4.4 The Slope of a Line

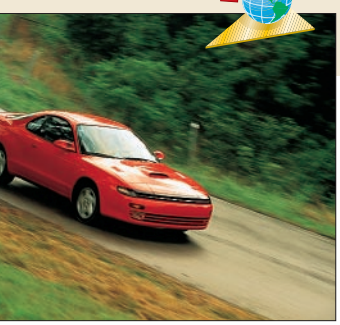
What you should learn

GOAL 1 Find the slope of a line using two of its points.

GOAL 2 Interpret slope as a rate of change in **real-life** situations like the parachute problem in **Example 6**.

Why you should learn it

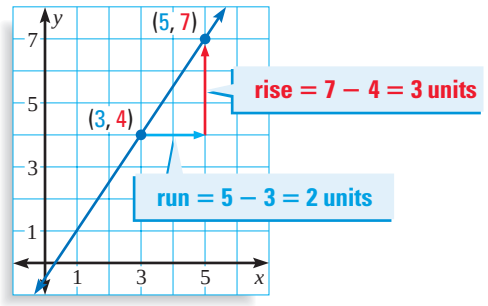
▼ To solve **real-life** problems about the steepness of a hill or the grade of a road in **Ex. 51**.



GOAL 1 FINDING THE SLOPE OF A LINE

The **slope** m of a nonvertical line is the number of units the line rises or falls for each unit of horizontal change from left to right.

The slanted line at the right rises 3 units for each 2 units of horizontal change from left to right. So, the slope m of the line is $\frac{3}{2}$.



$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{7-4}{5-3} = \frac{3}{2}$$

Two points on a line are all that is needed to find its slope.

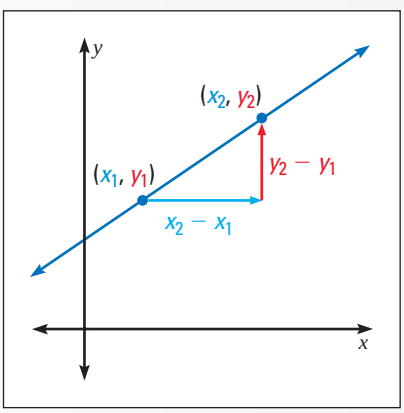
FINDING THE SLOPE OF A LINE

The slope m of the nonvertical line passing through the points (x_1, y_1) and (x_2, y_2) is

$$m = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } y}{\text{change in } x} = \frac{y_2 - y_1}{x_2 - x_1}.$$

Read x_1 as "x sub one." Think "x-coordinate of the first point."

Read y_2 as "y sub two." Think "y-coordinate of the second point."



When you use the formula for slope, the order of the subtraction is important. Given two points on a line, you can label either point as (x_1, y_1) and the other point as (x_2, y_2). After doing this, however, you must form the numerator and the denominator using the *same* order of subtraction.

Correct: $m = \frac{y_2 - y_1}{x_2 - x_1}$ ← Subtraction order is the same.

Incorrect: $m = \frac{y_2 - y_1}{x_1 - x_2}$ ← Subtraction order is different.

EXAMPLE 1 A Line with a Positive Slope Rises

Find the slope of the line passing through $(-2, 2)$ and $(3, 4)$.

SOLUTION

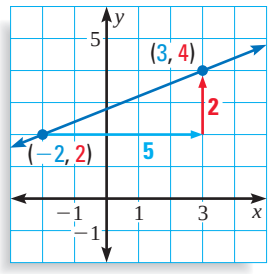
Let $(x_1, y_1) = (-2, 2)$ and $(x_2, y_2) = (3, 4)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$ ← Rise: Difference of *y*-values; ← Run: Difference of *x*-values

$$= \frac{4 - 2}{3 - (-2)}$$ Substitute values.

$$= \frac{2}{3 + 2}$$ Simplify.

$$= \frac{2}{5}$$ Slope is positive.



Positive slope: line rises from left to right.

STUDENT HELP

Look Back
For help with evaluating expressions, see p. 79.

EXAMPLE 2 A Line with a Zero Slope is Horizontal

Find the slope of the line passing through $(-1, 2)$ and $(3, 2)$.

SOLUTION

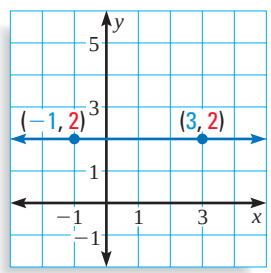
Let $(x_1, y_1) = (-1, 2)$ and $(x_2, y_2) = (3, 2)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$ ← Rise: Difference of *y*-values; ← Run: Difference of *x*-values

$$= \frac{2 - 2}{3 - (-1)}$$ Substitute values.

$$= \frac{0}{4}$$ Simplify.

$$= 0$$ Slope is zero.



Zero slope: line is horizontal.

EXAMPLE 3 A Line with a Negative Slope Falls

Find the slope of the line passing through $(0, 0)$ and $(3, -3)$.

SOLUTION

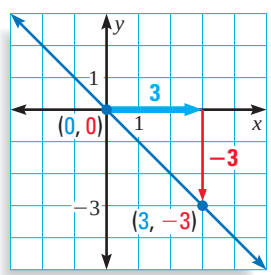
Let $(x_1, y_1) = (0, 0)$ and $(x_2, y_2) = (3, -3)$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$ ← Rise: Difference of *y*-values; ← Run: Difference of *x*-values

$$= \frac{-3 - 0}{3 - 0}$$ Substitute values.

$$= \frac{-3}{3}$$ Simplify.

$$= -1$$ Slope is negative.



Negative slope: line falls from left to right.

STUDENT HELP

Skills Review
For help with simplifying fractions, see p. 780.

EXAMPLE 4 Slope of a Vertical Line is Undefined

Find the slope of the line passing through (2, 4) and (2, 1).

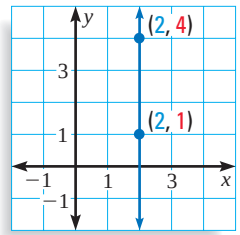
SOLUTION Let $(x_1, y_1) = (2, 1)$ and $(x_2, y_2) = (2, 4)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$ ← Rise: Difference of *y*-values ← Run: Difference of *x*-values

$$= \frac{4 - 1}{2 - 2}$$ Substitute values.

$$= \frac{3}{0}$$ Division by 0 is undefined.

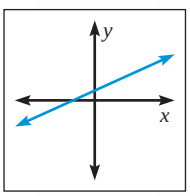
Because division by 0 is undefined, the expression $\frac{3}{0}$ has no meaning. The slope of a vertical line is undefined.



Undefined slope: line is vertical.

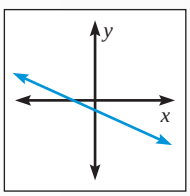
CONCEPT SUMMARY CLASSIFICATION OF LINES BY SLOPE

A line with positive slope *rises* from left to right.



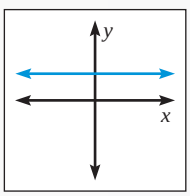
$m > 0$

A line with negative slope *falls* from left to right.



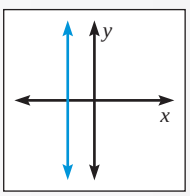
$m < 0$

A line with zero slope is *horizontal*.



$m = 0$

A line with undefined slope is *vertical*.



***m* is undefined.**

EXAMPLE 5 Given the Slope, Find a y-Coordinate

Find the value of y so that the line passing through the points $(-2, 1)$ and $(4, y)$ has a slope of $\frac{2}{3}$.

SOLUTION Let $(x_1, y_1) = (-2, 1)$ and $(x_2, y_2) = (4, y)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$ Write formula for slope.

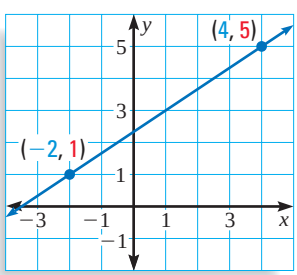
$$\frac{2}{3} = \frac{y - 1}{4 - (-2)}$$ Substitute values for m, x_1, y_1, x_2, and y_2.

$$\frac{2}{3} = \frac{y - 1}{6}$$ Simplify.

$$6 \cdot \frac{2}{3} = 6 \cdot \frac{y - 1}{6}$$ Multiply each side by 6.

$$4 = y - 1$$ Simplify.

$$5 = y$$ Add 1 to each side.



STUDENT HELP

HOMEWORK HELP

Visit our Web site www.mcdougallittell.com for extra examples.

GOAL 2 INTERPRETING SLOPE AS A RATE OF CHANGE

A **rate of change** compares two different quantities that are changing. For example, the rate at which distance is changing with time is called *velocity*. Slope provides an important way of visualizing a rate of change.

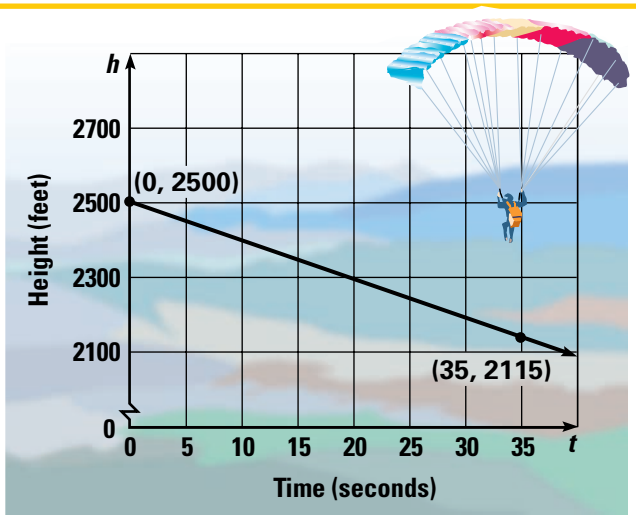
REAL LIFE Parachuting

EXAMPLE 6 *Slope as a Rate of Change*

You are parachuting. At time $t = 0$ seconds, you open your parachute at height $h = 2500$ feet above the ground. At time $t = 35$ seconds, you are at height $h = 2115$ feet.

a. What is your rate of change in height?

b. About when will you reach the ground?



SOLUTION

a. Use the formula for slope to find the rate of change. The change in time is $35 - 0 = 35$ seconds. Subtract in the same order. The change in height is $2115 - 2500 = -385$ feet.

PROBLEM SOLVING STRATEGY

VERBAL MODEL

$$\text{Rate of change} = \frac{\text{Change in height}}{\text{Change in time}}$$

LABELS

Rate of change $= m$ (ft/sec)

Change in height $= -385$ (ft)

Change in time $= 35$ (sec)

ALGEBRAIC MODEL

$$m = \frac{-385}{35}$$

▶ Your rate of change is -11 ft/sec. The negative value indicates that you are falling.

b. Falling at a rate of 11 ft/sec, find the time it will take you to fall 2500 feet.

$$\text{Time} = \frac{\text{Distance}}{\text{Rate}} = \frac{2500 \text{ ft}}{11 \text{ ft/sec}} \approx 227 \text{ sec}$$

▶ You will reach the ground about 227 seconds after opening your parachute.

UNIT ANALYSIS Check that *seconds* are the units of the solution.

$$\frac{\text{ft}}{\text{ft/sec}} = \cancel{\text{ft}} \cdot \frac{\text{sec}}{\cancel{\text{ft}}} = \text{sec}$$

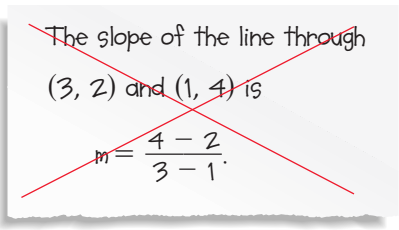
GUIDED PRACTICE

Vocabulary Check ✓

1. Draw a ramp and label its rise and run. Explain what is meant by the slope of the ramp.

Concept Check ✓

2. **ERROR ANALYSIS** Describe the error at the right. Then calculate the correct slope.



Ex. 2

3. Explain what happens when the formula for slope is applied to a vertical line.

4. **CRITICAL THINKING** How can you tell that the slope of the line through (2, 2) and (−3, 5) is negative without calculating?

Skill Check ✓

In Exercises 5–10, plot the points and draw a line through them. Find the slope of the line passing through the points.

5. (0, 0), (1, 2)
6. (0, 0), (−1, −1)
7. (1, 2), (2, 1)
8. (3, 2), (1, 4)
9. (2, 2), (3, 5)
10. (−3, −2), (1, 6)

11. Find the value of y so that the line passing through the points (0, 3) and (4, y) has a slope of −3.

PRACTICE AND APPLICATIONS

STUDENT HELP

Extra Practice to help you master skills is on p. 800.

DESCRIBING SLOPE **Plot the points and draw a line through them. Without calculating, state whether the slope of the line is *positive, negative, zero,* or *undefined*. Explain your reasoning.**

12. (6, 9), (4, 3)
13. (7, 4), (−1, 8)
14. (5, 10), (5, −4)
15. (1, 1), (4, −3)
16. (−2, 5), (3, 5)
17. (0, 0), (−5, 3)
18. (1, 3), (−2, 1)
19. (2, −2), (2, −6)

GRAPH AND CALCULATE **Plot the points and find the slope of the line passing through the points.**

20. (4, 5), (2, 3)
21. (1, 5), (5, 2)
22. (2, 3), (−3, 0)
23. (0, −6), (8, 0)
24. (0, 6), (8, 0)
25. (2, 4), (4, −4)
26. (−6, −1), (−6, 4)
27. (0, −10), (−4, 0)
28. (1, −2), (−2, 2)
29. (3, 6), (3, 0)
30. (−6, 2), (4, −2)
31. (−1, −1), (−3, −6)
32. $\left(0, \frac{1}{2}\right)$, (0, 0)
33. (2, 2), (−3, 5)
34. (4, 1), (6, 1)

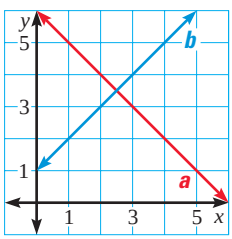
STUDENT HELP

HOMEWORK HELP

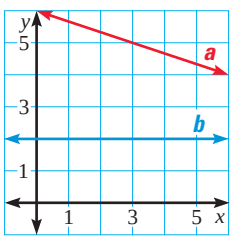
Example 1: Exs. 12–37
Example 2: Exs. 12–37
Example 3: Exs. 12–37
Example 4: Exs. 12–37
Example 5: Exs. 38–46
Example 6: Exs. 56, 57

GRAPHICAL REASONING **Find the slope of each line.**

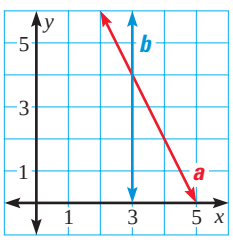
35.



36.



37.



FINDING A COORDINATE **Find the value of y so that the line passing through the two points has the given slope.**

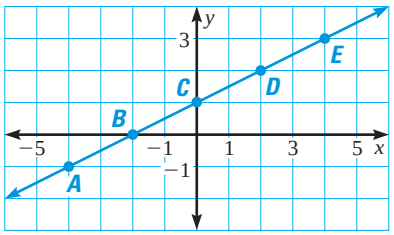
38. $(2, y), (4, 5), m = 2$ **39.** $(0, -2), (2, y), m = 3$ **40.** $(-5, 3), (3, y), m = -\frac{1}{2}$

41. $(0, y), (2, 5), m = 2$ **42.** $(-1, 5), (3, y), m = 5$ **43.** $(-1, 3), (5, y), m = -1$

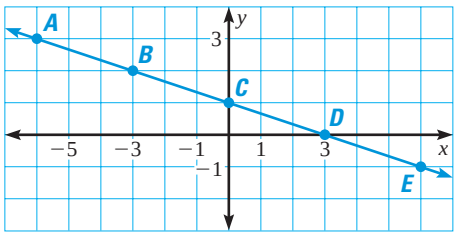
44. $(5, 7), (8, y), m = \frac{4}{3}$ **45.** $(3, y), (1, 4), m = -\frac{1}{2}$ **46.** $(2, -15), (5, y), m = \frac{4}{5}$

INDUCTIVE REASONING **Choose three different pairs of points on the line. Find the slope of the line using each pair. What do you notice?**

47.



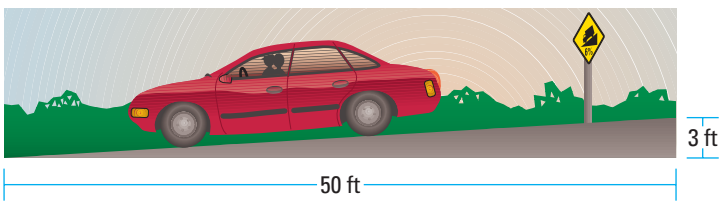
48.



49. **GEOMETRY CONNECTION** Use the concept of slope to decide whether the points $(-2, 4)$, $(2, -2)$, and $(6, 0)$ are on the same line. Explain your reasoning and include a diagram.

50. **FINDING SLOPE** To find the slope of a line, can you choose any two points on the line? Explain your reasoning. What are some guidelines that you could use to choose convenient points for calculating slope?

51. **ROAD GRADES** The U.S. Department of Transportation requires surveyors to place signs on steep sections of roads. The grade of a road is measured as a percent. For instance, the grade of the road shown below is 6%. What is the slope of the road? Explain the relationship between road grade and slope.



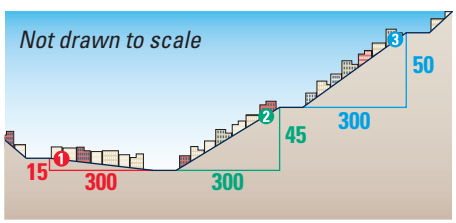
WHAT ARE THE UNITS? **In Exercises 52–54, find the rate of change between the two points. Give the units of measure for the rate.**

52. $(2, 2)$ and $(9, 23)$; x in minutes, y in inches

53. $(3, 5)$ and $(11, 69)$; x in years, y in dollars.

54. $(53, 44)$ and $(32, 14)$; x in seconds, y in liters.

55. **CABLE CARS** In the 1870s a cable car system was built in San Francisco to climb the steep streets. San Francisco was the site of the first cable car used for public transportation in America. Calculate each labeled slope from left to right in the diagram.



(cross section of side view)

FOCUS ON CAREERS

REAL LIFE SURVEYORS Surveying land is the responsibility of surveyors, survey technicians, and mapping scientists. They measure distances, angles between points, and elevations as in Ex. 51.

INTERNET CAREER LINK
www.mcdougallittell.com

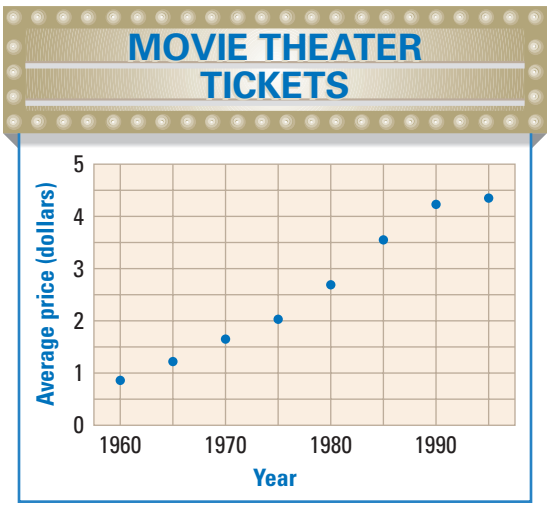
FOCUS ON APPLICATIONS

REAL LIFE MOVIE PRICES In 1954 when the Japanese movie *Gojira* was filmed, the average cost of a ticket in the United States was 49¢. In 1998 when the remake *Godzilla* was filmed, the average cost was $4.69, although some theaters charged more than $8.00.

56. SPACE SHUTTLE A space shuttle achieves orbit at 9:23 A.M. At 9:31 A.M. it has traveled 2,309.6 miles in orbit. Find the rate of change in miles per minute.

57. PROFIT In 1990, a company had a profit of $173,000,000. In 1996, the profit was $206,000,000. If the profit increased the same amount each year, find the rate of change of the company's profit in dollars per year.

MOVIE PRICES In Exercises 58–61, the graph shows the price of a movie ticket at the Midtown Theater for the years 1960–1995.



58. Estimate the rate of change from 1960 to 1995 in the price in dollars per year.

59. Estimate the rate of change from 1985 to 1990 in the price in dollars per year.

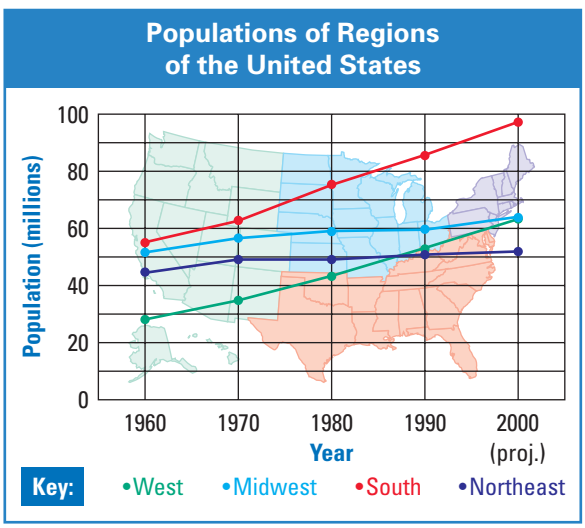
60. Which five-year period had the smallest price increase?

61. *Writing* Use the graph to estimate the cost of going to a movie at the Midtown Theater this year. Why might your estimate be different from the actual cost?

POPULATION RATES In Exercises 62–65, use the line graph below.

STUDENT HELP

INTERNET HOMEWORK HELP Visit our Web site www.mcdougallittell.com for help with problem solving in Exs. 62–64.



INTERNET DATA UPDATE of U.S. Bureau of the Census data at www.mcdougallittell.com

62. During which decade did the population of the South increase the most? Estimate the rate of change for this decade in people per year.

63. During which decade did the population of the Midwest increase the most? Estimate the rate of change for this decade in people per year.

64. INTERPRETING A GRAPH The line graph for the Northeast appears to be horizontal between 1970 and 1980. Write a sentence about what this means in terms of the real-life situation.

65. RESEARCH Find the population of your state in 1980 and in 1990. Find the annual rate of change of this population.

66. MATHEMATICAL REASONING When you use the formula $\frac{y_2 - y_1}{x_2 - x_1}$ to find the slope of a line passing through two points, does it matter which point you choose to use as (x_1, y_1)? Give three different examples to support your answer.

Test Preparation

67. MULTIPLE CHOICE The slope of the line passing through the points $(0, 5)$ and $(0, -3)$ is $\underline{?}$.

Ⓐ positive Ⓑ negative Ⓒ zero Ⓓ undefined

68. MULTIPLE CHOICE The slope of the line passing through the points $(8, 0)$ and $(0, 8)$ is $\underline{?}$.

Ⓐ positive Ⓑ negative Ⓒ zero Ⓓ undefined

69. MULTIPLE CHOICE What is the slope of the line passing through the points $(-3, 4)$ and $(5, -11)$?

Ⓐ $\frac{-7}{8}$ Ⓑ $\frac{-7}{2}$ Ⓒ $\frac{15}{-8}$ Ⓓ $\frac{15}{8}$

Challenge

70. FINDING AN *x*-COORDINATE Write an expression for the slope of a line passing through the points $(0, 3)$ and $(x, -9)$. What value of x will make the fraction equivalent to -2?

EXTRA CHALLENGE
www.mcdougallittell.com

71. Use your answers to Exercise 70 to find the value of x so that the line passing through the points $(0, 3)$ and $(x, -9)$ will have a slope of -3.

MIXED REVIEW

EVALUATING EXPRESSIONS **Evaluate the expression for the given value of the variable.** (Review 1.3)

72. $12x + 3$ when $x = 5$ **73.** $\frac{3}{4}p$ when $p = 16$ **74.** $8n$ when $n = \frac{3}{16}$

75. $\frac{7}{2}y - 3$ when $y = 4$ **76.** $5x + x$ when $x = 7$ **77.** $\frac{6}{5}z + 2$ when $z = 5$

SIMPLIFYING EXPRESSIONS **Simplify the variable expression.** (Review 2.5)

78. $(-6)(y)$ **79.** $(-1)(x)(-x)$ **80.** $-(-3)^2(-y)$

81. $\left(\frac{2}{5}\right)(5)(-x)(x)$ **82.** $(y)(-23)(-y^2)$ **83.** $\left(\frac{1}{8}\right)(-4)(-x)(-x)$

WRITING IN FUNCTION FORM **Solve for *y*.** (Review 3.7 for 4.5)

84. $9x - 2y = 14$ **85.** $5x + 9y = 18$ **86.** $-2x - 2y = 7$

87. $-x + 4y = 36$ **88.** $6x - 3y = 21$ **89.** $3x + 5y = 17$

FINDING INTERCEPTS **Find the *x*-intercept and the *y*-intercept of the graph of the equation.** (Review 4.3)

90. $y = x + 8$ **91.** $y = 2x + 12$ **92.** $y = 3x + 9$

93. $y = -6 + 2x$ **94.** $y = -2x + 16$ **95.** $3y = 6x + 3$