2.5 Multiplication of Real Numbers

What you should learn

GOAL 1 Multiply real numbers using properties of multiplication.

GOAL 2 Multiply real numbers to solve **real-life** problems like finding how much money is owed on a loan in **Exs. 64 and 65**.

Why you should learn it

To solve **real-life** problems like finding how much money a grocery store loses in **Example 5**.

GOAL 1 MULTIPLYING REAL NUMBERS

Remember that multiplication can be modeled as repeated addition. For example, $3(-2) = (-2) + (-2) + (-2) = -6$. The product of a positive number and a negative number is a negative number.

ACTIVITY Developing Concepts

Investigating Multiplication Patterns

Identify and extend the patterns to complete the lists. What generalizations can you make about the sign of a product of real numbers?

FACTOR OF −3	FACTOR OF −2	FACTOR OF −1
$(3)(-3) = -9$	$(3)(-2) = -6$	$(3)(-1) = -3$
$(2)(-3) = -6$	$(2)(-2) = -4$	$(2)(-1) = -2$
$(1)(-3) = -3$	$(1)(-2) = -2$	$(1)(-1) = -1$
$(0)(-3) = 0$	$(0)(-2) = 0$	$(0)(-1) = 0$
$(-1)(-3) = ?$	$(-1)(-2) = ?$	$(-1)(-1) = ?$
$(-2)(-3) = ?$	$(-2)(-2) = ?$	$(-2)(-1) = ?$

The results of the activity can be extended to determine the sign of a product of more than two factors.

STUDENT HELP

Study Tip

- A product is negative if it has an *odd* number of negative factors.
- A product is positive if it has an *even* number of negative factors.

EXAMPLE 1 *Multiplying Real Numbers*

a. $(-3)(4)(-2) = (-12)(-2) = 24$ — Two negative factors; positive product

b. $\left(-\frac{1}{2}\right)(-2)(-3) = (1)(-3) = -3$ — Three negative factors; negative product

c. $(-1)^4 = (-1)(-1)(-1)(-1) = (1)(-1)(-1) = (-1)(-1) = 1$ — Four negative factors; positive product

In Example 1(c), be sure you understand that $(-1)^4$ is not the same as -1^4.

MULTIPLYING REAL NUMBERS

The product of two real numbers with the same sign is the product of their absolute values. The product of two real numbers with different signs is the *opposite* of the product of their absolute values.

EXAMPLE 2 Products with Variable Factors

a. $(-2)(-x) = 2x$ — Two negative signs

b. $3(-n)(-n)(-n) = -3n^3$ — Three negative signs

c. $(-1)(-a)^2 = (-1)(-a)(-a) = -a^2$ — Three negative signs

d. $-(y)^4 = -(y \cdot y \cdot y \cdot y) = -y^4$ — One negative sign

CONCEPT SUMMARY PROPERTIES OF MULTIPLICATION

COMMUTATIVE PROPERTY The order in which two numbers are multiplied does not change the product.

$a \cdot b = b \cdot a$ **Example:** $3 \cdot (-2) = (-2) \cdot 3$

ASSOCIATIVE PROPERTY The way you group three numbers when multiplying does not change the product.

$(a \cdot b) \cdot c = a \cdot (b \cdot c)$ **Example:** $(-6 \cdot 2) \cdot 3 = -6 \cdot (2 \cdot 3)$

IDENTITY PROPERTY The product of a number and 1 is the number.

$1 \cdot a = a$ **Example:** $(-4) \cdot 1 = -4$

PROPERTY OF ZERO The product of a number and 0 is 0.

$a \cdot 0 = 0$ **Example:** $(-2) \cdot 0 = 0$

PROPERTY OF OPPOSITES The product of a number and -1 is the opposite of the number.

$(-1) \cdot a = -a$ **Example:** $(-1) \cdot (-3) = 3$

STUDENT HELP

INTERNET HOMEWORK HELP

Visit our Web site www.mcdougallittell.com for extra examples.

EXAMPLE 3 Evaluating a Variable Expression

Evaluate the expression when $x = -5$.

a. $-4(-1)(-x)$ **b.** $(-9.7 \cdot x)(-2)$

SOLUTION You can simplify the expression first, or substitute for x first.

a. $-4(-1)(-x) = -4x$ — Simplify expression first.

$= -4(-5)$ — Substitute -5 for x.

$= 20$ — Product of two negatives

b. $(-9.7 \cdot x)(-2) = [-9.7 \cdot (-5)] \cdot (-2)$ — Substitute -5 for x first.

$= -9.7 \cdot [-5 \cdot (-2)]$ — Use associative property.

$= -9.7 \cdot 10$ — $-5 \cdot (-2) = 10$

$= -97.0$ — Simplify.

FOCUS ON APPLICATIONS

REAL LIFE **FLYING SQUIRRELS** can't really fly as birds do. But they can glide through the air by using "gliding membranes," flaps of loose skin that extend from wrist to ankle.

GOAL 2 USING MULTIPLICATION IN REAL LIFE

Displacement is the change in the position of an object. Unlike distance, displacement can be positive, negative, or zero.

EXAMPLE 4 *Application of Products of Negatives*

FLYING SQUIRRELS A flying squirrel descends from a tree with a velocity of -6 feet per second. Find its vertical displacement in 3.5 seconds.

SOLUTION

VERBAL MODEL

$$\text{Vertical displacement} = \text{Velocity} \cdot \text{Time}$$

LABELS

Vertical displacement $= d$ (feet)

Velocity $= -6$ (feet per second)

Time $= 3.5$ (seconds)

ALGEBRAIC MODEL

$$d = -6 \cdot 3.5$$
$$= -21$$

▶ The negative displacement indicates downward motion. The squirrel traveled a vertical displacement of -21 feet or 21 feet downward.

UNIT ANALYSIS: Check that *feet* are the units of the solution.

$$\frac{\text{feet}}{\cancel{\text{second}}} \cdot \cancel{\text{seconds}} = \text{feet}$$

REAL LIFE *Grocery Stores*

EXAMPLE 5 *Application of Products of Negatives*

A grocery store sells pint baskets of strawberries as loss leaders, which means the store is willing to lose money selling them. The store hopes to make up the loss with additional sales to customers attracted to the store. The store loses \$.17 per pint. How much will the store lose if it sells 3450 pints?

SOLUTION

Use a calculator. Multiply the number of pints sold by the loss per pint to find the total loss.

3450 [×] [.] 17 [+/−] [=] −586.50

▶ The store loses \$586.50 on strawberry sales.

UNIT ANALYSIS: Check that *dollars* are the units of the solution.

$$\cancel{\text{pints}} \cdot \frac{\text{dollars}}{\cancel{\text{pint}}} = \text{dollars}$$

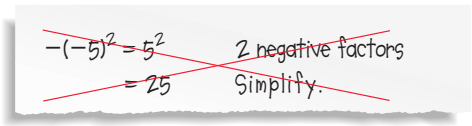
GUIDED PRACTICE

Vocabulary Check ✓

1. Multiplying -12 by -97 produces the same result as multiplying -97 by -12. What property is this?

Concept Check ✓

2. Is the product of an odd number of factors always a negative number?
3. Is the product of an even number of factors always a positive number?
4. **ERROR ANALYSIS** Describe the error shown at the right.



Skill Check ✓

Find the product.

5. $8 \cdot (-1)$
6. $-3 \cdot 0$
7. $-5 \cdot (-7)$
8. $-7 \cdot (-5)$
9. $-12 \cdot (-6)$
10. $-30 \cdot 8$
11. $-5 \cdot 2 \cdot (-7)$
12. $-(-1)^5$

Evaluate the expression.

13. $2(-6)(-x)$ when $x = 4$
14. $5(x - 4)$ when $x = -3$
15. **HAWKS** A hawk dives from a tree with a velocity of -10 feet per second. Find the vertical displacement in 4.7 seconds.

PRACTICE AND APPLICATIONS

STUDENT HELP

Extra Practice to help you master skills is on p. 798.

PRODUCTS **Find the product.**

16. $(-8)(3)$
17. $(4)(-4)$
18. $(20)(-65)$
19. $(-1)(-5)$
20. $(-7)(-1.2)$
21. $(-11)\left(\frac{1}{8}\right)$
22. $(-15)\left(\frac{3}{5}\right)$
23. $|(-12)(2)|$
24. $(-3)(-1)(-6)$
25. $(13)(-2)(-3)$
26. $(5)(-2)(7)$
27. $(-4)(-7)\left(\frac{3}{7}\right)$
28. $(-3)(-1)(4)(-6)$
29. $(-13)(-2)(-2)\left(-\frac{2}{13}\right)$

SIMPLIFYING EXPRESSIONS **Simplify the variable expression.**

30. $(-3)(-y)$
31. $(7)(-x)$
32. $5(-a)(-a)(-a)$
33. $(-4)(-x)(x)(-x)$
34. $-(-b)^3$
35. $-(-4)^2(y)$
36. $|(8)(-z)(-z)(-z)|$
37. $-(y^4)(y)$
38. $(-b^2)(-b^3)(-b^4)$
39. $-\frac{1}{2}(-2x)$
40. $\frac{2}{3}\left(-\frac{3}{2}x\right)$
41. $-\frac{3}{7}(-w^2)(7w)$

STUDENT HELP

HOMEWORK HELP

Example 1: Exs. 16–29
Example 2: Exs. 30–41
Example 3: Exs. 42–49
Example 4: Exs. 61–65
Example 5: Exs. 50–57, 61–63

EVALUATING EXPRESSIONS **Evaluate the expression.**

42. $-8x$ when $x = 6$
43. $y^3 - 4$ when $y = -2$
44. $3x^2 - 5x$ when $x = -2$
45. $4a + a^2$ when $a = -7$
46. $-4(|y - 12|)$ when $y = 5$
47. $-2(|x - 5|)$ when $x = -5$
48. $-2x^2 + 3x - 7$ when $x = 4$
49. $9r^3 - (-2r)$ when $r = 2$

EVALUATING EXPRESSIONS **Use a calculator to evaluate the expression. Round your answer to two decimal places.**

50. $(-7.39)(4.41)(-2.9)$

51. $(4.67)(-8.01)(1.89)$

52. $(3.6)(-2.67)^3(-9.41)$

53. $(-6.3)^2(9.5)(4.8)$

54. $x^3 - 8.29$ when $x = -2.47$

55. $8.3 + y^3$ when $y = -4.6$

56. $4.7b - (-b^2)$ when $b = 1.99$

57. $x^2 + x - 27.2$ when $x = -7$

STUDENT HELP

Look Back
For help with counterexamples, see p. 66.

COUNTEREXAMPLES **Decide whether the statement is *true* or *false*. If it is false, give a counterexample.**

58. $(-a) \cdot (-b) = (-b) \cdot (-a)$

59. The product $(-a) \cdot (-1)$ is always positive.

60. If $a > b$, then for any real number c, $a \cdot c > b \cdot c$.

LOSS LEADERS **To promote sales, a grocery store advertises bananas for $.25 per pound. The store loses $.11 on each pound of bananas it sells.**

61. Write a verbal model that you can use to find the amount of money that the store loses depending on the number of pounds of bananas it sells.

62. The store sells 2956 pounds of bananas. How much money does the store lose on banana sales?

63. The store also advertises apple juice for $1.19 per 64-ounce bottle, and loses $.08 per bottle sold. Use a verbal model to find how much the store loses on sales of 3107 bottles of apple juice.

PAYING BACK A LOAN **Your aunt lends you $175 to buy a guitar. She will decrease the amount you owe by $25 for each day you help her by doing odd jobs.**

64. Write a verbal model that you can use to find the decrease in the amount you owe your aunt depending on the number of days you help her out.

65. What is the change in the amount you owe your aunt after helping her out for 5 days? How much do you still owe her?

FOCUS ON APPLICATIONS

REAL LIFE **IRELAND** More people of Irish descent live in New England (4 million) than in Ireland (3.5 million)!
▶ Source: Irish Tourist Board

VACATION TRAVEL **You and your family take a summer vacation to Ireland. You discover that the number of Americans visiting Ireland is increasing by 80,000 visitors per year. Let *x* represent the number of visitors in 1997.**

66. Write an expression for the number of visitors in 2000.

67. If the number of visitors in 1997 was 700,000, how many visitors were expected in 2000? Use unit analysis to check your answer.

EXTENSION: SCALAR MULTIPLICATION **Multiply the matrix by the real number.**

Sample: $-3\begin{bmatrix} 1 & -2 \\ -4 & 0 \end{bmatrix} = \begin{bmatrix} -3(1) & -3(-2) \\ -3(-4) & -3(0) \end{bmatrix} = \begin{bmatrix} -3 & 6 \\ 12 & 0 \end{bmatrix}$

68. $-8\begin{bmatrix} -4 & -7 \\ 3 & 3 \end{bmatrix}$

69. $-7\begin{bmatrix} 6 & -4 & 3 \\ -1 & 2^2 & -9 \end{bmatrix}$

70. $-5x\begin{bmatrix} 2x & -6y \\ 4b & -8a \end{bmatrix}$

71. COMPARING METHODS As parts (a) and (b) of Example 3 show, it is sometimes easier to evaluate an expression by simplifying it before substituting, and sometimes easier if you substitute for the variable first.

a. Write an expression that is easier to evaluate if you simplify *before* substituting 12 for x.

b. Write an expression that is easier to evaluate if you substitute 12 for x first.

Test Preparation

72. MULTIPLE CHOICE Which of the following statements is *not* true?

Ⓐ The product of any number and zero is zero.

Ⓑ The order in which two numbers are multiplied does not change the product.

Ⓒ The product of any number and -1 is a negative number.

Ⓓ The product of any number and -1 is the opposite of the number.

73. MULTIPLE CHOICE Which of the following has the least value?

Ⓐ $\left[\frac{3}{8}(8-6)+\frac{1}{4}\right]\cdot(-12)$ Ⓑ $\frac{3}{8}\cdot 8-6+\frac{1}{4}\cdot(-12)$

Ⓒ $-\frac{3}{8}\cdot 8-6+\frac{1}{4}\cdot 12$ Ⓓ $-\frac{3}{8}\cdot\left(8-6+\frac{1}{4}\right)\cdot(-12)$

★ Challenge

GROUPING SYMBOLS **Evaluate the expression.**

74. $\frac{3}{4}\cdot[-7\cdot(-4-6)+30]-11$

75. $-3\cdot\left[\left(2\frac{9}{14}-3\frac{3}{7}\right)\cdot\frac{28}{11}\right]+5\left(-9\frac{1}{5}-9\right)$

MIXED REVIEW

MENTAL MATH **Write a question that can be represented by the equation. Then use mental math to solve the equation.** (Review 1.4)

76. $x+4=9$ **77.** $y-7=3$ **78.** $6x=18$

79. $\frac{y}{8}=4$ **80.** $2x+1=7$ **81.** $x^2=121$

GRAPHING NUMBERS **Graph the numbers on a number line. Then write two inequalities that compare the two numbers.** (Review 2.1)

82. 6 and -3 **83.** -4 and 9 **84.** $-\frac{1}{2}$ and $\frac{1}{3}$

85. -3.8 and -4.0 **86.** -2.8 and 0.5 **87.** -4.1 and -4.02

FINDING TERMS **Find the terms of the expression.** (Review 2.3 for 2.6)

88. $12-z$ **89.** $-t+5$ **90.** $4w-11$

91. $31-15n$ **92.** $-7+4x$ **93.** $m-2n-t^2$

94. c^2-3c-4 **95.** $y+6-8x$ **96.** $-9a^2+4-2a^3$

97. FEDERAL BUDGET In 1997 the federal government reported a budget deficit of $21.9 billion. In 1998 the deficit was $10 billion. What was the change in the deficit? ▶ Source: U.S. Office of Management and Budget (Review 2.3)