

6.1

Solving One-Step Linear Inequalities

GOAL 1 GRAPHING LINEAR INEQUALITIES

What you should learn

GOAL 1 Graph linear inequalities in one variable.

GOAL 2 Solve one-step linear inequalities.

Why you should learn it

▼ To describe real-life situations, such as the speeds of runners in Example 1.



The **graph** of a linear inequality in one variable is the set of points on a number line that represent all solutions of the inequality.

VERBAL PHRASE	INEQUALITY	GRAPH
All real numbers less than 2	$x < 2$	
All real numbers greater than -2	$x > -2$	
All real numbers less than or equal to 1	$x \leq 1$	
All real numbers greater than or equal to 0	$x \geq 0$	

An open dot is used for $<$ or $>$ and a solid dot for \leq or \geq .

EXAMPLE 1 Write and Graph a Linear Inequality

Sue ran a 2-kilometer race in 8 minutes. Write an inequality to describe the average speeds of runners who were faster than Sue. Graph the inequality.

SOLUTION Average speed is $\frac{\text{distance}}{\text{time}}$. A faster runner's average speed must be greater than Sue's average speed.

VERBAL MODEL	Faster average speed $>$ $\frac{\text{Distance}}{\text{Sue's time}}$
LABELS	<p>Faster average speed = s (kilometers per minute)</p> <p>Distance = 2 (kilometers)</p> <p>Sue's time = 8 (minutes)</p>
ALGEBRAIC MODEL	<p>$s > \frac{2}{8}$ Write linear inequality.</p> <p>$s > \frac{1}{4}$ Simplify.</p>



STUDENT HELP

Look Back
For help with inequalities, see p. 26.

STUDENT HELP**Look Back**

For help with solving equations with addition and subtraction, see pp. 132–134.

GOAL 2 SOLVING ONE-STEP LINEAR INEQUALITIES

Solving a linear inequality in one variable is much like solving a linear equation in one variable. To solve the inequality, you isolate the variable on one side using transformations that produce **equivalent inequalities**, which have the same solution(s).

TRANSFORMATIONS THAT PRODUCE EQUIVALENT INEQUALITIES

	Original Inequality	Equivalent Inequality
• Add the same number to <i>each</i> side.	$x - 3 < 5$	Add 3. $x < 8$
• Subtract the same number from <i>each</i> side.	$x + 6 \geq 10$	Subtract 6. $x \geq 4$

EXAMPLE 2 Using Subtraction to Solve an Inequality

Solve $x + 5 \geq 3$. Graph the solution.

SOLUTION

$$x + 5 \geq 3 \quad \text{Write original inequality.}$$

$$x + 5 - 5 \geq 3 - 5 \quad \text{Subtract 5 from each side.}$$

$$x \geq -2 \quad \text{Simplify.}$$

▶ The solution is all real numbers greater than or equal to -2 . Check several numbers that are greater than or equal to -2 in the original inequality.

**EXAMPLE 3 Using Addition to Solve an Inequality**

Solve $-2 > n - 4$. Graph the solution.

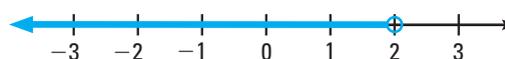
SOLUTION

$$-2 > n - 4 \quad \text{Write original inequality.}$$

$$-2 + 4 > n - 4 + 4 \quad \text{Add 4 to each side.}$$

$$2 > n \quad \text{Simplify.}$$

▶ The solution is all real numbers less than 2. Check several numbers that are less than 2 in the original inequality.

**STUDENT HELP****Study Tip**

Check several possible solutions by substituting them in the original inequality. Also try some numbers that are *not* solutions to make sure that they do *not* satisfy the original inequality.

STUDENT HELP**Look Back**

For help with solving equations with multiplication and division, see p. 138.

USING MULTIPLICATION AND DIVISION The operations used to solve linear inequalities are similar to those used to solve linear equations, but there are important differences. When you multiply or divide each side of an inequality by a *negative* number, you must *reverse* the inequality symbol to maintain a true statement. For instance, to reverse $>$, replace it with $<$.

TRANSFORMATIONS THAT PRODUCE EQUIVALENT INEQUALITIES

	Original inequality		Equivalent inequality
• Multiply each side by the same <i>positive</i> number.	$\frac{1}{2}x > 3$	Multiply by 2.	$x > 6$
• Divide each side by the same <i>positive</i> number.	$3x \leq 9$	Divide by 3.	$x \leq 3$
• Multiply each side by the same <i>negative</i> number and <i>reverse</i> the inequality symbol.	$-x < 4$	Multiply by (-1).	$x > -4$
• Divide each side by the same <i>negative</i> number and <i>reverse</i> the inequality symbol.	$-2x \leq 6$	Divide by (-2).	$x \geq -3$

EXAMPLE 4 *Using Multiplication or Division to Solve an Inequality*

a. $\frac{a}{3} \leq 12$ **Original inequality**

$3 \cdot \frac{a}{3} \leq 3 \cdot 12$ **Multiply each side by positive 3.**

$a \leq 36$ **Simplify.**

▶ The solution is all real numbers less than or equal to 36. Check several numbers that are less than or equal to 36 in the original inequality.

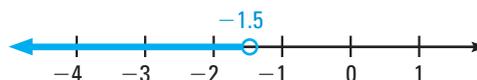


b. $-4.2m > 6.3$ **Original inequality**

$\frac{-4.2m}{-4.2} < \frac{6.3}{-4.2}$ **Divide each side by -4.2 and reverse inequality symbol.**

$m < -1.5$ **Simplify.**

▶ The solution is all real numbers less than -1.5 . Check several numbers that are less than -1.5 in the original inequality.

**STUDENT HELP**

INTERNET
HOMEWORK HELP
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for extra examples.

GUIDED PRACTICE

Vocabulary Check ✓

1. Since $x - 3 < 5$ and $x < 8$ have the same solution, they are ? inequalities.

Concept Check ✓

Write a verbal phrase that describes the inequality.

2. $x > -7$

3. $x < 1$

4. $9 \leq x$

5. $10 \geq x$

ERROR ANALYSIS Describe and correct the error.

6.

$$\begin{array}{r} -3x \geq 15 \\ -3x \geq 15 \\ \hline -3 \quad -3 \\ \hline x \geq -5 \end{array}$$

7.

$$\begin{array}{r} x + 4 < 1 \\ x + 4 - 4 < 1 - 4 \\ \hline x > -3 \end{array}$$

Tell whether you should use an *open dot* or a *closed dot* on the graph of the inequality.

8. $x < 3$

9. $x > 10$

10. $x \geq 5$

11. $3x + 5 < 4$

12. $5x - 3 \geq 12$

13. $-2x - 1 \leq 3$

Skill Check ✓

Solve the inequality and graph its solution.

14. $-2 + x < 5$

15. $-3 \leq y + 2$

16. $-2b \leq -8$

17. $x - 4 > 10$

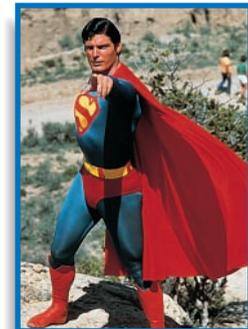
18. $5x > -45$

19. $-6y \leq 36$

 **COLLECTIBLES** In 1997 a software executive bid \$19,550 for Superman's cape from the 1978 movie *Superman*. This was the highest bid.

20. Let b represent the amount of a bid for the cape. Write an inequality for b .

21. Graph the inequality.



PRACTICE AND APPLICATIONS

STUDENT HELP

Extra Practice
to help you master
skills is on p. 802.

STUDENT HELP

HOMEWORK HELP

Example 1: Exs. 22–30,
61–63

Examples 2, 3:

Exs. 31–45, 55–60

Example 4: Exs. 46–60

GRAPHING Graph the inequality.

22. $x > -2$

23. $x < 16$

24. $x \leq 7$

25. $x \geq -6$

26. $2 \geq x$

27. $2.5 \leq x$

28. $1 \geq -x$

29. $-10 \leq -x$

30. $x < -0.5$

SOLVING AND GRAPHING Solve the inequality and graph its solution.

31. $x + 6 < 8$

32. $-5 < 4 + x$

33. $-4 + x < 20$

34. $8 + x \leq -9$

35. $p - 12 \geq -1$

36. $-2 > b - 5$

37. $x - 3 > 2$

38. $x - 5 \geq 1$

39. $6 \leq c + 2$

40. $-8 \leq x - 14$

41. $m + 7 \geq -10$

42. $-6 > x - 4$

43. $-2 + x < 0$

44. $-10 > a - 6$

45. $5 + x \geq -5$

SOLVING INEQUALITIES Solve the inequality.

46. $15p < 60$

47. $-10a > 100$

48. $-\frac{n}{5} < 17$

49. $11 \geq -2.2m$

50. $-18.2x \geq -91$

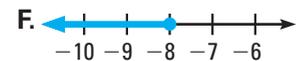
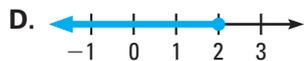
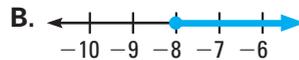
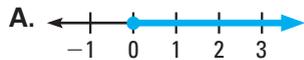
51. $2.1x \leq -10.5$

52. $13 \leq -\frac{x}{3}$

53. $-\frac{a}{10} \leq -2$

54. $\frac{x}{4} \leq -9$

SOLVING AND MATCHING In Exercises 55–60, solve the inequality. Then match its solution with its graph.



55. $x - 3.2 < 1.8$

56. $x + 4 \leq 6$

57. $10x > 50$

58. $x - 2 \geq -10$

59. $-\frac{x}{2} \leq 0$

60. $-3.5x \geq 28$

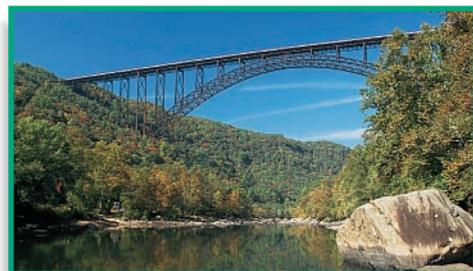
61. **WALKING RACE** You finish a three-mile walking race in 27.5 minutes. Write an inequality that describes the average speed of a walker who finished after you did.

62. **SCIENCE CONNECTION** Mercury is the metallic element with the lowest melting point, -38.87°C . Write an inequality that describes the melting point p (in degrees Celsius) of any other metallic element.

63. **LARGEST MARLIN** The world record for the largest Pacific blue marlin is 1376 pounds. It was caught in Kaaiwi Point, Kona, Hawaii. Let M represent the weight of a Pacific blue marlin that has been caught. Write an inequality for M . Graph the inequality. Source: International Game Fish Association

64. **BOWLING TOURNAMENT** After two games of bowling, Brenda has a total score of 475. To win the tournament, she needs a total score of 684 or higher. Let x represent the score she needs for her third game to win the tournament. Write an inequality for x . What is the lowest score she can get for her third game and win the tournament?

65. **STEEL ARCH BRIDGE** The longest steel arch bridge in the world is the New River Gorge Bridge near Fayetteville, West Virginia, at 1700 feet. Write an inequality that describes the length l (in feet) of any other steel arch bridge. Graph the inequality.



- MUSICAL INSTRUMENTS** Write an inequality to describe the frequency range f of the instrument.

66. The frequency range of a guitar is from 73 hertz to 698 hertz.

67. The frequency range of cymbals is from 131 hertz to 587 hertz.

FOCUS ON APPLICATIONS



MERCURY A melting point is the temperature at which a solid becomes a liquid. Mercury is used in thermometers. It is the only metal that is a liquid at room temperature.

APPLICATION LINK
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Test Preparation



68. MULTI-STEP PROBLEM You start your own business making wind chimes. Your equipment costs \$48. It costs \$3.50 to make each wind chime. You sell each wind chime for \$7.50. The graph shows your total costs.



- Copy the graph. On the same graph draw the line that represents the total revenue for different numbers of wind chimes sold.
- How many wind chimes must you sell before you start making a profit?
- Write an inequality that describes the number of wind chimes you must sell before you start making a profit.

★ Challenge

69. REVERSING INEQUALITIES Explain why you must reverse the direction of the inequality symbol when multiplying by a negative number. Give two examples to support your reasoning.

70. LOGICAL REASONING Explain why multiplying by x to solve the inequality $\frac{4}{x} > 2$ might lead to an error.

EXTRA CHALLENGE

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MIXED REVIEW

SOLVING EQUATIONS Solve the equation. (Review 3.3 for 6.2)

- | | | |
|-----------------------------|----------------------------|-------------------------------|
| 71. $45b - 3 = 2$ | 72. $-5x + 50 = 300$ | 73. $-3s - 2 = -44$ |
| 74. $\frac{1}{3}x + 5 = -4$ | 75. $\frac{x}{4} + 4 = 18$ | 76. $-8 = \frac{3}{5}a - 5$ |
| 77. $x + 2x + 5 = 14$ | 78. $3(x - 6) = 12$ | 79. $9 = -\frac{3}{2}(x - 2)$ |

WRITING EQUATIONS Write the slope-intercept form of the equation of the line that passes through the two points. Graph the line. Label the points where the line crosses the axes. (Review 4.3, 5.3)

- | | | |
|---------------------|----------------------|---------------------|
| 80. (1, 2), (4, -1) | 81. (2, 0), (-4, -3) | 82. (1, 1), (-3, 5) |
| 83. (-1, 4), (2, 4) | 84. (-1, -3), (2, 3) | 85. (5, 0), (5, -2) |

86. EXAM GRADES There are 20 questions on an exam, each worth 5 points. Your percent grade p varies directly with the number n of correct answers. Find an equation that relates p and n . What is your percent grade if you get 17 correct answers? (Review 4.5)

87. BIKE RIDING The table shows the time t (in minutes) and the distance d (in miles) that Maria rode her bike each week. Write a model that relates the variables d and t . (Review 4.5)

Time (minutes), t	3	5	10	12
Distance (miles), d	0.36	0.60	1.20	1.44