4.7 Solving Linear Equations Using Graphs

What you should learn

GOAL 1 Solve a linear equation graphically.

GOAL 2 Use a graph to solve **real-life** problems, such as estimating when there will be 2.5 million registered nurses in **Example 4.**

Why you should learn it

▼ To solve **real-life** problems such as finding production costs of a small business in **Ex. 44.**

GOAL 1 SOLVING LINEAR EQUATIONS GRAPHICALLY

Up to this point, you have been solving linear equations and checking the solutions *algebraically* (by using algebra). In this lesson you will learn how to solve linear equations and check solutions *graphically* (by using a graph).

STEPS FOR SOLVING LINEAR EQUATIONS GRAPHICALLY

The solution of a linear equation can be found graphically using the following steps:

STEP 1 Write the equation in the form $ax + b = 0$.

STEP 2 Write the related function $y = ax + b$.

STEP 3 Graph the equation $y = ax + b$.

The solution of $ax + b = 0$ is the x-intercept of $y = ax + b$.

EXAMPLE 1 *Using a Graphical Check for a Solution*

Solve $2x - 5 = 1$ algebraically. Check your solution graphically.

SOLUTION

$2x - 5 = 1$ **Write original equation.**

$2x = 6$ **Add 5 to each side.**

$x = 3$ **Divide each side by 2.**

✓**CHECK** Solve $2x - 5 = 1$ graphically as a check.

1 Write the equation in the form $ax + b = 0$.

$2x - 5 = 1$

$2x - 6 = 0$

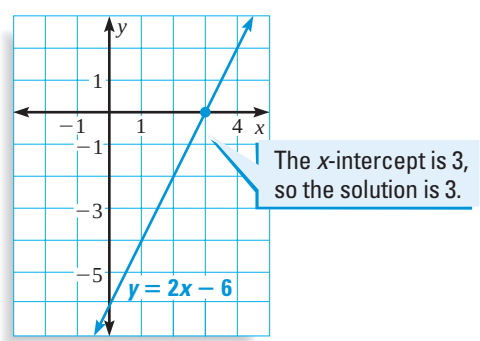
2 Write the related function in two variables.

$y = 2x - 6$

3 Graph the related function

$y = 2x - 6$.

Notice that the x-intercept is 3, which is the solution of $2x - 6 = 0$. This confirms that 3 is the solution of $2x - 5 = 1$.



STUDENT HELP

Study Tip

When you know two ways to solve a problem, it's a good idea to use one method to get a solution and the other method to check the solution. This is done in Examples 1 and 2.

EXAMPLE 2 Solving an Equation Graphically

Solve $\frac{4}{3}x + 3 = 2x$ graphically. Check your solution algebraically.

SOLUTION

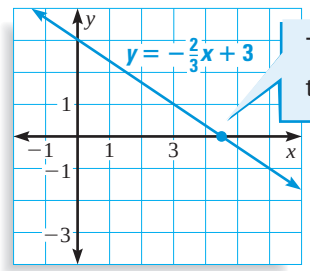
1 Write the equation in the form $ax + b = 0$.

$\frac{4}{3}x + 3 = 2x$ **Write original equation.**

$-\frac{2}{3}x + 3 = 0$ **Subtract $2x$ from each side.**

2 Write the related function $y = -\frac{2}{3}x + 3$.

3 Graph the equation $y = -\frac{2}{3}x + 3$. The x-intercept appears to be $\frac{9}{2}$.



The x-intercept is $\frac{9}{2}$, so the solution is $\frac{9}{2}$.

✓CHECK Use substitution.

$\frac{4}{3}x + 3 = 2x$

$\frac{4}{3}\left(\frac{9}{2}\right) + 3 \stackrel{?}{=} 2\left(\frac{9}{2}\right)$

$6 + 3 = 9$ ← **True statement**

▶ The solution of $\frac{4}{3}x + 3 = 2x$ is $\frac{9}{2}$.

EXAMPLE 3 Using a Graphing Calculator

Use a graphing calculator to approximate the solution of the linear equation $2.65(4x - 9) = -8.85 + 7.6x$.

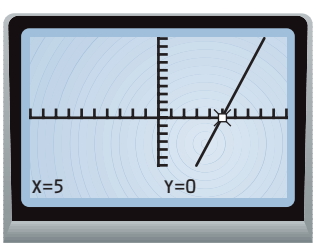
SOLUTION

When you graph an equation using a graphing calculator, you do not need to simplify the equation before entering it.

$2.65(4x - 9) + 8.85 - 7.6x = 0$ **Rewrite equation so one side is 0.**

Then use a graphing calculator (or a computer) to graph the related function

$y = 2.65(4x - 9) + 8.85 - 7.6x.$



▶ The x-intercept appears to be 5. Check that 5 is a solution of the original equation.

FOCUS ON CAREERS

NURSES Registered nurses are often responsible for measuring vital signs and giving correct doses of medication.

CAREER LINK
www.mcdougallittell.com

GOAL 2 APPROXIMATING SOLUTIONS IN REAL LIFE

Example 4 shows how to apply the graphical approach of Examples 1–3 to a real-life problem. Example 5 shows another graphical approach to this problem.

EXAMPLE 4 *Approximating a Real-Life Solution*

NURSES Based on data from 1980 to 1997, a model for the number n (in millions) of registered nurses in the United States is $n = 0.055t + 1.26$, where t is the number of years since 1980. According to this model, in what year will the United States have 2.5 million registered nurses? ▶ Source: U.S. Public Health Service

SOLUTION

Substitute 2.5 for n in the linear model. You can answer the question by solving the resulting linear equation, $2.5 = 0.055t + 1.26$.

Write the equation in the form $ax + b = 0$.

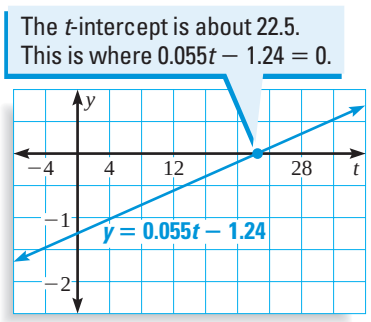
$$2.5 = 0.055t + 1.26$$

$$0 = 0.055t - 1.24$$

Graph the related function.

$$y = 0.055t - 1.24$$

The t-intercept is about 22.5.



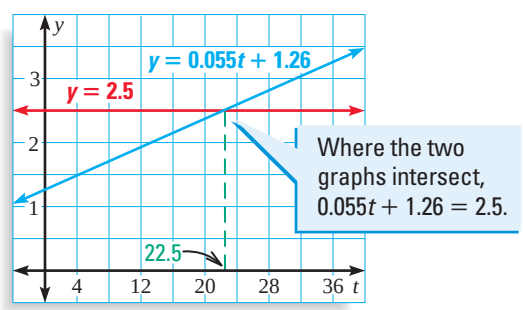
▶ Because t is the number of years since 1980, you can estimate that there will be 2.5 million registered nurses sometime in 2002.

EXAMPLE 5 *Solving by Graphing Both Sides*

Another graphical approach that you can use to solve the equation $2.5 = 0.055t + 1.26$ in Example 4 is to write and graph a function for each side of the equation.

$2.5 = 0.055t + 1.26$	**Original equation.**
$y = 2.5$	**Function for left side of equation.**
$y = 0.055t + 1.26$	**Function for right side of equation.**

Then graph both functions in the same coordinate plane. The graphs cross where the value of y for both functions is about 2.5.



▶ The t-coordinate where the graphs intersect is about 22.5, the same value found by the method in Example 4.

STUDENT HELP

HOMEWORK HELP
Visit our Web site www.mcdougallittell.com for extra examples.

GUIDED PRACTICE

Vocabulary Check ✓

1. Explain how to solve a linear equation in one variable *graphically*.

Concept Check ✓

2. In Example 2, explain why the x-intercept of the line $y = -\frac{2}{3}x + 3$ is the solution of the equation $0 = -\frac{2}{3}x + 3$.

Solve the equation graphically. Check your answer algebraically.

3. $x - 3 = 7$ **4.** $2 - x = -5$ **5.** $5x + 6 = -9$

Skill Check ✓

In Exercises 6–10, match the one-variable equation with its related function.

6. $2x = 10$

7. $2x - 10 = 6$

8. $5x + 4 = 3x + 5$

9. $-4x - 6 = -2x + 5$

10. $19x + 3 = x$

A. $y = 2x - 16$

B. $y = -2x - 11$

C. $y = 2x - 10$

D. $y = 2x - 1$

E. $y = 18x + 3$

PRACTICE AND APPLICATIONS

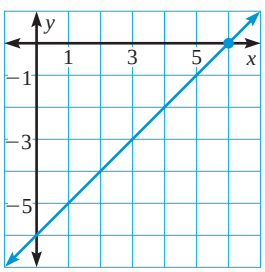
STUDENT HELP

Extra Practice to help you master skills is on p. 800.

MATCHING EQUATIONS AND GRAPHS **Match the equation with the line whose *x*-intercept is the solution of the equation.**

A. $2x - 4 = 0$ **B.** $x + 6 = 0$ **C.** $x - 6 = 0$

11.



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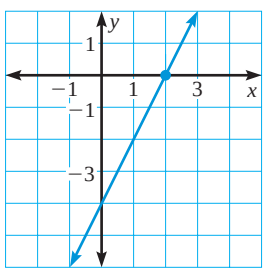

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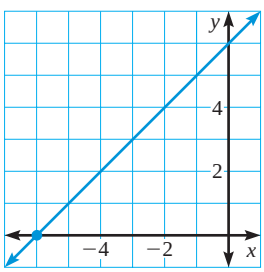


RELATED FUNCTIONS **Write the equation in the form $ax + b = 0$. Then write the related function $y = ax + b$.**

14. $7x - 3 = 3$ **15.** $6 - 4x = 13$ **16.** $9 + 5x = 19$

17. $12x + 5 = 8x$ **18.** $-4x - 5 = 3x + 8$ **19.** $6 - \frac{4}{7}x = 13 + \frac{3}{7}x$

STUDENT HELP

HOMEWORK HELP

Example 1: Exs. 11–28
Example 2: Exs. 11–37
Example 3: Exs. 38–43
Example 4: Exs. 44, 45
Example 5: Exs. 44, 45

SOLVING EQUATIONS **Solve the equation algebraically. Check your solution graphically.**

20. $5x + 3 = -2$ **21.** $-3x + 11 = 2$ **22.** $-x = -2$

23. $2x + 7 = 10$ **24.** $\frac{1}{4}x + 1 = -\frac{1}{2}$ **25.** $\frac{1}{2}x + 5 = 3$

26. $-\frac{2}{3}x - 6 = -4$ **27.** $-\frac{3}{4}x + 3 = \frac{9}{4}$ **28.** $\frac{2}{3}x - \frac{2}{3} = 2$

SOLVING GRAPHICALLY **Solve the equation graphically. Check your solution algebraically.**

29. $2x - 7 = -5$ **30.** $5x - 2 = 8$ **31.** $-4x = -12$

32. $6x + 9 = 3$ **33.** $7 - 9x = -11$ **34.** $3x + 10 = -2x$

35. $-5x + 4 = 12 - 3x$ **36.** $\frac{1}{3}x + 1 = 4$ **37.** $\frac{1}{2}x + 5 = 3$

SOLVING GRAPHICALLY **Use a graphing calculator to find the solution of the equation. Check your solution algebraically.**

38. $4(x + 2) = 3(x + 5)$ **39.** $-1.6(1.5x + 7.5) = 0.6(6x + 30)$

40. $\frac{1}{2}(x + 7) = \frac{1}{3}(10x + 2)$ **41.** $0.7(3x - 20) = 22 - 3.9x$

42. $\frac{3}{4}(4x - 15) = -\frac{3}{2}(4x - 18)$ **43.** $\frac{1}{3}(10x + 27) = -\frac{1}{5}\left(\frac{55}{3}x + 25\right)$

44. **PRODUCTION COSTS** Chai has a small business making decorated hats. She calculates her monthly cost y of producing x hats using the function $y = 1.9x + 350$. In January, her cost was \$458.30. How many hats did she make that month? Solve algebraically and graphically.

STUDENT HELP

APPLICATION LINK

Visit our Web site www.mcdougallittell.com for more information about tourism in Ex. 45.

45. **TOURISM** Based on data from 1991–1995, a model for the annual number of visitors v from the United States to Europe is

$$v = 559{,}100t + 6{,}423{,}000$$

where t is the number of years since 1991. According to this model, in what year will the number of visitors to Europe reach 11,000,000?

▶ Source: U.S. Bureau of the Census

Venice, Italy

VIEWING HABITS **In Exercises 46–49, use the models below. They can be used to estimate the average number of hours h per week that a household in the United States watches television programs. In all three models, t is the number of years since 1985.** ▶ Source: Media Dynamics, Inc.

Hours spent watching television: $h = 0.57t + 54.85$

Hours spent watching talk shows: $h = 0.35t + 4.06$

Hours spent watching game shows: $h = -0.2t + 3.4$

46. According to the model, in what year will a household watch television an average of 70 hours per week?

47. According to the model, in what year will a household watch an average of 10 hours of talk shows per week?

48. According to the model, in what year will the average number of hours per week that a household watches game shows be zero?

49. *Writing* Did you solve Exercises 46–48 algebraically or graphically? Give a reason for your choice.

Test Preparation

50. MULTI-STEP PROBLEM The U.S. Bureau of Labor Statistics projects job growth by using three models to make low, moderate, and high estimates. The equations below model the projected number of auto mechanics m from 1994 to 2005. In all three models, t is the number of years since 1994.

Model 1: $m = 13{,}272t + 736{,}000$

Model 2: $m = 9455t + 736{,}000$

Model 3: $m = 11{,}455t + 736{,}000$

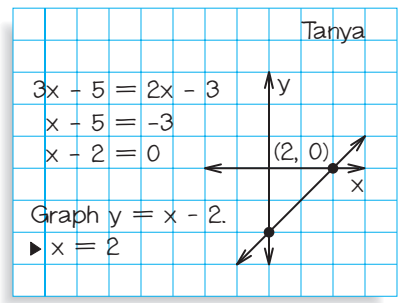
a. For each model, write an equation that enables you to predict the year in which the number of auto mechanics will reach 800,000.

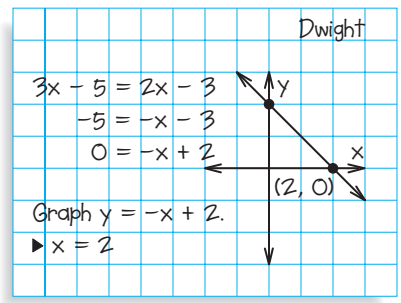
b. In the same coordinate plane, graph the related function for each equation that you found in part (a). According to each model, in what year will the number of auto mechanics reach 800,000?

c. VISUAL THINKING Which model gives a high estimate of the number of mechanics? a low estimate? How can you tell this from the graphs of the models?

Challenge

51. COMPARING METHODS Below, Tanya and Dwight solve $3x - 5 = 2x - 3$ graphically. Does each method give a correct solution? Explain.





EXTRA CHALLENGE
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MIXED REVIEW

CHECKING SOLUTIONS **Is the given number a solution of the inequality?** (Review 1.4)

52. $x + 3 > 5$; 7 **53.** $-2 - x \leq -9$; -7 **54.** $3x - 11 \geq 2$; 4

55. $8 + 5x < -2$; -2 **56.** $4x - 1 > 10$; 3 **57.** $2x + 4 \leq 3x - 3$; 5

DISTRIBUTIVE PROPERTY **Apply the distributive property.** (Review 2.6)

58. $4(x + 7)$ **59.** $-8(8 - y)$ **60.** $(3 - b)9$ **61.** $(q + 4)(-3q)$

62. $10(6x + 2)$ **63.** $-2(d - 5)$ **64.** $(7 - 2a)(4a)$ **65.** $-5w(-3 + 2w)$

66. FLOWER SERVICE You start a daily flower club and charge \$10 to join and \$.50 per day. Every day each member of the club gets a fresh flower. Let n represent the number of club members and let I represent your income for four weeks. A model for the situation is $I = [10 + 4 \cdot 7(0.5)]n$. Write an input-output table that shows your income for 2, 4, 6, 8, and 10 club members. (Review 1.7)