10.7 Factoring Special Products

What you should learn

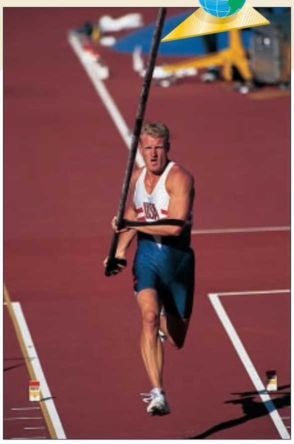
GOAL 1 Use special product patterns to factor quadratic polynomials.

GOAL 2 Solve quadratic equations by factoring.

Why you should learn it

▼ To solve **real-life** problems, such as finding the velocity of a pole-vaulter in **Exs. 68 and 69.**

REAL LIFE



GOAL 1 FACTORING QUADRATIC POLYNOMIALS

Each of the special product patterns you studied in Lesson 10.3 can be used to factor polynomials. Note that there are two forms of perfect square trinomials.

FACTORING SPECIAL PRODUCTS

DIFFERENCE OF TWO SQUARES PATTERN	**Example**
$a^2 - b^2 = (a + b)(a - b)$	$9x^2 - 16 = (3x + 4)(3x - 4)$
PERFECT SQUARE TRINOMIAL PATTERN	**Example**
$a^2 + 2ab + b^2 = (a + b)^2$	$x^2 + 8x + 16 = (x + 4)^2$
$a^2 - 2ab + b^2 = (a - b)^2$	$x^2 - 12x + 36 = (x - 6)^2$

EXAMPLE 1 *Factoring the Difference of Two Squares*

a. $m^2 - 4 = m^2 - 2^2$ — Write as $a^2 - b^2$.

$= (m + 2)(m - 2)$ — Factor using pattern.

b. $4p^2 - 25 = (2p)^2 - 5^2$ — Write as $a^2 - b^2$.

$= (2p + 5)(2p - 5)$ — Factor using pattern.

c. $50 - 98x^2 = 2(25 - 49x^2)$ — Factor out common factor.

$= 2[5^2 - (7x)^2]$ — Write as $a^2 - b^2$.

$= 2(5 + 7x)(5 - 7x)$ — Factor using pattern.

EXAMPLE 2 *Factoring Perfect Square Trinomials*

a. $x^2 - 4x + 4 = x^2 - 2(x)(2) + 2^2$ — Write as $a^2 - 2ab + b^2$.

$= (x - 2)^2$ — Factor using pattern.

b. $16y^2 + 24y + 9 = (4y)^2 + 2(4y)(3) + 3^2$ — Write as $a^2 + 2ab + b^2$.

$= (4y + 3)^2$ — Factor using pattern.

c. $3x^2 - 30x + 75 = 3(x^2 - 10x + 25)$ — Factor out common factor.

$= 3[x^2 - 2(x)(5) + 5^2]$ — Write as $a^2 - 2ab + b^2$.

$= 3(x - 5)^2$ — Factor using pattern.

GOAL 2 SOLVING QUADRATIC EQUATIONS

STUDENT HELP

HOMEWORK HELP Visit our Web site www.mcdougallittell.com for extra examples.

EXAMPLE 3 *Graphical and Analytical Reasoning*

Solve the equation $-2x^2 + 12x - 18 = 0$.

SOLUTION

$$-2x^2 + 12x - 18 = 0 \quad \text{Write original equation.}$$

$$-2(x^2 - 6x + 9) = 0 \quad \text{Factor out common factor.}$$

$$-2[x^2 - 2(3x) + 3^2] = 0 \quad \text{Write as } a^2 - 2ab + b^2.$$

$$-2(x - 3)^2 = 0 \quad \text{Factor using pattern.}$$

$$x - 3 = 0 \quad \text{Set repeated factor equal to 0.}$$

$$x = 3 \quad \text{Solve for } x.$$

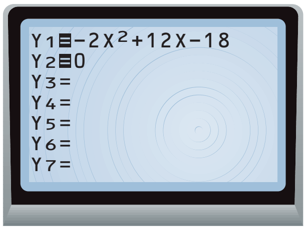
▶ The solution is 3.

✓**CHECK:** You can check your answer by substitution or by graphing.

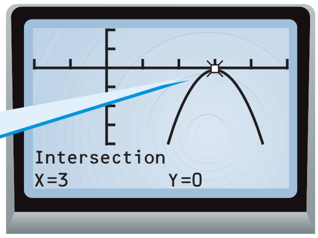
Graph $y = -2x^2 + 12x - 18$.

Graph the x-axis, $y = 0$.

Use your graphing calculator's *Intersect* feature to find the x-intercept, where $-2x^2 + 12x - 18 = 0$.



When $x = 3$, $-2x^2 + 12x - 18 = 0$, which appears to confirm your solution.



EXAMPLE 4 *Solving a Quadratic Equation*

Solve $x^2 + x + \frac{1}{4} = 0$.

SOLUTION

$$x^2 + x + \frac{1}{4} = 0 \quad \text{Write original equation.}$$

$$x^2 + 2\left(\frac{1}{2}x\right) + \left(\frac{1}{2}\right)^2 = 0 \quad \text{Write as } a^2 + 2ab + b^2.$$

$$\left(x + \frac{1}{2}\right)^2 = 0 \quad \text{Factor using pattern.}$$

$$x + \frac{1}{2} = 0 \quad \text{Set repeated factor equal to 0.}$$

$$x = -\frac{1}{2} \quad \text{Solve for } x.$$

▶ The solution is $-\frac{1}{2}$. Check this in the original equation.

STUDENT HELP

Study Tip
Another way to solve the equation in Example 4 is to first multiply each side of the equation by 4. This allows you to work less with fractions.

$$x^2 + x + \frac{1}{4} = 0$$
$$4x^2 + 4x + 1 = 0$$
$$(2x + 1)^2 = 0$$
$$2x + 1 = 0$$
$$x = -\frac{1}{2}$$

FOCUS ON APPLICATIONS

REAL LIFE **BLOCK AND TACKLE**
A block and tackle makes it easier to lift a heavy object. For instance, using a block and tackle with 4 pulleys, you can lift 1000 pounds with only 250 pounds of applied force.

EXAMPLE 5 *Writing and Using a Quadratic Model*

BLOCK AND TACKLE An object lifted with a rope or wire should not weigh more than the *safe working load* for the rope or wire. The safe working load S (in pounds) for a natural fiber rope is a function of C, the circumference of the rope in inches.

Safe working load model: $150 \cdot C^2 = S$

You are setting up a block and tackle to lift a 1350-pound safe. What size natural fiber rope do you need to have a safe working load?

SOLUTION Use the model to find a safe rope size. Substitute 1350 for S.

$150C^2 = S$	Write model.
$150C^2 = 1350$	Substitute.
$150C^2 - 1350 = 0$	Subtract 1350 from each side.
$150(C^2 - 9) = 0$	Factor out common factor.
$150(C - 3)(C + 3) = 0$	Factor.
$(C - 3) = 0$ or $(C + 3) = 0$	Use zero-product property.
$C - 3 = 0$	Set first factor equal to 0.
$C = 3$	Solve for C.
$C + 3 = 0$	Set second factor equal to 0.
$C = -3$	Solve for C.

▶ The negative solution makes no sense. You need a rope with a circumference of at least 3 inches.

EXAMPLE 6 *Writing and Using a Quadratic Model*

If you project a ball straight up from the ground with an initial velocity of 64 feet per second, will the ball reach a height of 64 feet? If it does, how long will it take to reach that height?

SOLUTION Use a vertical motion model where $v = 64$, $s = 0$, and $h = 64$.

$-16t^2 + vt + s = h$	Vertical motion model
$-16t^2 + 64t + 0 = 64$	Substitute for v, s, and h.
$-16t^2 + 64t - 64 = 0$	Write in standard form.
$-16(t^2 - 4t + 4) = 0$	Factor out common factor.
$-16(t - 2)^2 = 0$	Factor.
$t - 2 = 0$	Set repeated factor equal to 0.
$t = 2$	Solve for t.

▶ Because there is a solution, you know that the ball will reach a height of 64 feet. The solution is $t = 2$, so it will take 2 seconds to reach that height.

GUIDED PRACTICE

Vocabulary Check ✓

Concept Check ✓

1. What is the difference of two squares pattern?

2. ERROR ANALYSIS Describe the error at the right. Then solve the equation correctly.

$$9x^2 - 6x + 1 = 0$$
$$(3x + 1)(3x - 1) = 0$$
$$3x + 1 = 0 \qquad x = -\frac{1}{3}$$
$$3x - 1 = 0 \qquad x = \frac{1}{3}$$
$$x = -\frac{1}{3} \text{ or } \frac{1}{3}$$

3. Look back to Lesson 10.3. Describe the relationship between the special products shown in Lesson 10.3 and the polynomials you factored in this lesson.

4. Write the three special product factoring patterns. Give an example of each pattern.

5. Factor out the common factor in $3x^2 - 6x + 9$.

Skill Check ✓

Factor the expression.

6. $x^2 - 9$	**7.** $t^2 + 10t + 25$	**8.** $w^2 - 16w + 64$
9. $16 - t^2$	**10.** $6y^2 - 24$	**11.** $18 - 2z^2$

Use factoring to solve the equation.

12. $x^2 + 6x + 9 = 0$	**13.** $144 - y^2 = 0$	**14.** $s^2 - 14s + 49 = 0$
15. $-25 + x^2 = 0$	**16.** $7x^2 + 28x + 28 = 0$	**17.** $-4y^2 + 24y - 36 = 0$

PRACTICE AND APPLICATIONS

STUDENT HELP

Extra Practice to help you master skills is on p. 806.

DIFFERENCE OF TWO SQUARES **Factor the expression.**

18. $n^2 - 16$	**19.** $100x^2 - 121$	**20.** $6m^2 - 150$
21. $60y^2 - 540$	**22.** $16 - 81r^2$	**23.** $98 - 2t^2$
24. $w^2 - y^2$	**25.** $9t^2 - 4q^2$	**26.** $-28y^2 + 7t^2$

PERFECT SQUARES **Factor the expression.**

27. $x^2 + 8x + 16$	**28.** $x^2 - 20x + 100$	**29.** $y^2 + 30y + 225$
30. $b^2 - 14b + 49$	**31.** $9x^2 + 6x + 1$	**32.** $4r^2 + 12r + 9$
33. $25n^2 - 20n + 4$	**34.** $36m^2 - 84m + 49$	**35.** $18x^2 + 12x + 2$
36. $48y^2 - 72xy + 27x^2$	**37.** $-16w^2 - 80w - 100$	**38.** $-3k^2 + 42k - 147$

STUDENT HELP

HOMEWORK HELP
Example 1: Exs. 18–26, 39–50
Example 2: Exs. 27–50
Example 3: Exs. 51–62
Example 4: Exs. 51–62
Example 5: Exs. 63–66
Example 6: Ex. 67

FACTORING EXPRESSIONS **Factor the expression. Tell which special product factoring pattern you used.**

39. $z^2 - 25$	**40.** $y^2 + 12y + 36$	**41.** $4n^2 - 36$
42. $32 - 18x^2$	**43.** $4b^2 - 40b + 100$	**44.** $-27t^2 - 18t - 3$
45. $-2x^2 + 52x - 338$	**46.** $169 - x^2$	**47.** $x^2 - 10{,}000w^2$
48. $-108 + 147x^2$	**49.** $x^2 + \frac{2}{3}x + \frac{1}{9}$	**50.** $\frac{3}{4} - 12x^2$

SOLVING EQUATIONS **Use factoring to solve the equation. Use a graphing calculator to check your solution if you wish.**

51. $2x^2 - 72 = 0$

52. $3x^2 - 24x + 48 = 0$

53. $25x^2 - 4 = 0$

54. $\frac{1}{5}x^2 - 2x + 5 = 0$

55. $27 - 12x^2 = 0$

56. $50x^2 + 60x + 18 = 0$

57. $\frac{1}{3}x^2 - 6x + 27 = 0$

58. $90x^2 - 120x + 40 = 0$

59. $x^2 - \frac{5}{3}x + \frac{25}{36} = 0$

60. $112x^2 - 252 = 0$

61. $-16x^2 + 56x - 49 = 0$

62. $-\frac{4}{5}x^2 - \frac{4}{5}x - \frac{1}{5} = 0$

SAFE WORKING LOAD **In Exercises 63 and 64, the safe working load *S* (in tons) for a *wire* rope is a function of *D*, the diameter of the rope in inches.**

Safe working load model for wire rope: $4 \cdot D^2 = S$

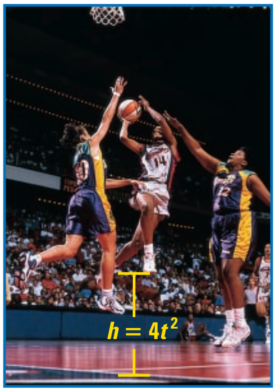
63. What diameter of wire rope do you need to lift a 9-ton load and have a safe working load?

64. When determining the safe working load S of a rope that is old or worn, decrease S by 50%. Write a model for S when using an old wire rope. What diameter of old wire rope do you need to safely lift a 9-ton load?

HANG TIME **In Exercises 65 and 66, use the following information about *hang time*, the length of time a basketball player is in the air after jumping.**

The maximum height h jumped (in feet) is a function of t, where t is the hang time (in seconds).

Hang time model: $h = 4t^2$

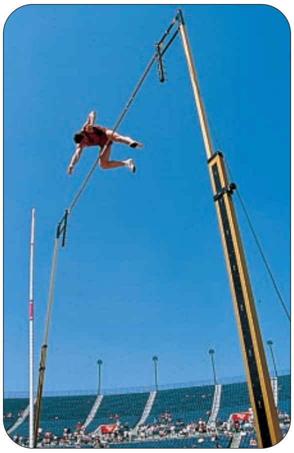


65. If you jump 1 foot into the air, what is your hang time?

66. If a professional player jumps 4 feet into the air, what is the hang time?

67. **VERTICAL MOTION** An object is propelled from the ground with an initial upward velocity of 224 feet per second. Will the object reach a height of 784 feet? If it does, how long will it take the object to reach that height? Solve by factoring.

FOCUS ON APPLICATIONS



REAL LIFE **POLE-VAULTERS** must practice to perfect their technique in order to maximize the height vaulted.

POLE-VAULTING **In Exercises 68 and 69, use the following information.** In the sport of pole-vaulting, the height h (in feet) reached by a pole-vaulter is a function of v, the velocity of the pole-vaulter, as shown in the model below. The constant g is approximately 32 feet per second per second.

Pole-vaulter height model: $h = \frac{v^2}{2g}$

68. To reach a height of 9 feet, what is the pole-vaulter's velocity?

69. To reach a height of 16 feet, what is the pole-vaulter's velocity?

Test Preparation

70. MULTIPLE CHOICE Which of the following is a correct factorization of $-12x^2 + 147$?

(A) $-3(2x + 7)^2$ (B) $3(2x - 7)(2x + 7)$

(C) $-2(2x - 7)(2x + 7)$ (D) $-3(2x - 7)(2x + 7)$

71. MULTIPLE CHOICE Which of the following is a correct factorization of $72x^2 - 24x + 2$?

(A) $-9(3x - 1)^2$ (B) $8\left(9x - \frac{1}{2}\right)^2$

(C) $8\left(3x - \frac{1}{2}\right)\left(3x - \frac{1}{2}\right)$ (D) $-8\left(3x - \frac{1}{2}\right)^2$

72. MULTIPLE CHOICE Which of the following is the solution of the equation $-4x^2 + 24x - 36 = 0$?

(A) -6 (B) -3 (C) 2 (D) 3

★ **Challenge**

73. SAFE WORKING LOAD Example 5 on page 621 models the safe working load of a natural fiber rope as a function of its *circumference*. Exercises 63 and 64 use a model for the safe working load of a wire rope as a function of its *diameter*. Find a way to use these models to compare the strength of a wire rope to the strength of a natural fiber rope.

EXTRA CHALLENGE
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MIXED REVIEW

FINDING THE GCF Find the greatest common factor of the numbers. (Skills Review, p. 777)

74. 9 and 12 **75.** 15 and 45 **76.** 55 and 132 **77.** 14 and 18

CHECKING FOR SOLUTIONS Decide whether or not the ordered pair is a solution of the system of linear equations. (Review 7.1)

78. $x + 9y = -11$, $-4x + y = -30$ $(7, -2)$

79. $2x + 6y = 22$, $-x - 4y = -13$ $(-5, -2)$

80. $-2x + 7y = -41$, $3x + 5y = 15$ $(-10, 3)$

81. $-5x - 8y = 28$, $9x - 2y = 48$ $(4, -6)$

SOLVING LINEAR SYSTEMS Use the substitution method to solve the linear system. (Review 7.2)

82. $x - y = 2$, $2x + y = 1$

83. $x - 2y = 10$, $3x - y = 0$

84. $-x + y = 0$, $2x + y = 0$

85. $2x + 3y = -5$, $x - 2y = -6$

SIMPLIFYING RADICAL EXPRESSIONS Simplify the expression. (Review 9.2)

86. $\sqrt{216}$ **87.** $\sqrt{5} \cdot \sqrt{15}$ **88.** $\sqrt{10} \cdot \sqrt{20}$ **89.** $\sqrt{4} \cdot 3\sqrt{9}$

90. $\sqrt{\frac{28}{49}}$ **91.** $\frac{10\sqrt{8}}{\sqrt{25}}$ **92.** $\frac{12\sqrt{4}}{\sqrt{9}}$ **93.** $\frac{-6\sqrt{12}}{\sqrt{4}}$

SOLVING EQUATIONS Use the quadratic formula to solve the equation. (Review 9.5 for 10.8)

94. $9x^2 - 14x - 7 = 0$ **95.** $9d^2 - 58d + 24 = 0$ **96.** $7y^2 - 9y - 17 = 0$