9.6 Applications of the Discriminant

What you should learn

GOAL 1 Use the discriminant to find the number of solutions of a quadratic equation.

GOAL 2 Apply the discriminant to solve **real-life** problems, such as financial analysis in **Exs. 29 and 30**.

Why you should learn it

▼ To model a **real-life** situation, such as storing your food while camping in **Example 4**.

REAL LIFE

GOAL 1 NUMBER OF SOLUTIONS OF A QUADRATIC

In the quadratic formula, the expression inside the radical is the **discriminant**.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \longleftarrow \text{Discriminant}$$

The discriminant $b^2 - 4ac$ of a quadratic equation can be used to find the number of solutions of the quadratic equation.

THE NUMBER OF SOLUTIONS OF A QUADRATIC EQUATION

Consider the quadratic equation $ax^2 + bx + c = 0$.

- If $b^2 - 4ac$ is positive, then the equation has two solutions.
- If $b^2 - 4ac$ is zero, then the equation has one solution.
- If $b^2 - 4ac$ is negative, then the equation has no real solution.

EXAMPLE 1 *Finding the Number of Solutions*

Find the value of the discriminant and use the value to tell if the equation has *two solutions*, *one solution*, or *no solution*.

a. $x^2 - 3x - 4 = 0$ **b.** $-x^2 + 2x - 1 = 0$ **c.** $2x^2 - 2x + 3 = 0$

SOLUTION

a. $b^2 - 4ac = (-3)^2 - 4(1)(-4)$ Substitute 1 for a, −3 for b, −4 for c.

$= 9 + 16$ Simplify.

$= 25$ Discriminant is positive.

▶ The discriminant is positive, so the equation has two solutions.

b. $b^2 - 4ac = (2)^2 - 4(-1)(-1)$ Substitute −1 for a, 2 for b, −1 for c.

$= 4 - 4$ Simplify.

$= 0$ Discriminant is zero.

▶ The discriminant is zero, so the equation has one solution.

c. $b^2 - 4ac = (-2)^2 - 4(2)(3)$ Substitute 2 for a, −2 for b, 3 for c.

$= 4 - 24$ Simplify.

$= -20$ Discriminant is negative.

▶ The discriminant is negative, so the equation has no real solution.

The number of x-intercepts of the graph of $y = ax^2 + bx + c$ is the same as the number of real solutions of the equation $ax^2 + bx + c = 0$.

EXAMPLE 2 *Finding the Number of x-Intercepts*

Use the related equation to find the number of x-intercepts of the graph of the function.

a. $y = x^2 + 2x - 2$ **b.** $y = x^2 + 2x + 1$ **c.** $y = x^2 + 2x + 3$

STUDENT HELP

HOMEWORK HELP Visit our Web site www.mcdougallittell.com for extra examples.

SOLUTION

For each function, let $y = 0$. Then find the value of the discriminant.

a. $b^2 - 4ac = 2^2 - 4(1)(-2)$ $\quad$ **$a = 1$, $b = 2$, $c = -2$**

$= 4 + 8$ $\quad$ **Simplify.**

$= 12$ $\quad$ **Discriminant is positive.**

▶ The discriminant is positive, so the equation has two solutions *and* the graph has two x-intercepts.

b. $b^2 - 4ac = 2^2 - 4(1)(1)$ $\quad$ **$a = 1$, $b = 2$, $c = 1$**

$= 4 - 4$ $\quad$ **Simplify.**

$= 0$ $\quad$ **Discriminant is zero.**

▶ The discriminant is zero, so the equation has one solution *and* the graph has one x-intercept.

c. $b^2 - 4ac = 2^2 - 4(1)(3)$ $\quad$ **$a = 1$, $b = 2$, $c = 3$**

$= 4 - 12$ $\quad$ **Simplify.**

$= -8$ $\quad$ **Discriminant is negative.**

▶ The discriminant is negative, so the equation has no real solutions *and* the graph has no x-intercepts.

EXAMPLE 3 *Changing the Value of c*

Sketch the graphs of the equations in Example 2 to check the number of x-intercepts. What effect does changing the value of c have on the graph?

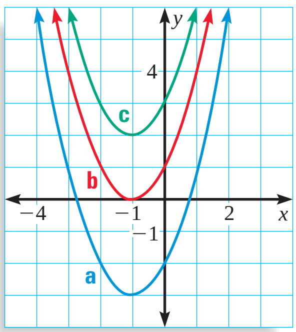
STUDENT HELP

Look Back For help with sketching a quadratic function, see p. 518.

SOLUTION

By changing the value of c, you can move the graph of $y = x^2 + 2x + c$ up or down in the coordinate plane.

If the graph is moved high enough, it will not have an x-intercept and the equation $x^2 + 2x + c = 0$ will have no real solution.



GOAL 2 USING THE DISCRIMINANT IN REAL LIFE

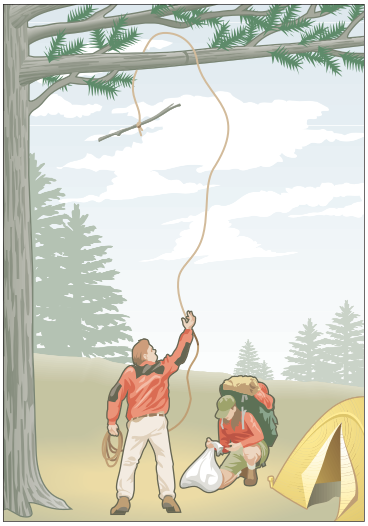
REAL LIFE Camping

EXAMPLE 4 Using the Discriminant

You and a friend are camping in Glacier National Park in Montana. You want to hang a food pack from a high tree branch in order to protect your food from bears. You attach a stick to a rope and your friend is preparing to throw it over a tree branch that is 20 feet from the ground.

a. Your friend can throw the stick upward with an initial velocity of 29 feet per second from an initial height of 6 feet. Will the stick reach the branch when it is thrown?

b. You can throw a stick upward with an initial velocity of 32 feet per second from the same initial height as your friend. Will the stick reach the branch when it is thrown?



Not drawn to scale

SOLUTION

In both situations, you can use a vertical motion model $h = -16t^2 + vt + s$ where h is the height you are trying to reach, t is the time in motion, v is the initial velocity, and s is the initial height.

Because the initial height for you and your friend is 6 feet, the vertical motion model is $h = -16t^2 + vt + 6$. After you substitute for the initial velocity, evaluating the discriminant will tell you whether the stick reaches the tree branch.

a. Solve for an initial velocity v of 29. Use the discriminant to decide if the equation has a solution when $h = 20$.

$h = -16t^2 + vt + 6$ **Write vertical motion model.**

$20 = -16t^2 + 29t + 6$ **Substitute 20 for *h* and 29 for *v*.**

$0 = -16t^2 + 29t - 14$ **Rewrite in standard form.**

Evaluate $b^2 - 4ac = 29^2 - 4(-16)(-14)$. The discriminant is -55.

▶ Because the discriminant is negative, the equation has no solution. The stick thrown by your friend *will not* reach the branch.

b. Solve for an initial velocity v of 32. Use the discriminant to decide if the equation has a solution when $h = 20$.

$h = -16t^2 + vt + 6$ **Write vertical motion model.**

$20 = -16t^2 + 32t + 6$ **Substitute 20 for *h* and 32 for *v*.**

$0 = -16t^2 + 32t - 14$ **Rewrite in standard form.**

Evaluate $b^2 - 4ac = 32^2 - 4(-16)(-14)$. The discriminant is 128.

▶ Because the discriminant is positive, the equation has two solutions. The stick that you throw *will* reach the branch.

STUDENT HELP
INTERNET
APPLICATION LINK
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Guided Practice

Vocabulary Check ✓

1. Write the quadratic formula and circle the part that is the discriminant.

Concept Check ✓

2. How can you use the discriminant to tell the number of solutions of $ax^2 + bx + c = 0$ and the number of x-intercepts of the graph of the equation?

Skill Check ✓

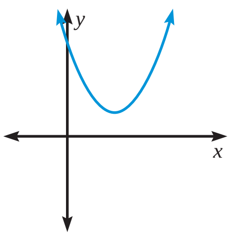
Find the discriminant for the equation. Then tell if the equation has *two solutions, one solution,* or *no real solution.*

3. $3x^2 - 2x + 5 = 0$ **4.** $-3x^2 + 6x - 3 = 0$ **5.** $x^2 - 5x - 10 = 0$

Match the discriminant with the graph.

A. $b^2 - 4ac = 2$ **B.** $b^2 - 4ac = 0$ **C.** $b^2 - 4ac = -3$

6.



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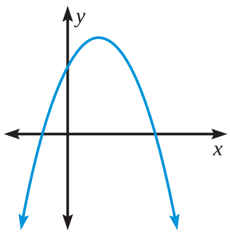

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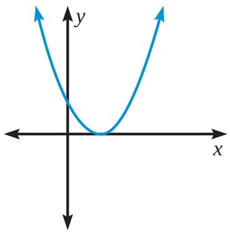


Practice and Applications

Student Help

Extra Practice to help you master skills is on p. 805.

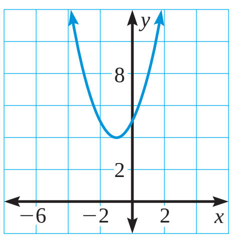
Using the Discriminant **Tell if the equation has *two solutions, one solution,* or *no real solution.***

9. $x^2 - 3x + 2 = 0$ **10.** $2x^2 - 4x + 3 = 0$ **11.** $-3x^2 + 5x - 1 = 0$

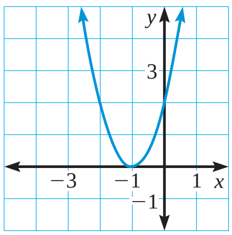
12. $-\frac{1}{3}x^2 + x + 4 = 0$ **13.** $6x^2 - 2x + 4 = 0$ **14.** $3x^2 - 6x + 3 = 0$

Number of x-Intercepts **Use the related equation to find the number of x-intercepts the graph of the function has. Then match the function with its graph.**

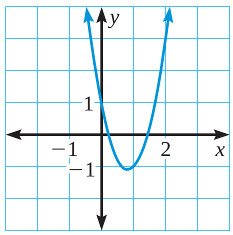
A.



B.



C.



15. $y = 2x^2 + 4x + 2$ **16.** $y = 3x^2 - 5x + 1$ **17.** $y = x^2 + 2x + 5$

Student Help

Homework Help
Example 1: Exs. 9–14
Example 2: Exs. 15–20
Example 3: Exs. 21–23
Example 4: Exs. 24–26

Interpreting the Discriminant **Consider the equation $\frac{1}{2}x^2 + \frac{2}{3}x - 3 = 0$.**

18. Evaluate the discriminant.

19. How many solutions does the equation have?

20. What does the discriminant tell you about the graph of $y = \frac{1}{2}x^2 + \frac{2}{3}x - 3$? Does the graph cross the x-axis?

CHANGING *c*-VALUES **In Exercises 21–23, find values of *c* so that the equation will have two solutions, one solution, and no real solution. Then sketch the graph of the equation for each value of *c* that you chose.**

21. $x^2 + 4x + c = 0$ **22.** $x^2 - 2x + c = 0$ **23.** $2x^2 + 3x + c = 0$

24. **FIREFIGHTING** You see a firefighter aim a fire hose from 4 feet above the ground at a window that is 26 feet above the ground. The equation $h = -0.01d^2 + 1.06d + 4$ models the path of the water when h equals height in feet. Estimate, to the nearest whole number, the possible horizontal distances d (in feet) between the firefighter and the building.

BASKETBALL **In Exercises 25 and 26, use the vertical motion model $h = -16t^2 + vt + s$ (p. 535) and the following information.**
You and a friend are playing basketball. You can jump with an initial velocity of 12 feet per second. You need to jump 2.2 feet to dunk a basketball. Your friend can jump with an initial velocity of 14 feet per second. Your friend needs to jump 3.4 feet to dunk a basketball.

25. Can you dunk the ball? Can your friend? Justify your answers.

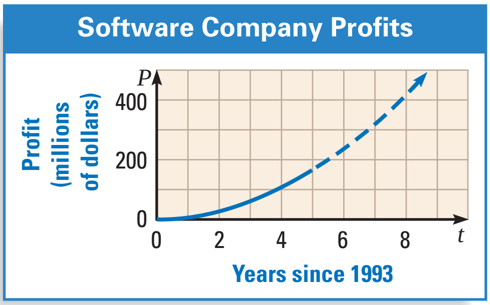
26. Suppose you can jump with an initial velocity of 11.5 feet per second and your friend can jump with an initial velocity of 15.5 feet per second. How, if at all, would this change your answers to Exercise 25?

GOVERNMENT PAYROLL **In Exercises 27 and 28, use a graphing calculator and the following information.**
For a recent 12-year period, the total government payroll (local, state, and federal) in the United States can be modeled by $P = 26t^2 + 1629t + 19{,}958$, where P is the payroll in millions of dollars and t is the number of years since the beginning of the 12-year period. ▶ Source: U.S. Bureau of the Census

27. Use the discriminant to show that the payroll will reach 80 billion dollars in the future according to the model.

28. Use a graphing calculator to find out how many years it will take for the total payroll to reach 80 billion dollars according to the model.

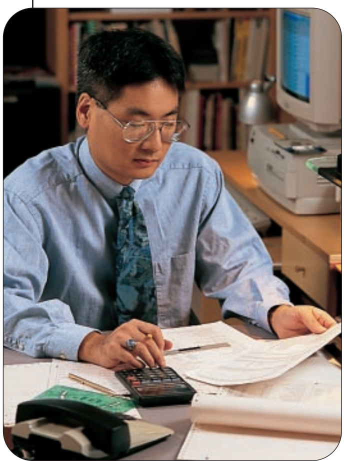
FINANCIAL ANALYSIS **In Exercises 29 and 30, use a graphing calculator and the following information.**
You are a financial analyst for a software company. You have been asked to project the net profit of your company. The net profit of the company from 1993 to 1998 can be modeled by $P = 6.84t^2 - 3.76t + 9.29$ where P is the profit in millions of dollars and t represents the number of years since 1993.



29. Use the model to predict whether the net profit will reach 650 million dollars.

30. Use a graphing calculator to estimate how many years it will take for the company's net profit to reach 475 million dollars according to the model.

FOCUS ON CAREERS



REAL LIFE **FINANCIAL ANALYSTS**
Mathematical models help financial analysts analyze and predict a company's future earnings.

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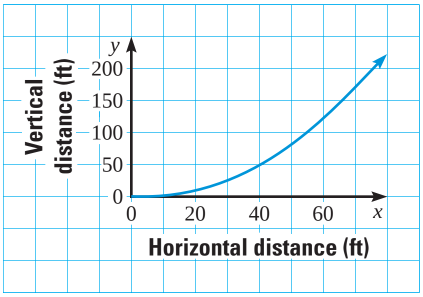
Test Preparation

31. MULTI-STEP PROBLEM You are on a team that is building a roller coaster. The vertical height of the first hill of the roller coaster is supposed to be 220 feet. According to the design, the path of the first hill can be modeled by $y = 0.039x^2 - 0.331x + 1.850$, where y is the vertical height in feet and x is the horizontal distance in feet. The first hill can use only 75 feet of horizontal distance.



a. Use the model to determine whether the first hill will reach a height of 220 feet.

b. What minimum horizontal distance is needed for the first hill to reach a vertical height of 220 feet?

c. *Writing* Can you build the first hill high enough? Explain your findings.

d. CRITICAL THINKING Is the shape of the graph the same as the shape of the hill? Why or why not?

★ Challenge

LOGICAL REASONING Consider the equation $ax^2 + bx + c = 0$ and use the quadratic formula to justify the statement.

32. If $b^2 - 4ac$ is positive, then the equation has two solutions.

33. If $b^2 - 4ac$ is zero, then the equation has one solution.

34. If $b^2 - 4ac$ is negative, then the equation has no real solution.

EXTRA CHALLENGE
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MIXED REVIEW

GRAPHING FUNCTIONS Graph the function. (Review 4.8)

35. $f(x) = -x + 1$ **36.** $f(x) = -6x + 1$ **37.** $f(x) = 3x - 9$

SOLVING INEQUALITIES Solve the inequality. (Review 6.1)

38. $\frac{x}{6} \leq -2$ **39.** $-\frac{x}{3} \geq 15$ **40.** $-12.3x > 86.1$ **41.** $11.2x \leq 134.4$

SKETCHING GRAPHS Sketch the graph of the inequality. (Review 6.5 for 9.7)

42. $3x + y > 9$ **43.** $y - 4x < 0$ **44.** $-2x - y \geq 4$ **45.** $-y - 3x \leq 6$

COMPUTER MODEMS In Exercises 46–48, use the following data which list the prices of several computer modems. (Review 6.6, 6.7)

\$230, \$220, \$170, \$215, \$190, \$200, \$200, \$150, \$170

46. Use a stem-and-leaf plot to order the data from least to greatest.

47. Make a box-and-whisker plot of the data.

48. Use your results to describe the prices.

QUIZ 2

Self-Test for Lessons 9.4–9.6

SOLVING GRAPHICALLY **Solve the equation graphically. Check the results algebraically.** (Lesson 9.4)

1. $x^2 - 3x = 10$ **2.** $x^2 - 12x = -36$ **3.** $3x^2 + 12x = -9$

USING THE FORMULA **Use the quadratic formula to solve the equation.** (Lesson 9.5)

4. $x^2 + 6x + 9 = 0$ **5.** $2x^2 + 13x + 6 = 0$ **6.** $-x^2 + 6x + 16 = 0$

7. $-2x^2 + 7x - 6 = 0$ **8** $-3x^2 - 5x + 12 = 0$ **9** $5x^2 + 8x + 3 = 0$

USING THE DISCRIMINANT **Tell if the equation has *two solutions, one solution,* or *no real solution.*** (Lesson 9.6)

10. $x^2 - 15x + 56 = 0$ **11.** $x^2 + 8x + 16 = 0$ **12.** $x^2 - 3x + 4 = 0$

13. **THROWING A BASEBALL** Your friend is standing on a 45-foot balcony. He asks you to throw a baseball up to him. You throw the baseball with an initial upward velocity of 50 feet per second. Assuming that you released the baseball 6 feet above the ground, did it reach your friend? Explain. (Lesson 9.6)

MATH & History — History of the Quadratic Formula

APPLICATION LINK www.mcdougallittell.com

THEN

OVER 3700 YEARS AGO, in Mesopotamia, math exercises using quadratic equations were written on cuneiform tablets. A cuneiform tablet contains the math exercise:

A region consists of two nonoverlapping squares of total area 1000. The side of one square is two-thirds of the side of the other square, diminished by ten. What are the sides of the two squares?

1. Use the equations $x^2 + y^2 = 1000$ and $y = \frac{2}{3}x - 10$ to write a quadratic equation to find the length of a side x of the figure.

2. Use the discriminant to find the number of solutions. How many reasonable solutions are there?

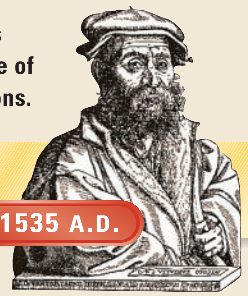
3. Solve the quadratic equation. What are the lengths of both sides of the figure?

NOW

TODAY, there are many modern applications of quadratic equations. NASA relies on aircraft that fly in a parabolic arc in which to train astronauts.

Cuneiform tablets record knowledge of quadratic equations.

c. 1775 B.C.



1535 A.D.

Mathematician Niccolo Tartaglia solves cubic equations.

Astronaut Mary Cleave trains in a low gravity environment.

1985