Adding and Subtracting Matrices

GOAL ① ORGANIZING DATA IN A MATRIX

What you should learn

GOAL ① Organize data in a matrix.

GOAL ② Add and subtract two matrices.

Why you should learn it

To organize data, such as the number of male and female members of the United States Congress in **Example 3**.

A **matrix** is a rectangular arrangement of numbers into horizontal rows and vertical columns. Each number in the matrix is called an **entry** or an **element**. (The plural of *matrix* is *matrices*.)

$$\begin{bmatrix} 3 & 1 & 0 & 8 \\ -1 & 2 & 4 & -2 \\ 5 & -7 & 3 & 6 \end{bmatrix}$$

column

row

The entry in the second row and third column is 4.
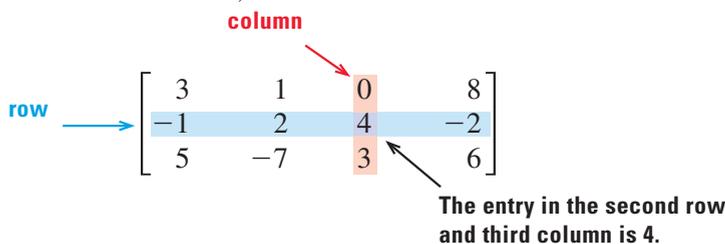
The size of a matrix is described as follows.

(the number of **rows**) $\times$ (the number of **columns**)

The matrix above is a 3×4 (read "3 by 4") matrix, because it has three rows and four columns. Think of a matrix as a type of table that can be used to organize data.

Two matrices are equal if the entries in corresponding positions are equal.

$$\begin{bmatrix} 3 & -2 \\ \frac{1}{2} & 0 \end{bmatrix} = \begin{bmatrix} 3 & -2 \\ 0.5 & 0 \end{bmatrix} \qquad \begin{bmatrix} -4 & 7 \\ 0 & -1 \end{bmatrix} \neq \begin{bmatrix} 7 & -4 \\ 0 & -1 \end{bmatrix}$$

EXAMPLE 1 *Writing a Matrix*

Write a matrix to organize the following information about your CD collection.

Country: 4 groups, 6 solo artists, 0 collections

Rock: 8 groups, 3 solo artists, 3 collections

Blues: 1 group, 5 solo artists, 2 collections

SOLUTION

Country, *Rock*, and *Blues* can be labels for the rows or for the columns.

AS ROW LABELS:

	Group	Solo artist	Collection
Country	4	6	0
Rock	8	3	3
Blues	1	5	2

AS COLUMN LABELS:

	Country	Rock	Blues
Group	4	8	1
Solo artist	6	3	5
Collection	0	3	2

To add or subtract matrices, you add or subtract corresponding entries. Each matrix must have the same number of rows and columns. For instance, you cannot add a matrix that has three rows to a matrix that has only two rows.

EXAMPLE 2 *Adding and Subtracting Matrices*

a.
$$\begin{bmatrix} 4 & 2 \\ 0 & -3 \\ -5 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 2 & -1 \\ 6 & -4 \end{bmatrix} = \begin{bmatrix} 4+1 & 2+0 \\ 0+2 & -3+(-1) \\ -5+6 & 1+(-4) \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 2 \\ 2 & -4 \\ 1 & -3 \end{bmatrix}$$

b.
$$\begin{bmatrix} 10 & -6 \\ 5 & 0 \end{bmatrix} - \begin{bmatrix} 4 & 5 \\ -3 & 2 \end{bmatrix} = \begin{bmatrix} 10-4 & -6-5 \\ 5-(-3) & 0-2 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & -11 \\ 8 & -2 \end{bmatrix}$$

EXAMPLE 3 *Political Composition of U.S. Congress*

CONGRESS The United States Congress is composed of the House of Representatives and the Senate. The matrices below show the number of men and women in the Senate and the House at the 1999 start of the 106th Congress. Write and label a single matrix that shows the number of men and women in Congress in 1999. **DATA UPDATE** of United States Congress data at www.mcdougallittell.com

HOUSE	Men	Women
Democrat	172	39
Republican	206	17
Other	1	0

SENATE	Men	Women
Democrat	39	6
Republican	52	3
Other	0	0

SOLUTION Add the two matrices. Then label the result.

$$\begin{bmatrix} 172 & 39 \\ 206 & 17 \\ 1 & 0 \end{bmatrix} + \begin{bmatrix} 39 & 6 \\ 52 & 3 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 211 & 45 \\ 258 & 20 \\ 1 & 0 \end{bmatrix}$$

▶ The result can be written as follows.

CONGRESS	Men	Women
Democrat	211	45
Republican	258	20
Other	1	0

ORGANIZING DATA As you learned in Example 1, matrices are a useful way to organize and keep track of data. For example, if you have a business, it is important to keep track of revenue (or income) and expenses (or costs). You can find out how much profit has been made by subtracting expenses from revenue. If the profit is a negative number, you lost money.

REAL LIFE

Business

EXAMPLE 4 *Finding a Profit Matrix*

You own two stores that sell household appliances. The matrices below show revenue and expenses for three months at each store.

	REVENUE ($)	
	Store 1	Store 2
January	78,432	109,345
February	82,529	120,429
March	94,311	118,782

	EXPENSES ($)	
	Store 1	Store 2
January	59,426	98,459
February	64,372	104,972
March	85,456	120,833

a. Write a matrix that shows the monthly profit for each store.

b. Which store had higher overall profits during the three-month period?

c. Which store lost money? In which month?

SOLUTION

Profit is the difference of revenue and expenses.

a. To find the *profit matrix,* you can subtract the *expenses matrix* from the *revenue matrix.*

$$
\begin{bmatrix} 78,432 & 109,345 \\ 82,529 & 120,429 \\ 94,311 & 118,782 \end{bmatrix} - \begin{bmatrix} 59,426 & 98,459 \\ 64,372 & 104,972 \\ 85,456 & 120,833 \end{bmatrix} = \begin{bmatrix} 19,006 & 10,886 \\ 18,157 & 15,457 \\ 8,855 & -2,051 \end{bmatrix}
$$

▶ Label the matrix to identify the monthly profit at each store.

	PROFIT ($)	
	Store 1	Store 2
January	19,006	10,886
February	18,157	15,457
March	8,855	−2,051

b. Add the entries in each column of the profit matrix to find the total profit for each store during the three-month period.

Store 1: $19{,}006 + 18{,}157 + 8{,}855 = \$46{,}018$

Store 2: $10{,}886 + 15{,}457 + (-2{,}051) = \$24{,}292$

▶ Store 1 had higher overall profits.

c. Store 2 had a negative profit of $-2{,}051$ in March. This means the store lost $2,051 during March.

GUIDED PRACTICE

Vocabulary Check ✓

1. How many rows are there in the matrix at the right? How many columns?

$$\begin{bmatrix} 5 & -7 & 3 \\ 2 & -2 & -4 \end{bmatrix}$$

Concept Check ✓

2. Is the matrix at the right a 3×2 matrix or a 2×3 matrix?

3. 🌐 **CONGRESS** Use the matrix showing the number of Democratic and Republican members of the House of Representatives from Arkansas, Delaware, and North Dakota. What is the entry in the first row and second column? What does the number represent?

$$\begin{array}{c} \\ \text{AR} \\ \text{DE} \\ \text{ND} \end{array} \begin{array}{cc} \text{Democrat} & \text{Republican} \\ \begin{bmatrix} 2 & 2 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \end{array}$$

Skill Check ✓

4. 🌐 **VIDEO RENTALS** Write a matrix to organize the information about a video store's movies. Label each row and column.

Comedy: 25 new releases, 215 regular selections

Drama: 30 new releases, 350 regular selections

Horror: 26 new releases, 180 regular selections

Find the sum and the difference of the matrices.

5. $\begin{bmatrix} -3 & 0 \\ -6 & 4 \\ 1 & -4 \end{bmatrix}, \begin{bmatrix} 2 & -4 \\ 1 & -3 \\ -1 & 9 \end{bmatrix}$

6. $\begin{bmatrix} 1 & 8 & -2 \\ -4 & -5 & 6 \end{bmatrix}, \begin{bmatrix} -1 & 9 & 2 \\ 3 & 3 & -5 \end{bmatrix}$

PRACTICE AND APPLICATIONS

MATRIX OPERATIONS Tell whether the matrices can be added.

7. $\begin{bmatrix} 4 & -1 \\ 7 & 5 \end{bmatrix}, \begin{bmatrix} 2 & -2 \\ 5 & -6 \end{bmatrix}$

8. $\begin{bmatrix} 3 & -2 & 0 \\ -4 & 1 & -8 \end{bmatrix}, \begin{bmatrix} -4 & 5 \\ 10 & 5 \end{bmatrix}$

9. $\begin{bmatrix} 4 & 2 \\ -6 & 3 \\ -1 & -2 \end{bmatrix}, \begin{bmatrix} 6 & 4 & -3 \\ 7 & -8 & 1 \end{bmatrix}$

10. $\begin{bmatrix} 8 & 5 & -8 \\ 4 & -1 & 2 \end{bmatrix}, \begin{bmatrix} -2 & -9 & 1 \\ -6 & 0 & 4 \end{bmatrix}$

ADDING MATRICES Find the sum of the matrices.

11. $\begin{bmatrix} 3 & -2 \\ 5 & 1 \end{bmatrix} + \begin{bmatrix} 4 & -3 \\ -8 & -2 \end{bmatrix}$

12. $\begin{bmatrix} 4 & -1 \\ -5 & -9 \end{bmatrix} + \begin{bmatrix} -6 & -3 \\ 2 & -3 \end{bmatrix}$

13. $\begin{bmatrix} 1 & -2 & 2 \\ 0 & -3 & 4 \end{bmatrix} + \begin{bmatrix} 3 & -4 & 5 \\ -8 & 1 & 6 \end{bmatrix}$

14. $\begin{bmatrix} -2.4 & 1.6 & -7.8 \\ 14.3 & 1.1 & -3.9 \end{bmatrix} + \begin{bmatrix} -2.8 & 5.4 & 2.3 \\ -1.7 & 4.2 & 5.6 \end{bmatrix}$

15. $\begin{bmatrix} 6.2 & -1.2 \\ -2.5 & -4.4 \\ 3.4 & -5.8 \end{bmatrix} + \begin{bmatrix} 1.5 & 9.2 \\ 6.6 & -2.2 \\ 5.7 & -7.1 \end{bmatrix}$

16. $\begin{bmatrix} 2 & 9 & -3 \\ 1 & 8 & -2 \\ -3 & -1 & -7 \end{bmatrix} + \begin{bmatrix} -2 & -6 & 4 \\ -1 & -2 & 5 \\ 2 & 0 & 8 \end{bmatrix}$

SUBTRACTING MATRICES Find the difference of the matrices.

17. $\begin{bmatrix} 8 & -3 \\ 4 & -1 \end{bmatrix} - \begin{bmatrix} 7 & 7 \\ -2 & -5 \end{bmatrix}$

18. $\begin{bmatrix} 4 & 3 \\ -12 & -10 \end{bmatrix} - \begin{bmatrix} -6 & 1 \\ -4 & 2 \end{bmatrix}$

19. $\begin{bmatrix} -4 & 1 \\ 0 & -13 \\ 2 & -8 \end{bmatrix} - \begin{bmatrix} -6 & 3 \\ -5 & 8 \\ 2 & -7 \end{bmatrix}$

20. $\begin{bmatrix} -5 & 11 & -2 \\ -10 & 4 & 6 \end{bmatrix} - \begin{bmatrix} -3 & 0 & 2 \\ 8 & -5 & -1 \end{bmatrix}$

MENTAL MATH Use mental math to find *a, b, c,* and *d.*

21. $\begin{bmatrix} 3a & 5b \\ c-6 & d \end{bmatrix} = \begin{bmatrix} -12 & -5 \\ 1 & -3 \end{bmatrix}$

22. $\begin{bmatrix} 4a & b+3 \\ c & d-3 \end{bmatrix} = \begin{bmatrix} 8 & -1 \\ 0 & -6 \end{bmatrix}$

WRITING A MATRIX Write and label a matrix to organize the information.

23. Music Store Inventory:

   CDs: 52 sale price titles, 3300 regular price titles

   Tapes: 28 sale price titles, 1600 regular price titles

24. Team Uniform Order:

   Shirts: 3 small, 7 medium, 10 large, 5 extra large

   Shorts: 7 small, 4 medium, 2 large, 2 extra large

DOG KENNEL The owner of a kennel keeps records of all the dogs she cares for each year. In Exercises 25 and 26, use the matrices below, which show her records for 3 breeds of dogs cared for in 1999 and 2000.

1999

	Male	Female
Beagle	36	28
Dalmatian	24	26
Bulldog	51	32

2000

	Male	Female
Beagle	40	31
Dalmatian	26	20
Bulldog	46	34

25. Find the sum of the two matrices.

26. *Writing* Explain what the data represent in the matrix you wrote in Exercise 25.

WAGES AND RAISES In Exercises 27 and 28, use the matrix, which shows employees' hourly wage rates at a grocery store. The wage rates depend on the job and the number of years of experience.

WAGE RATES

	0–1 year	2–3 years	4+ years
Service Clerk	5.50	6.50	7.00
Cashier	6.50	8.00	9.50
Deli Clerk	7.50	8.75	11.00

27. The store is giving $.20 raises to all services clerks, $.35 raises to all cashiers, and $.45 raises to all deli clerks. Write a matrix that you can add to the matrix above to find the new wage rates after raises are given.

28. Write a matrix that shows the new wage rates after raises are given.

29. MULTI-STEP PROBLEM Use the table, which shows the monthly average high and low temperatures in degrees Fahrenheit in three cities.

	May		June		July	
City	**High**	**Low**	**High**	**Low**	**High**	**Low**
Atlanta, GA	79.6	58.7	85.8	66.2	88.0	69.5
San Francisco, CA	66.5	49.7	70.3	52.6	71.6	53.9
Anchorage, AK	54.4	38.8	61.6	47.2	65.2	51.7

▶ Source: National Climatic Data Center

a. Write a matrix for the average high temperatures in each city during May, June, and July.

b. Write a matrix for the average low temperatures in each city during May, June, and July.

c. CRITICAL THINKING Explain how to find the difference of average temperatures in each city during May, June, and July. Use the matrices you wrote in parts (a) and (b). Then write a matrix for the difference of average temperatures in each city during May, June, and July.

★ **Challenge**

PROPERTIES OF MATRIX ADDITION In Exercises 30 and 31, recall the properties of addition you learned on page 73.

30. a. Does $\begin{bmatrix} 5 & -9 \\ -4 & 1 \\ -1 & 4 \end{bmatrix} + \begin{bmatrix} 2 & 8 \\ 1 & 9 \\ -3 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 8 \\ 1 & 9 \\ -3 & -1 \end{bmatrix} + \begin{bmatrix} 5 & -9 \\ -4 & 1 \\ -1 & 4 \end{bmatrix}$? Explain.

b. Does the commutative property apply when adding matrices?

EXTRA CHALLENGE
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31. Does the associative property apply when adding matrices? Give an example to support your answer.

MIXED REVIEW

EXPONENTIAL FORM Write the expression in exponential form. **(Review 1.2)**

32. $2y \cdot 2y \cdot 2y$ **33.** five squared   **34.** four to the sixth power

35. $x \cdot x \cdot y \cdot y \cdot y$ **36.** two to the xth power   **37.** $3 \cdot (t \cdot t \cdot t \cdot t)$

FINDING ABSOLUTE VALUES Evaluate the expression. **(Review 2.1)**

38. $|-82|$ **39.** $-|43.7|$ **40.** $-|-4.5|$

41. $|-29| + 7$ **42.** $-|13 - 12.1|$ **43.** $14 + |-11| - 10$

RULES OF ADDITION Find the sum. **(Review 2.2 for 2.5)**

44. $-19 + (-6)$ **45.** $-12 + (-9)$ **46.** $-3 + 0 + (-29)$

47. $0 + (-5) + 2$ **48.** $-3 + (-6) + (-2)$ **49.** $-5 + (-6) + (-3)$

50. $-7 + (-8) + (-9)$ **51.** $-1 + (-1) + (-1)$ **52.** $-4 + (-4) + (-4)$