

5.6

The Standard Form of a Linear Equation

What you should learn

GOAL 1 Write a linear equation in standard form.

GOAL 2 Use the standard form of an equation to model real-life situations, such as the amount of food to purchase for a barbecue in Example 4.

Why you should learn it

▼ To model real-life situations, such as buying combinations of bird seeds in Exs. 64–66.



GOAL 1 WRITING AN EQUATION IN STANDARD FORM

Most of the linear equations in this chapter are written in slope-intercept form.

Slope-intercept form: $y = mx + b$

Another commonly used form is the **standard form** of the equation of a line.

Standard (or general) form: $Ax + By = C$

In this form, notice that the variable terms are on the left side of the equation and the constant term is on the right. A , B , and C represent real numbers and A and B are not both zero.

ACTIVITY Developing Concepts

Investigating Forms of Equations

1 Graph each of these equations.

a. $y = \frac{4}{3}x - 2$

b. $-4x + 3y = -6$

c. $4x - 3y = 6$

2 Which, if any, of the equations are equivalent? How do you know?

EXAMPLE 1 Writing an Equation in Standard Form

Write $y = \frac{2}{5}x - 3$ in standard form with integer coefficients.

SOLUTION

To write the equation in standard form, isolate the variable terms on the left and the constant term on the right.

$$y = \frac{2}{5}x - 3 \quad \text{Write original equation.}$$

$$5y = 5\left(\frac{2}{5}x - 3\right) \quad \text{Multiply each side by 5.}$$

$$5y = 2x - 15 \quad \text{Use distributive property.}$$

$$-2x + 5y = -15 \quad \text{Subtract } 2x \text{ from each side.}$$

.....

A linear equation can have more than one standard form. For instance, if you multiply each side of the standard form above by -1 , you get $2x - 5y = 15$, which is also in standard form.

STUDENT HELP**Look Back**

For help with solving linear equations, see p. 155.

EXAMPLE 2 *Writing a Linear Equation*

Write the standard form of an equation of the line passing through $(-4, 3)$ with a slope of -2 .

SOLUTION

You are given a point on the line and its slope, so you can write the point-slope form of the equation of the line.

$$\begin{array}{ll}
 y - y_1 = m(x - x_1) & \text{Write point-slope form.} \\
 y - 3 = -2[x - (-4)] & \text{Substitute for } y_1, m, \text{ and } x_1. \\
 y - 3 = -2(x + 4) & \text{Simplify.} \\
 y - 3 = -2x - 8 & \text{Use distributive property.} \\
 2x + y = -5 & \text{Add } 2x \text{ and } 3 \text{ to each side.}
 \end{array}$$

▶ The equation $2x + y = -5$ is in standard form.

EXAMPLE 3 *Horizontal and Vertical Lines*

Write the standard form of an equation

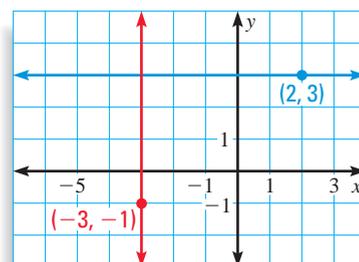
- of the horizontal line.
- of the vertical line.

SOLUTION

- The horizontal line has a point with a y -coordinate of 3. The standard form of the equation is $y = 3$.
- The vertical line has a point with an x -coordinate of -3 . The standard form of the equation is $x = -3$.

.....

You have now studied all of the commonly used forms of linear equations. They are summarized in the following list.

**CONCEPT
SUMMARY****EQUATIONS OF LINES**

SLOPE-INTERCEPT FORM:	$y = mx + b$
POINT-SLOPE FORM:	$y - y_1 = m(x - x_1)$
VERTICAL LINE (UNDEFINED SLOPE):	$x = a$
HORIZONTAL LINE (ZERO SLOPE):	$y = b$
STANDARD FORM:	$Ax + By = C$, where A , B , and C are real numbers; A and B are not both zero.

GOAL 2 MODELING A REAL-LIFE SITUATION

EXAMPLE 4 Writing and Using a Linear Model

BARBECUE You are in charge of buying the hamburger and boned chicken for a barbecue. The hamburger costs \$2 per pound and the boned chicken costs \$3 per pound. You have \$30 to spend.

- Write an equation that models the different amounts of hamburger and chicken that you can buy.
- Make a table and a graph that illustrate several different amounts of hamburger and chicken that you can buy.

SOLUTION

- Model the possible combinations of hamburger and chicken.



VERBAL MODEL

$$\boxed{\text{Price of hamburger}} \cdot \boxed{\text{Weight of hamburger}} + \boxed{\text{Price of chicken}} \cdot \boxed{\text{Weight of chicken}} = \boxed{\text{Total cost}}$$

LABELS

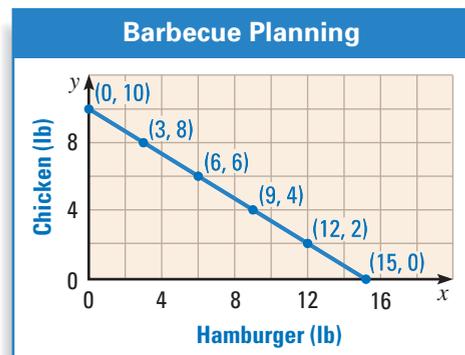
Price of hamburger = 2 (dollars per pound)
 Weight of hamburger = x (pounds)
 Price of chicken = 3 (dollars per pound)
 Weight of chicken = y (pounds)
 Total cost = 30 (dollars)

ALGEBRAIC MODEL $2x + 3y = 30$ **Linear model**

- Use the algebraic model to make a table.

Hamburger (lb), x	0	3	6	9	12	15
Chicken (lb), y	10	8	6	4	2	0

The points corresponding to these different amounts are shown in the graph. Note that as the number of pounds of hamburger you purchase increases, the number of pounds of chicken you purchase decreases, and vice versa.



FOCUS ON APPLICATIONS



REAL LIFE BARBECUE
 The International Bar-B-Que Festival in Owensboro, Kentucky, uses over 5000 chickens in its annual cookout.

GUIDED PRACTICE

Vocabulary Check ✓

1. Name the following form of the equation of a line: $y = mx + b$.
2. Name the following form of the equation of a line: $Ax + By = C$.

Concept Check ✓

3. The equations $y = -\frac{2}{3}x + 3$ and $2x + 3y = 9$ are equivalent. Describe how you would use each equation to graph the equation.

Skill Check ✓

Write the equation in standard form with integer coefficients.

- | | | |
|---------------------------|-----------------|----------------------------|
| 4. $y = 2x - 9$ | 5. $y = 6 - 5x$ | 6. $y = 9 + 1x$ |
| 7. $y = \frac{1}{2}x + 8$ | 8. $y = 7x + 3$ | 9. $y = -2 + \frac{3}{2}x$ |

Write an equation in standard form of the line that passes through the given point and has the given slope.

- | | | |
|-----------------------|-----------------------|-----------------------|
| 10. $(-3, 4), m = -4$ | 11. $(1, -2), m = 5$ | 12. $(2, 5), m = 3$ |
| 13. $(5, -8), m = -1$ | 14. $(-5, -6), m = 3$ | 15. $(4, -3), m = -2$ |
16.  **VEGETABLES** You have \$10 to buy tomatoes and carrots for a salad. Tomatoes cost \$2 per pound and carrots cost \$1.25 per pound. Write a linear equation that models the different amounts of tomatoes x and carrots y that you can buy.

17. Copy and complete the table using the linear model in Exercise 16.

Tomatoes (lb), x	0	1.25	2.5	3.75	5
Carrots (lb), y	?	?	?	?	?

PRACTICE AND APPLICATIONS

STUDENT HELP

► **Extra Practice**
to help you master
skills is on p. 801.

EQUATIONS Write the equation in standard form with integer coefficients.

- | | | |
|---------------------------------------|---------------------------------------|-----------------------------|
| 18. $4x - y - 7 = 0$ | 19. $x + 3y - 4 = 0$ | 20. $-4x + 5y + 16 = 0$ |
| 21. $2x - 3y - 14 = 0$ | 22. $x - 5 = 0$ | 23. $y + 3 = 0$ |
| 24. $y = -5x + 2$ | 25. $y = 3x - 8$ | 26. $y = -0.4x + 1.2$ |
| 27. $3x + 9 = \frac{7}{2}y$ | 28. $y = 9x + \frac{1}{2}$ | 29. $y = \frac{5}{2}x + 9$ |
| 30. $y = -\frac{3}{4}x + \frac{5}{4}$ | 31. $y = -\frac{1}{7}x + \frac{6}{7}$ | 32. $y = -\frac{1}{3}x - 4$ |

WRITING EQUATIONS Write an equation in standard form of the line that passes through the given point and has the given slope.

- | | | |
|------------------------|-----------------------|-----------------------|
| 33. $(-8, 3), m = 2$ | 34. $(-2, 7), m = -4$ | 35. $(-1, 4), m = -3$ |
| 36. $(-6, -7), m = -1$ | 37. $(3, -2), m = 5$ | 38. $(10, 6), m = 7$ |
| 39. $(2, 9), m = -7$ | 40. $(5, -8), m = 10$ | 41. $(7, 3), m = -2$ |
| 42. $(-4, 2), m = -2$ | 43. $(0, 3), m = 1$ | 44. $(-3, 3), m = 4$ |

STUDENT HELP

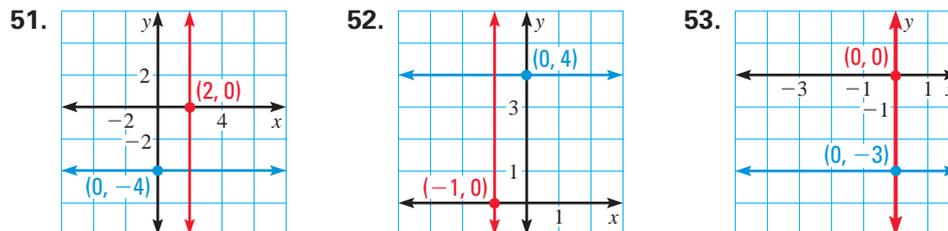
HOMEWORK HELP

Example 1: Exs. 18–32
Example 2: Exs. 33–50
Example 3: Exs. 51–61
Example 4: Exs. 64–69

WRITING EQUATIONS Write an equation in standard form of the line that passes through the two points.

45. (1, 4), (5, 7) 46. (-3, -3), (7, 2) 47. (-4, 1), (2, -5)
 48. (9, -2), (-3, 2) 49. (0, 0), (2, 0) 50. (4, -7), (5, -1)

HORIZONTAL AND VERTICAL LINES Write an equation in standard form of the horizontal line and the vertical line.



HORIZONTAL AND VERTICAL LINES Write an equation in standard form of the horizontal line and the vertical line that pass through the point.

54. (1, 3) 55. (-4, 4) 56. (-2, -5) 57. (6, -1)
 58. (0, 7) 59. (-9, 0) 60. (-3, 7) 61. (10, -3)

ERROR ANALYSIS In Exercises 62 and 63, describe the error.

62. ~~$y = \frac{1}{3}x - 2$
 $y = 3\left(\frac{1}{3}x - 2\right)$
 $y = x - 6$~~

63. ~~$y - 4 = -3[x - (-6)]$
 $y - 4 = -3(x - 6)$
 $y - 4 = -3x + 18$
 $y = -3x + 22$~~

FOCUS ON APPLICATIONS



REAL LIFE FEEDING HABITS

The American Goldfinch uses its feet to extract seeds. Before a storm, American Goldfinches increase the amount they eat tremendously and gain weight. ▶ Source: Canadian Wildlife Service

BIRD SEED MIXTURE In Exercises 64–66, use the following information.

You are buying \$20 worth of bird seed that consists of two types of seed. Thistle seed attracts finches and costs \$2 per pound. Dark oil sunflower seed attracts many kinds of song birds and costs \$1.50 per pound.

64. Write an equation that represents the different amounts of \$2 thistle seed x and \$1.50 dark oil sunflower seed y that you could buy.
 65. Graph the line representing the possible seed mixtures in Exercise 64.
 66. Copy and complete the table.

Label the points from the table on the graph created in Exercise 65.

Thistle seed (lb), x	0	2	4	6	8
Dark oil sunflower seed (lb), y	?	?	?	?	?

PRIZES In Exercises 67–69, you are in charge of buying prizes for a school contest. A one-strand rosette ribbon costs \$2.00 and a three-strand rosette ribbon costs \$3.00. You have \$10 to spend.

67. Write an equation that represents the different numbers of one-strand and three-strand rosette ribbons that you can purchase.
 68. What is the greatest number of students that can receive a ribbon?
 69. What is the least number of students that can receive a ribbon?

STUDENT HELP

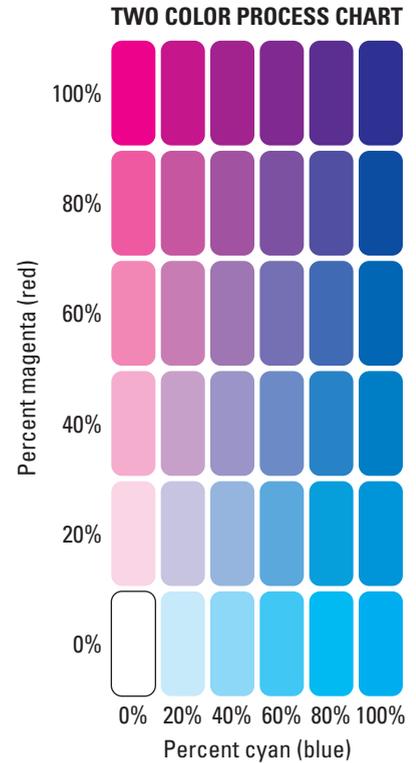
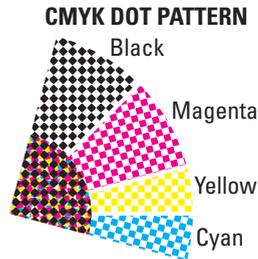


HOMEWORK HELP
Visit our Web site
www.mcdougallittell.com
for help with Exs. 70–74.



COLOR PRINTING In Exercises 70–74, use the following information.

In printing, different shades of color are often formed by *overprinting* different percents of two or more primary colors such as magenta (red) and cyan (blue). For instance, combining a 40% magenta screen with a 20% cyan screen produces a more reddish purple than combining a 20% magenta screen with a 40% cyan screen. This process reduces the number of colors of ink needed.

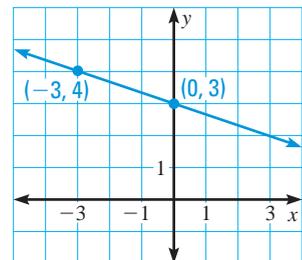


70. You are designing an advertising piece. You want the *combined* percent of magenta and cyan screens for a background color to be 60%. Write a linear equation that models the different percents of magenta and cyan you can use. Let x represent the percent of magenta, and let y represent the percent of cyan.
71. Trace the magenta-cyan color palette matrix at the right and identify the possible shades you can use for a combined percent of 60%.
72. Write a linear equation that models the different percents of magenta and cyan you can use for a background color of 80%.
73. Trace the magenta-cyan color palette matrix and identify the possible shades you can use for a combined percent of 80%.
74. How do the patterns formed by the possible shades relate to the lines whose equations you found in Exercises 70 and 72?
75. **MULTIPLE CHOICE** Choose the equation in standard form of the line that passes through the point $(-1, -4)$ and has a slope of 2.

- (A) $2x + y = -2$ (B) $-2x + y = -2$
(C) $2x + y = 2$ (D) $2x - y = -2$

76. **MULTIPLE CHOICE** Choose the standard form of the equation shown in the graph.

- (A) $3x + y = 3$
(B) $-3y = x + (-9)$
(C) $-x - y = 9$
(D) $-x - 3y = -9$



Test Preparation



EXPLORING EQUATIONS In Exercises 77–80, use the equation $2x + 7y = 14$.

77. What is the x -intercept?
78. What is the y -intercept?
79. Multiply each term in the equation by $\frac{1}{14}$.
80. Find a relationship between the denominators on the left side of the equation and the x -intercept and the y -intercept.

★ Challenge

FINDING EQUATIONS In Exercises 81–83, use the following information.

The equation represents the intercept form of the equation of a line. In the equation, the x -intercept is a and the y -intercept is b .

$$\frac{x}{a} + \frac{y}{b} = 1$$

81. Write the intercept form of the equation of the line whose x -intercept is 2 and y -intercept is 3.
82. Write the equation in Exercise 81 in standard form.
83. Write the equation in Exercise 82 in slope-intercept form.

EXTRA CHALLENGE

www.mcdougallittell.com

MIXED REVIEW

SOLVING EQUATIONS Solve the equation. Check the result. (Review 3.4)

84. $8 + y = 3$ 85. $y - 9 = 2$ 86. $-6(q + 22) = 5q$
87. $2(x + 5) = 18$ 88. $7 - 2a = -14$ 89. $-2 + 4c = 19$

GRAPHING EQUATIONS Use a table of values to graph the equation. Label the x -intercept and the y -intercept. (Review 4.2, 4.3)

90. $y = -x + 8$ 91. $y = 4x - 4$ 92. $y = x + 5$
93. $y = -9 + 3x$ 94. $y = -x - 1$ 95. $y = 10 - x$

SLOPE-INTERCEPT FORM Graph the line that passes through the points. Write its equation in slope-intercept form. (Review 5.3 for 5.7)

96. $(2, 3), (-4, 1)$ 97. $(7, 6), (5, -9)$ 98. $(-5, -2), (-1, 12)$
99. $(6, -2), (-5, -8)$ 100. $(2, 9), (4, 7)$ 101. $(19, -2), (4, -1)$
102. $(5, -8), (7, -3)$ 103. $(14, -9), (-5, -3)$ 104. $(-5, 9), (5, 4)$

105.  **BUYING A CD PLAYER** You have \$125.79 to spend for a CD player. The sales tax is 6%. What is your price limit for the CD player? (Review 3.6)
106.  **BUYING A CALENDAR** You have \$15 to spend for a calendar. The sales tax is 8.5%. What is your price limit for the calendar? (Review 3.6)
107.  **LEAVING A TIP** You leave a \$1.50 tip for a lunch of \$6.00. What percent tip did you leave? (Review 3.8)
108.  **SHOPPING** A coat regularly sells for \$84. You buy the coat on sale and save \$10. What percent is the discount? (Review 3.8)