CHAPTER 10

Chapter Summary

WHAT did you learn?	*WHY did you learn it?*
Classify a polynomial by degree and by number of terms. (10.1)	Understand how to use polynomials to model real-life situations. (p. 578)
Add and subtract polynomials. (10.1)	Represent the growth in the number of Internet sites. (p. 579)
Multiply polynomials	
• using the FOIL pattern. (10.2)	Evaluate a polynomial expression to find the area of a window. (p. 586)
• using special product patterns. (10.3)	Find products of some polynomials more easily. (p. 591)
Solve polynomial equations in factored form. (10.4)	Find the dimensions of an archway. (p. 599)
Factor quadratic trinomials. (10.5–10.6)	Find the time it takes a diver to reach the water. (p. 613)
Factor a polynomial	
• using special product patterns. (10.7)	Find the size of rope needed to lift a safe. (p. 621)
• using the distributive property. (10.8)	Find the dimensions of a box. (p. 627)
Solve polynomial equations by factoring. (10.5–10.8)	Apply factoring skills to write models for real-life situations. (pp. 604–632)

How does Chapter 10 fit into the BIGGER PICTURE of algebra?

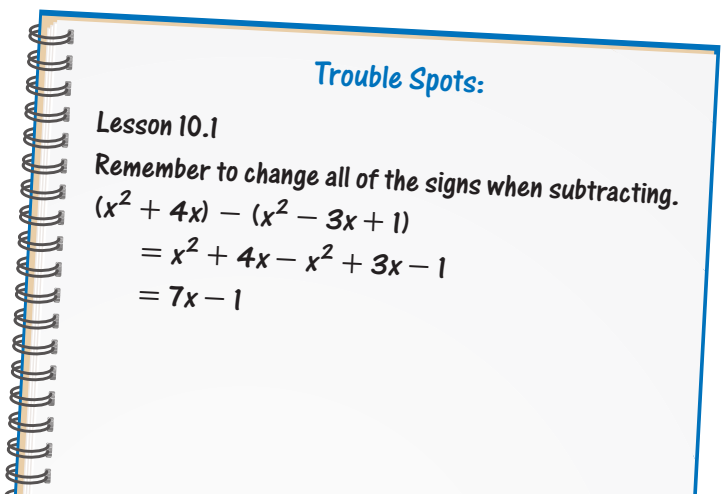
Polynomials can model a great variety of real-life situations. You were already familiar with three types of polynomial models: $y = a$ (constant model), $y = ax + b$ (linear model), and $y = ax^2 + bx + c$ (quadratic model).

In this chapter you learned how to add, subtract and multiply polynomials. You also learned how to "undo" multiplication by a process called factoring. This chapter has many connections with the mathematics you studied in Chapter 9. For instance, in this chapter you learned how to use factoring to solve quadratic equations.

STUDY STRATEGY

How did identifying trouble spots help you review?

One of your notes about trouble spots, using the **Study Strategy** on p. 574, may look like this.



Trouble Spots:

Lesson 10.1

Remember to change all of the signs when subtracting.

$(x^2 + 4x) - (x^2 - 3x + 1)$

$= x^2 + 4x - x^2 + 3x - 1$

$= 7x - 1$

CHAPTER 10

Chapter Review

VOCABULARY

- polynomial, p. 576
- standard form of a polynomial, p. 576
- degree of a term, p. 576
- degree of a polynomial, p. 576
- leading coefficient, p. 576
- monomial, p. 576
- binomial, p. 576
- trinomial, p. 576
- FOIL pattern, p. 585
- factored form of a polynomial, p. 597
- zero-product property, p. 597
- factor a quadratic expression, p. 604
- prime factor, p. 625
- factor a polynomial completely, p. 625

10.1 Adding and Subtracting Polynomials

Examples on pp. 576–579

EXAMPLES To add or subtract polynomials, add or subtract like terms.

Horizontal Format

$$(4x^3 + 6x - 8) - (-x^2 + 7x - 2)$$
$$= 4x^3 + 6x - 8 + x^2 - 7x + 2$$
$$= 4x^3 + x^2 - x - 6$$

Vertical Format

$$\begin{array}{rr} & -2x^3 - 4x^2 - x + 5 \\ & 3x^3 + 2x^2 - 4x + 9 \\ + & -x^3 + 5x^2 - x - 1 \\ \hline & 3x^2 - 6x + 13 \end{array}$$

Find the sum or the difference.

1. $(-x^2 + x + 2) + (3x^2 + 4x + 5)$
2. $(x^2 + 3x - 1) - (4x^2 - 5x + 6)$
3. $(x^3 + 5x^2 - 4x) - (3x^2 - 6x + 2)$
4. $(4x^3 + x^2 - 1) + (2 - x - x^2)$

10.2 Multiplying Polynomials

Examples on pp. 584–586

EXAMPLES To multiply polynomials, use the distributive property or the FOIL pattern.

a. $(3x + 2)(5x^2 - 4x + 1) = 5x^2(3x + 2) - 4x(3x + 2) + 1(3x + 2)$
$= 15x^3 + 10x^2 - 12x^2 - 8x + 3x + 2$
$= 15x^3 - 2x^2 - 5x + 2$

b. $(4x + 5)(-3x - 6) = -12x^2 - 24x - 15x - 30$ (F, O, I, L)
$= -12x^2 - 39x - 30$

Find the product.

5. $(-x)(8x^3 - 12x^2)$
6. $4x^3(-x^2 + 2x - 7)$
7. $(x + 3)(x + 11)$
8. $(7x - 1)(5x + 2)$
9. $(x + 5)(2x^2 + x - 10)$
10. $(4x^2 + 9x)(-x^2 - 3)$

10.3 Special Products of Polynomials

Examples on pp. 590–592

EXAMPLES Use special product patterns to multiply some polynomials.

$(a + b)(a - b) = a^2 - b^2$

$(3x + 7)(3x - 7) = (3x)^2 - 7^2$

$= 9x^2 - 49$

$(a + b)^2 = a^2 + 2ab + b^2$

$(5t + 4)^2 = (5t)^2 + 2(5t)(4) + 4^2$

$= 25t^2 + 40t + 16$

Find the product.

11. $(x + 15)(x - 15)$ **12.** $(5x - 2)(5x + 2)$ **13.** $(x + 2)^2$ **14.** $(7m - 6)^2$

10.4 Solving Polynomial Equations in Factored Form

Examples on pp. 597–599

EXAMPLE Solve the equation $(x + 1)(x - 5) = 0$.

$(x + 1) = 0$ or $(x - 5) = 0$ Use zero-product property.

$x + 1 = 0$ $x - 5 = 0$

$x = -1$ $x = 5$

▶ The solutions are -1 and 5. Check these in the original equation.

Solve the equation.

15. $(x + 1)(x + 10) = 0$ **16.** $(x - 3)(x - 2) = 0$

17. $(x + 9)^2 = 0$ **18.** $(3x + 6)(4x - 1)(x - 4) = 0$

10.5 Factoring $x^2 + bx + c$

Examples on pp. 604–606

EXAMPLE Solve the equation $x^2 - 9x = -14$ by factoring.

$x^2 - 9x = -14$ Write original equation.

$x^2 - 9x + 14 = 0$ Write in standard form.

$(x - 2)(x - 7) = 0$ Factor.

$(x - 2) = 0$ or $(x - 7) = 0$ Use zero-product property.

$x - 2 = 0$ $x - 7 = 0$

$x = 2$ $x = 7$

▶ The solutions are 2 and 7. Check these in the original equation.

Solve the equation by factoring.

19. $x^2 - 21x + 108 = 0$ **20.** $x^2 + 10x - 200 = 0$ **21.** $x^2 + 26x = -169$

10.6 FACTORING $ax^2 + bx + c$

Examples on pp. 611–613

EXAMPLE To factor $3x^2 - x - 2$, test the possible factors of a (1 and 3) and of c (-2 and 1 or 2 and -1).

$(x - 2)(3x + 1) = 3x^2 - 5x - 2$ **Not correct**

$(x + 1)(3x - 2) = 3x^2 + x - 2$ **Not correct**

$(x + 2)(3x - 1) = 3x^2 + 5x - 2$ **Not correct**

$(x - 1)(3x + 2) = 3x^2 - x - 2$ **Correct**

Factor the trinomial.

22. $12x^2 + 7x + 1$ **23.** $2x^2 + 5x - 12$ **24.** $6x^2 + 4x - 10$ **25.** $4x^2 - 12x + 9$

10.7 FACTORING SPECIAL PRODUCTS

Examples on pp. 619–621

EXAMPLES Factor using special product patterns to solve the equations.

$a^2 - b^2 = (a + b)(a - b)$

$4x^2 - 64 = 0$

$(2x + 8)(2x - 8) = 0$

$2x + 8 = 0$ or $2x - 8 = 0$

▶ The solutions are -4 and 4.

$a^2 - 2ab + b^2 = (a - b)^2$

$x^2 - 4x + 4 = 0$

$x^2 - 2(x)(2) + 2^2 = 0$

$(x - 2)^2 = 0$

$x - 2 = 0$

▶ The solution is 2.

Use factoring to solve the equation.

26. $100x^2 - 121 = 0$ **27.** $16x^2 + 24x + 9 = 0$ **28.** $9x^2 - 18x + 9 = 0$

10.8 FACTORING USING THE DISTRIBUTIVE PROPERTY

Examples on pp. 625–628

EXAMPLES Use the distributive property to factor out common variable factors.

$3x^3 - 27x = 3x(x^2 - 9)$

$= 3x(x + 3)(x - 3)$

$x^4 - 4x^3 + x^2 - 4x = (x^4 + x^2) + (-4x^3 - 4x)$

$= x^2(x^2 + 1) - 4x(x^2 + 1)$

$= (x^2 - 4x)(x^2 + 1)$

$= x(x - 4)(x^2 + 1)$

Factor the expression completely.

29. $-2x^5 - 2x^4 + 4x^3$ **30.** $2x^4 - 32x^2$

31. $3x^3 + x^2 + 15x + 5$ **32.** $x^3 + 3x^2 - 4x - 12$

CHAPTER 10

Chapter Test

Use a vertical format or a horizontal format to add or subtract.

1. $(x^2 + 4x - 1) + (5x^2 + 2)$

2. $(5t^2 - 9t + 1) - (8t + 13)$

3. $(7n^3 + 2n^2 - n - 4) - (4n^3 - 3n^2 + 8)$

4. $(x^4 + 6x^2 + 7) + (2x^4 - 3x^2 + 1)$

Find the product.

5. $(x + 3)(2x + 3)$

6. $(9x - 1)(7x + 4)$

7. $(w - 6)(4w^2 + w - 7)$

8. $(5t^3 + 2)(4t^2 + 8t - 7)$

9. $(3z^3 - 5z^2 + 8)(z + 2)$

10. $(4x + 1)\left(\frac{1}{2}x - \frac{3}{8}\right)$

11. $(x - 12)^2$

12. $(7x + 2)^2$

13. $(8x + 3)(8x - 3)$

Use the zero-product property to solve the equation.

14. $(6x - 5)(x + 2) = 0$

15. $(x + 8)^2 = 0$

16. $(4x + 3)(x - 1)(3x + 9) = 0$

Find the *x*-intercepts and the vertex of the graph. Then sketch the graph.

17. $y = (x + 1)(x - 5)$

18. $y = (x + 2)(x + 6)$

19. $y = (-x - 4)(x + 7)$

Solve the equation by factoring.

20. $x^2 + 13x + 30 = 0$

21. $x^2 - 19x + 84 = 0$

22. $x^2 - 34x - 240 = 0$

23. $2x^2 + 15x - 108 = 0$

24. $9x^2 - 9x = 28$

25. $18x^2 - 57x = -35$

Factor the expression.

26. $x^2 - 196$

27. $16x^2 - 36$

28. $128 - 50x^2$

29. $x^2 - 6x + 9$

30. $4x^2 + 44x + 121$

31. $2x^2 + 28x + 98$

32. $9t^2 - 54$

33. $4x^2 + 38x + 34$

34. $x^3 + 2x^2 - 16x - 32$

Solve the equation by a method of your choice.

35. $x^2 - 60 = -11$

36. $2x^2 + 15x - 8 = 0$

37. $x^2 - 13x = -40$

38. $x(x - 16) = 0$

39. $12x^2 + 3x = 0$

40. $8x^2 + 6x = 5$

41. $12x^2 + 17x = 7$

42. $81x^2 - 6 = 30$

43. $16x^2 - 34x - 15 = 0$

44. **TIC-TAC-TOE** You are making a tic-tac-toe board. Each square will have sides of x inches. The board will have a border with a width of 1 inch. Draw a diagram and label the dimensions. Write a polynomial expression for the area of the board.

45. **ROOM DIMENSIONS** A room's length is 3 feet less than twice its width. The area of the room is 135 square feet. What are the room's dimensions?

46. **KICKING A FOOTBALL** You kick a football into the air. The vertical component of this motion can be modeled by the vertical motion model. Suppose the football has an initial upward velocity of 31 feet per second. The ball makes contact with your foot about 2 feet above the ground. Find the time t (in seconds) for the football to reach the ground. Use the vertical motion model on page 621.