ and $7^2 \div 6$.

59. LOGICAL REASONING Describe any pattern you observe in Exercises 56–58. Use the pattern to predict the quotient and remainder when x^2 is divided by $x - 1$.

EXTRA CHALLENGE
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60. Use long division to find $x^2 \div (x - 1)$. Did you get the result you predicted in Exercise 59?

MIXED REVIEW

SOLVING EQUATIONS Solve the equation. (Review 3.2 for 11.8)

61. $\frac{x}{5} = 3$ **62.** $\frac{a}{-3} = 7$ **63.** $\frac{c}{4} = \frac{6}{8}$ **64.** $\frac{y}{-2} = \frac{5}{4}$

SOLVING PROPORTIONS In Exercises 65–70, solve the proportion. Check for extraneous solutions. (Review 11.1 for 11.8)

65. $\frac{7}{5} = \frac{2}{x}$ **66.** $\frac{2}{x} = \frac{x - 1}{6}$ **67.** $\frac{6x - 7}{4} = \frac{5}{x}$

68. $\frac{8b^2 + 4b}{4b} = \frac{2b - 5}{3}$ **69.** $\frac{5p^2 - 9}{5} = \frac{2p^2 + 3p}{2}$ **70.** $\frac{a^2 - 4}{a - 2} = \frac{a + 2}{10}$

71. GRAPHING INVERSE VARIATION Use the model $y = \frac{12}{x}$. Make a table of values for $x = 1, 2, 3, 4$, and 5. Sketch the graph. (Review 11.3 for 11.8)

72. BIOLOGY CONNECTION The largest mammal, a blue whale, has a weight of 1.3×10^5 kilograms. The smallest mammal, a pygmy shrew, has a weight of 2.0×10^{-3} kilogram. What is the ratio of the weight of a blue whale to the weight of a pygmy shrew? (Review 8.4)