

1.2

Exponents and Powers

What you should learn

GOAL 1 Evaluate expressions containing exponents.

GOAL 2 Use exponents in real-life problems such as finding the volume of an aquarium in **Example 6**.

Why you should learn it

▼ To solve real-life problems, such as finding the volume of a glass cube in **Ex. 64**.



GOAL 1 EXPRESSIONS CONTAINING EXPONENTS

An expression like 4^6 is called a **power**. The **exponent** 6 represents the number of times the **base** 4 is used as a factor.

$$\begin{array}{c} \text{exponent} \swarrow \\ \text{base} \longrightarrow 4^6 = 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \\ \underbrace{\hspace{1.5cm}}_{\text{power}} \quad \underbrace{\hspace{2.5cm}}_{\text{6 factors of 4}} \end{array}$$

EXAMPLE 1 Reading and Writing Powers

Express the meaning of the power in words and then with numbers or variables.

SOLUTION

EXPONENTIAL FORM	WORDS	MEANING
a. 10^1	ten to the first power	10
b. 4^2	four to the second power, or four squared	$4 \cdot 4$
c. 5^3	five to the third power, or five cubed	$5 \cdot 5 \cdot 5$
d. 7^6	seven to the sixth power	$7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7$
e. x^n	x to the n th power	$x \cdot x \cdot x \cdot x \cdot \dots \cdot x$
.....		and so on

For a number raised to the first power, you usually do not write the exponent 1. For instance, you write 5^1 simply as 5.

EXAMPLE 2 Evaluating Powers

Evaluate the expression x^3 when $x = 5$.

SOLUTION

$$\begin{array}{ll} x^3 = 5^3 & \text{Substitute 5 for } x. \\ = 5 \cdot 5 \cdot 5 & \text{Write factors.} \\ = 125 & \text{Multiply.} \end{array}$$

► The value of the expression is 125.

GROUPING SYMBOLS For problems that have more than one operation, it is important to know which operation to do first. **Grouping symbols**, such as parentheses () or brackets [], indicate the order in which the operations should be performed. Operations within the innermost set of grouping symbols are done first. For instance, the value of the expression $(3 \cdot 4) + 7$ is not the same as the value of the expression $3 \cdot (4 + 7)$.

Multiply. Then add.

$$(3 \cdot 4) + 7 = 12 + 7 = 19$$

Add. Then multiply.

$$3 \cdot (4 + 7) = 3 \cdot 11 = 33$$

EXAMPLE 3 *Evaluating an Exponential Expression*

Evaluate the expression when $a = 1$ and $b = 2$.

a. $(a + b)^2$

b. $(a^2) + (b^2)$

SOLUTION

$$\begin{aligned} \text{a. } (a + b)^2 &= (1 + 2)^2 \\ &= 3^2 \\ &= 3 \cdot 3 \\ &= 9 \end{aligned}$$

Substitute 1 for a and 2 for b .

Add within parentheses.

Write factors.

Multiply.

$$\begin{aligned} \text{b. } (a^2) + (b^2) &= (1^2) + (2^2) \\ &= 1 + 4 \\ &= 5 \end{aligned}$$

Substitute 1 for a and 2 for b .

Evaluate power.

Add.

.....

An exponent applies only to the number, variable, or expression immediately to its left. In the expression $2x^3$, the base is x , not $2x$. In the expression $(2x)^3$, the base is $2x$, as indicated by the parentheses.

EXAMPLE 4 *Exponents and Grouping Symbols*

Evaluate the expression when $x = 4$.

a. $2x^3$

b. $(2x)^3$

SOLUTION

$$\begin{aligned} \text{a. } 2x^3 &= 2(4^3) \\ &= 2(64) \\ &= 128 \end{aligned}$$

Substitute 4 for x .

Evaluate power.

Multiply.

$$\begin{aligned} \text{b. } (2x)^3 &= (2 \cdot 4)^3 \\ &= 8^3 \\ &= 512 \end{aligned}$$

Substitute 4 for x .

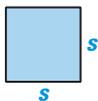
Multiply within parentheses.

Evaluate power.

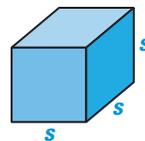
STUDENT HELP
 **HOMEWORK HELP**
 Visit our Web site
www.mcdougallittell.com
 for extra examples.

GOAL 2 REAL-LIFE APPLICATIONS OF EXPONENTS

Exponents often are used in the formulas for area and volume. In fact, the words *squared* and *cubed* come from the formula for the area of a square, $A = s^2$, and the formula for the volume of a cube, $V = s^3$.



Area of Square: $A = s^2$



Volume of Cube: $V = s^3$

Units of area, such as square feet, ft^2 , can be written using a second power. Units of volume, such as cubic centimeters, cm^3 , can be written using a third power.

EXAMPLE 5 Making a Table

You can find the volume of cubes that have edge lengths of 1 inch, 2 inches, 3 inches, 4 inches, and 5 inches by using the formula $V = s^3$.

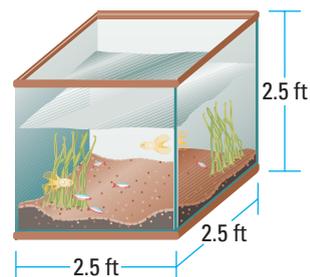
Edge, s	1	2	3	4	5
s^3	1^3	2^3	3^3	4^3	5^3
Volume, V	1 in.^3	8 in.^3	27 in.^3	64 in.^3	125 in.^3



EXAMPLE 6 Finding Volume

The aquarium has the shape of a cube. Each edge is 2.5 feet long.

- Find the volume in cubic feet.
- How many gallons of water will the cubic aquarium hold? Convert to liquid volume, where one cubic foot holds 7.48 gallons.



SOLUTION

$$\begin{aligned} \text{a. } V &= s^3 && \text{Write formula for volume.} \\ &= 2.5^3 && \text{Substitute 2.5 for } s. \\ &= 15.625 && \text{Evaluate power.} \end{aligned}$$

- The volume of the aquarium is 15.625 ft^3 .

$$\begin{aligned} \text{b. } V &= 15.625 \text{ ft}^3 (7.48 \text{ gal/1 ft}^3) && \text{Write conversion factor.} \\ &= 116.875 \text{ gal} && \text{Multiply.} \end{aligned}$$

- A 15.625 cubic foot aquarium will hold 116.875 gallons of water.

STUDENT HELP

KEYSTROKE HELP

Your calculator may have a y^x key or a \wedge key that you can use to evaluate powers.

GUIDED PRACTICE

Vocabulary Check ✓

1. In the expression 15^3 , what is 15 called? What is 3 called? What is the expression called?

Concept Check ✓

2. The expressions $3x^2$ and $(3x)^2$ do not have the same meaning. Explain the difference.

3. Evaluate the expressions $3x^2$ and $(3x)^2$ when $x = 4$.

Skill Check ✓

Match the power with the words that describe it.

A. five to the sixth power

B. two to the fifth power

C. five squared

D. five cubed

4. 5^2

5. 5^3

6. 2^5

7. 5^6

Evaluate the expression when $x = 3$.

8. x^2

9. $(x + 1)^3$

10. $2x^2$

11. $(2x)^3$

12. $(x - 1)^4$

13. 5^x

14. $(3x)^4$

15. 10^x

16.  **STEREO SPEAKERS** A cubical stereo speaker measures 35 centimeters along each edge. The expression for finding the surface area of a cube is $6s^2$, where s is the length of each edge. Find the surface area of the stereo speaker.

PRACTICE AND APPLICATIONS

STUDENT HELP

▶ **Extra Practice**
to help you master
skills is on p. 797.

EXPONENTIAL FORM Write the expression in exponential form.

17. two cubed

18. p squared

19. nine to the y th power

20. b to the eighth power

21. $3 \cdot 3 \cdot 3 \cdot 3 \cdot y$

22. $t \cdot t$

23. $c \cdot c \cdot c \cdot c \cdot c \cdot c$

24. $5 \cdot x \cdot x \cdot x \cdot x \cdot x$

25. $4x \cdot 4x \cdot 4x$

EVALUATING POWERS Evaluate the power.

26. 10^2

27. 5^2

28. 8^2

29. 6^4

30. 10^5

31. 7^4

32. 4^6

33. 9^3

34. 2^5

EVALUATING EXPRESSIONS Evaluate the expression for the given value of the variable.

35. 4^n when $n = 5$

36. b^4 when $b = 9$

37. x^6 when $x = 10$

38. c^6 when $c = 2$

39. w^3 when $w = 13$

40. p^2 when $p = 2.5$

EXPONENTIAL EXPRESSIONS Evaluate the expression for the given values of the variables.

41. $(x + y)^2$ when $x = 5$ and $y = 3$

42. $m - n^2$ when $m = 25$ and $n = 4$

43. $(a - b)^4$ when $a = 4$ and $b = 2$

44. $c^3 + d$ when $c = 4$ and $d = 16$

45. $(d - 3)^2$ when $d = 13$

46. $16 + x^3$ when $x = 2$

STUDENT HELP

▶ HOMEWORK HELP

Example 1: Exs. 17–25

Example 2: Exs. 26–40

Example 3: Exs. 41–46

Example 4: Exs. 55–60

Example 5: Ex. 61

Example 6: Exs. 63–66



EVALUATING POWERS Use a calculator to evaluate the power. For keystroke help see Student Help box on page 11.

47. 9^5 48. 2^{10} 49. 5^9 50. 3^{11}
 51. 8^6 52. 12^7 53. 6^8 54. 13^5

EXPONENTIAL EXPRESSIONS Evaluate the expression for the given value of the variable.

55. $(5w)^3$ when $w = 5$ 56. $6t^4$ when $t = 3$ 57. $7b^2$ when $b = 7$
 58. $2x^2$ when $x = 15$ 59. $(8x)^3$ when $x = 2$ 60. $5y^5$ when $y = 2$

61. **USING A TABLE** The area of a square is s^2 . Show the relationship between the side length of a square and its area by copying and completing the following table.

Side length, s	1	2	3	4	5
Area, s^2	?	?	?	?	?

62. **CRITICAL THINKING** Copy and complete the table. What pattern do you see?

Power	10^2	100^2	1000^2	$10,000^2$
Evaluate	100	?	?	?

63. **INTERIOR DESIGN** One room in Jean's apartment is a square measuring 12.2 feet along the base of each wall. How many square feet of wall-to-wall carpet does Jean need to carpet the room?

64. **ART CONNECTION** In 1997 the artist Jon Kuhn of North Carolina created a cubic sculpture called Crystal Victory, shown at the left. Each edge of the solid glass cube is 9.5 inches in length. How much liquid glass did Kuhn need to make the cube?

65. **VOLUME OF A SAFE** Each dimension of the cubical storage space inside a fireproof safe is 12 inches. What is the volume of the storage space?

66. **SWIMMING POOL** A swimming pool is 50 meters long, 19.5 meters wide, and 3 meters deep. Use the formula for the volume of a rectangular prism to find the volume of water in the pool. The formula is the length times the width times the height.

67. **RAIN FOREST PYRAMID**

The formula for the volume of a pyramid is $\frac{1}{3}$ times the height times the area of the base. The Rain Forest Pyramid in Moody Gardens near Galveston, Texas, is 100 feet high and 200 feet along each side of its square base. What is the volume of space inside the Rain Forest Pyramid?

► Source: Morris Architects



FOCUS ON APPLICATIONS



GLASS

SCULPTURE

Artist John Kuhn used lead crystal and colored glass powders to make the Crystal Victory.

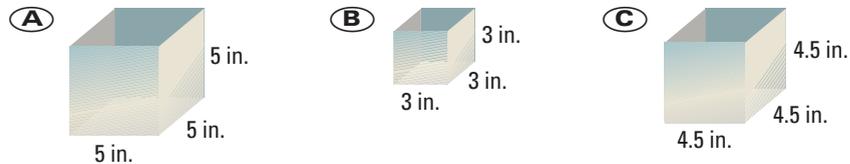


APPLICATION LINK

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68. MULTI-STEP PROBLEM You are making candles to sell at your school's art festival. You melt paraffin wax in a cubic container. Each edge of the container is 6 inches in length. The container is one half full.

- What is the volume of the wax in the container?
- Which of the candle molds could hold all of the melted wax?



- Writing** Design a cubic candle mold different from those given that will hold all of the melted wax. Draw a diagram of the mold. Explain why your mold will hold all of the melted wax.

★ Challenge

FINDING A PATTERN Copy the table.

Power	9^1	9^2	9^3	9^4	9^5	9^6	9^7	9^8
Evaluate	?	?	?	?	?	?	?	?

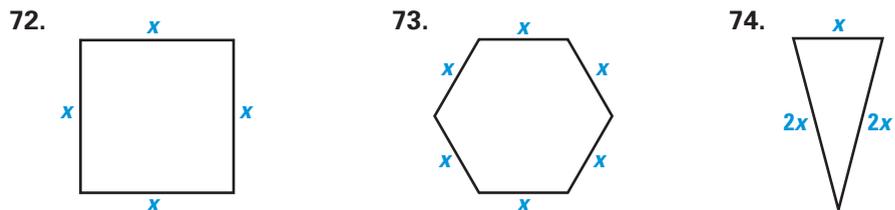
- Evaluate the powers of 9 in the table. What pattern do you see for the last digit of each product?
- Make a table like the one shown for powers of 8. Describe any patterns.
- Make a table for powers of 7. Describe any patterns.

EXTRA CHALLENGE

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MIXED REVIEW

GEOMETRY CONNECTION Find the perimeter of the figure when $x = 1.7$. (Skills Review, pp. 790–791)



FRACTIONS, DECIMALS, AND PERCENTS Write the fraction as a decimal and as a percent. (Skills Review, pp. 784–785)

- $\frac{5}{8}$
- $\frac{3}{4}$
- $\frac{11}{20}$
- $\frac{4}{25}$

EVALUATING VARIABLE EXPRESSIONS Evaluate the expression for the given value of the variable. (Review 1.1 for 1.3)

- $7x$ when $x = 3$
- $y + 2$ when $y = 10$
- $\frac{a}{2}$ when $a = 8$
- $m - 5$ when $m = 17$
- $\frac{9}{b}$ when $b = 4$
- $9b$ when $b = 4$