# 12.3 Solving Radical Equations

*What you should learn*

GOAL 1 Solve a radical equation.

GOAL 2 Use radical equations to solve **real-life** problems, such as finding the centripetal force a person experiences on a ride in **Exs. 66 and 67**.

*Why you should learn it*

▼ To solve **real-life** problems such as finding the nozzle pressure of antifreeze used to de-ice a plane in **Example 5**.


## GOAL 1 SOLVING A RADICAL EQUATION

In solving an equation involving radicals, the following property can be extremely useful.

**SQUARING BOTH SIDES OF AN EQUATION**

Suppose $a$ and $b$ are algebraic expressions.

If $a = b$, then $a^2 = b^2$. **Example:** $\sqrt{4} = 2$, so $(\sqrt{4})^2 = 2^2$.

### EXAMPLE 1 *Solving a Radical Equation*

Solve $\sqrt{x} - 7 = 0$.

**SOLUTION** Isolate the radical expression on one side of the equation.

$\sqrt{x} - 7 = 0$ **Write original equation.**

$\sqrt{x} = 7$ **Add 7 to each side.**

$(\sqrt{x})^2 = 7^2$ **Square each side.**

$x = 49$ **Simplify.**

▶ The solution is 49. Check the solution in the original equation.

### EXAMPLE 2 *Solving a Radical Equation*

To solve the equation $\sqrt{2x - 3} + 3 = 4$, you need to isolate the radical expression first.

$\sqrt{2x - 3} + 3 = 4$ **Write original equation.**

$\sqrt{2x - 3} = 1$ **Subtract 3 from each side.**

$(\sqrt{2x - 3})^2 = 1^2$ **Square each side.**

$2x - 3 = 1$ **Simplify.**

$2x = 4$ **Add 3 to each side.**

$x = 2$ **Divide each side by 2.**

▶ The solution is 2. Check the solution in the original equation.

**EXTRANEOUS SOLUTIONS** Squaring both sides of an equation often introduces an extraneous solution of $a^2 = b^2$ that is *not* a solution of $a = b$. When you square both sides of an equation, check each solution in the *original* equation.

## EXAMPLE 3 Checking for Extraneous Solutions

Solve the equation.

**a.** $\sqrt{x + 2} = x$ **b.** $\sqrt{x} + 13 = 0$

**SOLUTION**

**a.** $\sqrt{x + 2} = x$ Write original equation.

$(\sqrt{x + 2})^2 = x^2$ Square each side.

$x + 2 = x^2$ Simplify.

$0 = x^2 - x - 2$ Write in standard form.

$0 = (x - 2)(x + 1)$ Factor.

$x = 2$ or $x = -1$ Zero-product property

✓**CHECK** Substitute 2 and $-1$ in the original equation.

$\sqrt{2 + 2} \stackrel{?}{=} 2$ $\qquad$ $\sqrt{-1 + 2} \stackrel{?}{=} -1$

$2 = 2$ ✓ $\qquad$ $1 \neq -1$

▶ The only solution is 2, because $x = -1$ is not a solution.

**STUDENT HELP**

**Study Tip**

In part (b) of Example 3 you can also use critical thinking to find that there is no solution. Because $\sqrt{x}$ represents a non-negative number, there is *no* value for $\sqrt{x}$ that is the opposite of 13.

**b.** $\sqrt{x} + 13 = 0$ Write original equation.

$\sqrt{x} = -13$ Subtract 13 from each side.

$(\sqrt{x})^2 = (-13)^2$ Square each side.

$x = 169$ Simplify.

▶ $\sqrt{169} + 13 \neq 0$, so the equation has no solution.

## EXAMPLE 4 Using a Geometric Mean

The *geometric mean* of $a$ and $b$ is $\sqrt{ab}$. If the geometric mean of $a$ and 6 is 12, what is the value of $a$?

**SOLUTION** Geometric mean $= \sqrt{ab}$

$12 = \sqrt{6a}$ Substitute.

$12^2 = (\sqrt{6a})^2$ Square each side.

$144 = 6a$ Simplify.

$24 = a$ Solve for $a$.

## GOAL 2 SOLVING RADICAL EQUATIONS IN REAL LIFE

REAL LIFE
*Plane De-icing*

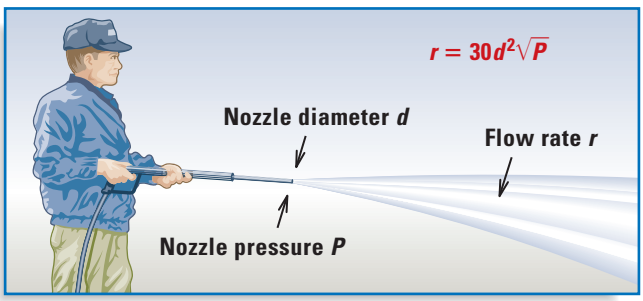
### EXAMPLE 5 *Using a Radical Model*

You work for a commercial airline and remove ice from planes. The relationship among the flow rate $r$ (in gallons per minute) of the antifreeze for de-icing, the nozzle diameter $d$ (in inches), and the nozzle pressure $P$ (in pounds per square inch) is shown in the diagram. You want a flow rate of 250 gallons per minute.

**a.** Find the nozzle pressure for a nozzle whose diameter is 1.25 inches.

**b.** Find the nozzle pressure for a nozzle whose diameter is 1.75 inches.

**c.** When you increase the diameter of the nozzle by 40%, do you also change the nozzle pressure by 40%? Explain.




**SOLUTION**

**a.**

$r = 30d^2\sqrt{P}$ — Write model for water flow.

$250 = 30(1.25)^2\sqrt{P}$ — Substitute 1.25 for $d$ and 250 for $r$.

$250 = 46.875\sqrt{P}$ — Simplify.

$\frac{250}{46.875} = \sqrt{P}$ — Divide each side by 46.875.

$\left(\frac{250}{46.875}\right)^2 = P$ — Square each side.

$28.4 \approx P$ — Use a calculator.

▶ The nozzle pressure will be about 28.4 pounds per square inch.

**b.**

$r = 30d^2\sqrt{P}$ — Write model for water flow.

$250 = 30(1.75)^2\sqrt{P}$ — Substitute 1.75 for $d$ and 250 for $r$.

$\frac{250}{91.875} = \sqrt{P}$ — Simplify and divide each side by 91.875.

$\left(\frac{250}{91.875}\right)^2 = P$ — Square each side.

$7.4 \approx P$ — Use a calculator.

▶ The nozzle pressure will be about 7.4 pounds per square inch.

**c.** No, when you increase the nozzle diameter by 40%, the pressure decreases by about 74%.

# GUIDED PRACTICE

**Vocabulary Check** ✓

1. Explain what an *extraneous solution* is.

**Concept Check** ✓

2. One reason for checking a solution in the original equation is to look for an error in one of the steps of the solution. Give another reason.

3. Is 36 a solution of $\sqrt{x} = -6$? Why or why not?

**Skill Check** ✓

**Solve the equation. Check for extraneous solutions.**

4. $\sqrt{x} - 20 = 0$
5. $\sqrt{5x + 1} + 8 = 12$
6. $\sqrt{4x} - 1 = 3$
7. $\sqrt{x} + 6 = 0$
8. $\sqrt{4x + 5} = x$
9. $\sqrt{x + 6} - x = 0$
10. $x = \sqrt{x + 12}$
11. $-5 + \sqrt{x} = 0$
12. $x = \sqrt{5x + 24}$
13. Find $a$ if the geometric mean of 12 and $a$ is 6.
14. **PLANE DE-ICING** Use Example 5 to answer the question. What will the nozzle pressure be if the nozzle diameter is 2 inches?

# PRACTICE AND APPLICATIONS

**STUDENT HELP**

**Extra Practice**
to help you master skills is on p. 808.

**SOLVING RADICAL EQUATIONS** **Solve the equation. Check for extraneous solutions.**

15. $\sqrt{x} - 9 = 0$
16. $\sqrt{x} - 1 = 0$
17. $\sqrt{x} + 5 = 0$
18. $\sqrt{x} - 10 = 0$
19. $\sqrt{x} - 15 = 0$
20. $\sqrt{x} - 0 = 0$
21. $\sqrt{6x} - 13 = 23$
22. $\sqrt{4x + 1} + 5 = 10$
23. $\sqrt{9 - x} - 10 = 14$
24. $\sqrt{5x + 1} + 2 = 6$
25. $\sqrt{6x - 2} - 3 = 7$
26. $4 = 7 - \sqrt{33x - 2}$
27. $10 = 4 + \sqrt{5x + 11}$
28. $-5 - \sqrt{10x - 2} = 5$
29. $\sqrt{-x} - \frac{3}{2} = \frac{3}{2}$
30. $\sqrt{x} + \frac{1}{3} = \frac{13}{3}$
31. $\sqrt{\frac{1}{5}x - 2} - \frac{1}{10} = \frac{7}{10}$
32. $x = \sqrt{\frac{3}{2}x + \frac{5}{2}}$
33. $\sqrt{\frac{1}{4}x - 4} - 3 = 5$
34. $6 - \sqrt{7x - 9} = 3$
35. $\sqrt{\frac{1}{9}x + 1} - \frac{2}{3} = \frac{5}{3}$
36. $x = \sqrt{35 + 2x}$
37. $x = \sqrt{-4x - 4}$
38. $x = \sqrt{6x - 9}$
39. $x = \sqrt{30 - x}$
40. $x = \sqrt{11x - 28}$
41. $\sqrt{-10x - 4} = 2x$
42. $x = \sqrt{200 - 35x}$
43. $\sqrt{110 - x} = x$
44. $2x = \sqrt{-13x - 10}$
45. $x = \sqrt{4x + 45}$
46. $\frac{1}{5}x = \sqrt{x - 6}$
47. $\frac{2}{3}x = \sqrt{24x - 128}$

**STUDENT HELP**

**HOMEWORK HELP**

**Example 1:** Exs. 15–21
**Example 2:** Exs. 22–47
**Example 3:** Exs. 15–47
**Example 4:** Exs. 48–53
**Example 5:** Exs. 69, 70

**GEOMETRIC MEAN** **Two numbers and their geometric mean are given. Find the value of *a*.**

48. 12 and $a$; 27
49. 4 and $a$; 16
50. 6 and $a$; 27
51. 4 and $a$; 14
52. 6 and $a$; 72
53. 8 and $a$; 104

**ERROR ANALYSIS** **Find and correct the error.**

**54.**

$\sqrt{x} = 7$

$(\sqrt{x})^2 = (\sqrt{7})^2$

$x^2 = 7$

$x = \sqrt{7}$

**55.**

$\sqrt{x} - 15 = 0$

$\sqrt{x} = 15$

$(\sqrt{x})^2 = (15)^2$

$x = 225$ and $-225$

STUDENT HELP

**Look Back**
For help with solving an equation using a graphing calculator, see p. 359.

**SOLVING GRAPHICALLY** **In Exercises 56–64, use a graphing calculator to graphically solve the radical equation. Check the solution algebraically.**

**56.** $\sqrt{x + 4} = 3$

**57.** $\sqrt{x - 5.6} = 2.5$

**58.** $\sqrt{9.2 - x} = 1.8$

**59.** $\sqrt{6x - 2} - 3 = 7$

**60.** $4 + \sqrt{x} = 9$

**61.** $\sqrt{7x - 12} = x$

**62.** $\sqrt{2x + 7} = x + 2$

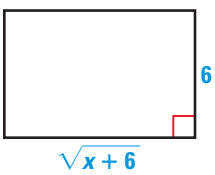
**63.** $\sqrt{3x - 2} = 4 - x$

**64.** $\sqrt{15 - 4x} = 2x$

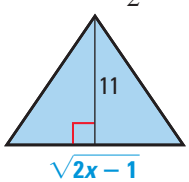
**65.** **VISUAL THINKING** Use a graphing calculator or a computer to show that the radical equation $\sqrt{11x - 30} = -x$ has no solution. Explain.

**GEOMETRY CONNECTION** **Find the value of *x*.**

**66.** Perimeter = 30




**67.** Area = $82\frac{1}{2}$




**68.** **LOGICAL REASONING** Write a radical equation that has a solution of 18.

**AMUSEMENT PARK RIDE** **In Exercises 69 and 70, use the following information.**

A ride at an amusement park spins in a circle of radius $r$ (in meters). The centripetal force $F$ experienced by a passenger on the ride is modeled by the equation below, where $t$ is the number of seconds the ride takes to complete one revolution and $m$ is the mass (in kilograms) of the passenger.

$$t = \sqrt{\frac{4\pi^2 mr}{F}}$$

STUDENT HELP

INTERNET **APPLICATION LINK**
Visit our Web site www.mcdougallittell.com for more information about centripetal force.

**69.** A person whose mass is 67.5 kilograms is on a ride that is spinning in a circle at a rate of 10 seconds per revolution. The radius of the circle is 6 meters. How much centripetal force does the person experience?

**70.** **CRITICAL THINKING** A person whose mass is 54 kilograms is on the ride. A mass of 54 kilograms is 80% of 67.5 kilograms. Is the centripetal force experienced by the person 80% of the force in Exercise 69? Explain.

**BLOTTING PAPER** **In Exercises 71 and 72, use the following information.** Blotting paper is a thick, soft paper used for absorbing fluids such as water or ink. The distance $d$ (in centimeters) that tap water is absorbed up a strip of blotting paper at a temperature of 28.4°C is given by the equation

$d = 0.444\sqrt{t}$ where $t$ is the time (in seconds).

**71.** Approximately how many minutes would it take for the water to travel a distance of 28 centimeters up the strip of blotting paper?

**72.** How far up the blotting paper would the water be after $33\frac{1}{3}$ seconds?

**GEOMETRIC MEAN** **In Exercises 73–76, use the proportion $\frac{a}{b} = \frac{b}{d}$, where $a$, $b$, and $d$ are positive numbers.**

**73.** In the proportion $\frac{a}{b} = \frac{b}{d}$, $b$ is called the geometric mean of $a$ and $d$. Use the cross product property to show that $b = \sqrt{ad}$.

**74.** Two numbers have a geometric mean of 4. One number is 6 more than the other.

**a.** Use the proportion in Exercise 73. Rewrite the proportion substituting the given value for the geometric mean.

**b.** Let $x$ represent one of the numbers. How can you represent "one number is 6 more than the other" in the proportion? Rewrite the proportion using $x$.

**c.** Solve the proportion in part (b) to find the numbers.

**75.** Two numbers have a geometric mean of 10. One number is 21 less than the other. Find the numbers.

**76.** Two numbers have a geometric mean of 12. One number is 32 more than the other. Find the numbers.

## Test Preparation

**QUANTITATIVE COMPARISON** **In Exercises 77–80, choose the statement below that is true about the given quantities.**

Ⓐ The quantity in column A is greater.

Ⓑ The quantity in column B is greater.

Ⓒ The two quantities are equal.

Ⓓ The relationship cannot be determined from the given information.

| | Column A | Column B |
|---|---|---|
| **77.** | The geometric mean of 3 and 12 | The geometric mean of 1 and 36 |
| **78.** | The solution of $\sqrt{x} + 4 = 5$ | The solution of $\sqrt{x} - 4 = 5$ |
| **79.** | The solution of $\sqrt{x} - 3 = 5$ | The solution of $\sqrt{x - 3} = 5$ |
| **80.** | The geometric mean of $-1$ and $-64$ | The geometric mean of $-2$ and $-32$ |

## Challenge

**81. LOGICAL REASONING** Using $4\sqrt{x} = 2x + k$, find three different expressions that can be substituted for $k$ so that the equation has two solutions, one solution, and no solution. Describe how you found the equations.

# MIXED REVIEW

**SOLVING EQUATIONS** **Solve the equation.** (Review 9.1, 9.2 for 12.4)

**82.** $x^2 = 36$ **83.** $x^2 = 11$ **84.** $7x^2 = 700$

**85.** $25x^2 - 9 = -5$ **86.** $\frac{1}{7}x^2 - 7 = -7$ **87.** $-16t^2 + 48 = 0$

**FINDING PRODUCTS** **Multiply.** (Review 10.3 for 12.4)

**88.** $(x + 5)^2$ **89.** $(2x - 3)^2$ **90.** $(3x + 5y)(3x - 5y)$

**91.** $(6y - 4)(6y + 4)$ **92.** $(x + 7y)^2$ **93.** $(2a - 9b)^2$

**FACTORING TRINOMIALS** **Factor the trinomial.** (Review 10.7 for 12.4)

**94.** $x^2 + 18x + 81$ **95.** $x^2 - 12x + 36$ **96.** $4x^2 + 28x + 49$

**UNDEFINED VALUES** **For what values of the variable is the rational expression undefined?** (Review 11.4)

**97.** $\frac{5}{x - 4}$ **98.** $\frac{x + 2}{x^2 - 4}$ **99.** $\frac{x + 4}{x^2 + x - 6}$

**100.** **COMPARING COSTS** A discount grocery store offers two types of memberships with annual membership fees of \$25 and \$100. If you pay the \$25 membership fee, you get regular discounted prices. If you pay the \$100 membership fee, you get an additional 10% discount. How much money would you have to spend on groceries for the two memberships to cost the same? (Review 7.2)

# QUIZ 1

*Self-Test for Lessons 12.1–12.3*

**Identify the domain and the range of the function. Then sketch the graph of the function.** (Lesson 12.1)

**1.** $y = 10\sqrt{x}$ **2.** $y = \sqrt{x - 9}$ **3.** $y = \sqrt{2x - 1}$ **4.** $y = \frac{\sqrt{x} - 2}{3}$

**Simplify the expression.** (Lesson 12.2)

**5.** $7\sqrt{10} + 11\sqrt{10}$ **6.** $2\sqrt{7} - 5\sqrt{28}$ **7.** $4\sqrt{5} + \sqrt{125} - \sqrt{80}$

**8.** $\sqrt{3}(3\sqrt{2} + \sqrt{3})$ **9.** $(\sqrt{7} + 5)^2$ **10.** $\frac{15}{8 + \sqrt{7}}$

**Solve the equation.** (Lesson 12.3)

**11.** $\sqrt{x} - 12 = 0$ **12.** $\sqrt{x} - 8 = 0$ **13.** $\sqrt{3x + 2} + 2 = 3$

**14.** $\sqrt{3x - 2} + 3 = 7$ **15.** $\sqrt{77 - 4x} = x$ **16.** $x = \sqrt{2x + 3}$

**17.** **NOZZLE PRESSURE** The nozzle pressure $P$ (in pounds per square inch) of a hose with diameter $d$ (in inches) and water-flow rate $r$ (in gallons per minute) is given by the equation $r = 30d^2\sqrt{P}$. Find the nozzle pressure in a hose that has a water-flow rate of 250 gallons per minute and a diameter of 2.5 inches. (Lesson 12.3)