

3.6

Solving Decimal Equations

GOAL 1 SOLVING DECIMAL EQUATIONS

What you should learn

GOAL 1 Find exact and approximate solutions of equations that contain decimals.

GOAL 2 Solve real-life problems that use decimals such as calculating sales tax in Example 5.

Why you should learn it

▼ To solve real-life problems, such as finding the expansion gap for a bridge in Exs. 56–58.



So far in this chapter you have worked with equations that have exact solutions. In real life, exact solutions are not always practical and you must use rounded solutions. Rounded solutions can lead to **round-off error**, as in Example 1.

EXAMPLE 1 Rounding for a Practical Answer

Three people want to share equally in the cost of a pizza. The pizza costs \$12.89. You can find each person's share by solving $3x = 12.89$.

$$3x = 12.89$$

Write original equation.

$$x = 4.29666\dots$$

Exact answer is a repeating decimal.

$$x \approx 4.30$$

Round to nearest cent.

▶ The exact answer is not practical, but three times the rounded answer does not correspond to the price of the pizza. It is one cent too much due to round-off error.

EXAMPLE 2 Rounding for the Final Answer

Solve $-38x - 39 = 118$. Round to the nearest hundredth.

SOLUTION

$$-38x - 39 = 118$$

Write original equation.

$$-38x = 157$$

Add 39 to each side.

$$x = \frac{157}{-38}$$

Divide each side by -38 .

$$x \approx -4.131578947$$

Use a calculator.

$$x \approx -4.13$$

Round to nearest hundredth.

▶ The solution is approximately -4.13 .

✓ **CHECK** When you substitute a rounded answer into the original equation, the two sides of the equation may not be exactly equal, but they should be almost equal.

$$-38x - 39 = 118$$

Write original equation.

$$-38(-4.13) - 39 \stackrel{?}{=} 118$$

Substitute -4.13 for x .

$$117.94 \approx 118$$

Rounded answer is reasonable.

STUDENT HELP**Study Tip**

When rounding, think about the situation. For example, dollar amounts are usually rounded to hundredths. If no situation is given, use the same number of decimal places in the answer as are used in the original equation.

EXAMPLE 3 *Original Equation Involving Decimals*

Solve $3.58x - 37.40 = 0.23x + 8.32$. Round to the nearest hundredth.

SOLUTION

Use the same methods you learned for solving equations without decimals.

$$3.58x - 37.40 = 0.23x + 8.32$$

Write original equation.

$$3.35x - 37.40 = 8.32$$

Subtract $0.23x$ from each side.

$$3.35x = 45.72$$

Add 37.40 to each side.

$$x = \frac{45.72}{3.35}$$

Divide each side by 3.35 .

$$x \approx 13.64776119$$

Use a calculator.

$$x \approx 13.65$$

Round to nearest hundredth.

▶ The solution is approximately 13.65.

✓CHECK

$$3.58x - 37.40 = 0.23x + 8.32$$

Write original equation.

$$3.58(\mathbf{13.65}) - 37.40 \stackrel{?}{=} 0.23(\mathbf{13.65}) + 8.32$$

Substitute 13.65 for each x .

$$11.467 \approx 11.4595$$

Rounded answer is reasonable.

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An equation involving decimals can be rewritten as an equivalent equation with only integer terms and coefficients.

EXAMPLE 4 *Changing Decimal Coefficients to Integers*

Solve $4.5 - 7.2x = 3.4x - 49.5$. Round to the nearest tenth.

SOLUTION

Because the coefficients and constant terms each have only one decimal place, you can rewrite the equation without decimals by multiplying each side by 10.

$$4.5 - 7.2x = 3.4x - 49.5$$

Write original equation.

$$45 - 72x = 34x - 495$$

Multiply each side by 10.

$$45 = 106x - 495$$

Add $72x$ to each side.

$$540 = 106x$$

Add 495 to each side.

$$\frac{540}{106} = x$$

Divide each side by 106.

$$5.094339623 \approx x$$

Use a calculator.

$$5.1 \approx x$$

Round to nearest tenth.

▶ The solution is approximately 5.1. Check this in the original equation.

GOAL 2 SOLVING PROBLEMS WITH DECIMALS



EXAMPLE 5 Using a Verbal Model

You are shopping for earrings. The sales tax is 5%. You have a total of \$18.37 to spend. What is your price limit for the earrings?

SOLUTION

The total cost is your price limit plus the tax figured on the price limit.



VERBAL MODEL	Price limit + Sales tax rate • Price limit = Total cost
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LABELS	Price limit = x (dollars)
	Sales tax rate = 0.05 (no units)
	Total cost = 18.37 (dollars)

ALGEBRAIC MODEL

$$x + 0.05 \cdot x = 18.37$$

$$1.05x = 18.37$$

$$x = \frac{18.37}{1.05}$$

$$x \approx 17.4952381$$

$$x \approx 17.49$$

Write algebraic model.

Combine like terms.

Divide each side by 1.05.

Use a calculator.

Round down.

▶ Because you have a limited amount to spend, the answer is rounded *down* to \$17.49. If you round up to \$17.50, you will be a penny short because the sales tax is rounded up to \$.88.

ACTIVITY

Developing Concepts

Investigating Round-Off Error

- 1 Work with a partner. Each of you will solve the equation below in a different way.

$$18.42x - 12.75 = (5.32x - 6.81)3.46$$

Student 1: Round to the nearest hundredth in intermediate steps.

Student 2: Round to the nearest hundredth only in the last step.

- 2 Compare the answers you get using both methods. Which method do you think is more accurate? Why?

When solving an equation that has decimals, you need to realize that rounding in the intermediate steps can increase the round-off error.

STUDENT HELP

Skills Review

For help with finding a percent of a number, see p. 786.

GUIDED PRACTICE

Vocabulary Check ✓

1. What special notation do you need to use when you are giving an approximate answer?

Concept Check ✓

2. Look back at Example 3. What number would you multiply the equation by to rewrite the equation without decimals?

 **FIELD TRIP** In Exercises 3 and 4, school buses that hold 68 people will be used to transport 377 students and 65 teachers.

3. Write and solve an equation to find the number of buses needed.

4. Is the exact answer in Exercise 3 practical? Explain.

Skill Check ✓

Round to the nearest tenth.

5. 23.4459

6. 108.2135

7. -13.8953

8. 62.9788

Solve the equation. Round the result to the nearest hundredth. Check the rounded solution.

9. $2.2x = 15$

10. $14 - 9x = 37$

11. $2(3b - 14) = -9$

12. $2.69 - 3.64x = 8.37 + 23.78x$

13.  **BUYING A SWEATSHIRT** You have \$35.72 to spend for a sweatshirt. The sales tax is 7%. What is your price limit for the sweatshirt?

PRACTICE AND APPLICATIONS

STUDENT HELP

▶ **Extra Practice**
to help you master
skills is on p. 799.

ROUNDING Perform any indicated operation. Round the result to the nearest tenth and then to the nearest hundredth.

14. -35.1923

15. $5.34(6.79)$

16. $-7.895 + 4.929$

17. $47.0362 - 39.7204$

18. $5.349 \div 46.597$

19. $-25.349(-1.369)$

SOLVING AND CHECKING Solve the equation. Round the result to the nearest hundredth. Check the rounded solution.

20. $13x - 7 = 27$

21. $18 - 3y = 5$

22. $-7n + 17 = -6$

23. $38 = -14 + 9a$

24. $47 = 28 - 12x$

25. $14r + 8 = 32$

26. $358 = 39c - 17$

27. $37 - 58b = 204$

28. $3(31 - 12t) = 82$

29. $4(-7y + 13) = 49$

30. $2(-5a + 7) = -a$

31. $-(d - 3) = 2(3d + 1)$

SOLVING EQUATIONS Solve the equation. Round the result to the nearest hundredth.

32. $12.67 + 42.35x = 5.34x + 26.58$

33. $4.65x - 4.79 = 13.57 - 6.84x$

34. $7.45x - 8.81 = 5.29 + 9.47x$

35. $39.21x + 2.65 = -31.68 + 42.03x$

36. $5.86x - 31.94 = 27.51x - 3.21$

37. $-2(4.36 - 6.92x) = 9.27x + 3.87$

38. $6.1(3.1 + 2.5x) = 15.3x - 3.9$

39. $4.21x + 5.39 = 12.07(2.01 - 4.72x)$

STUDENT HELP

▶ HOMEWORK HELP

Example 1: Exs. 14–39, 44

Example 2: Exs. 14–39

Example 3: Exs. 32–39

Example 4: Exs. 40–43

Example 5: Exs. 49–53

FOCUS ON APPLICATIONS



REAL LIFE **COCOA** is the principle ingredient in chocolate. Typically, it takes an entire year's harvest from a mature cocoa tree to produce 1 kilogram of cocoa.

CHANGING TO INTEGER COEFFICIENTS Multiply the equation by a power of 10 to write an equivalent equation with integer coefficients.

40. $2.5x + 0.7 = 4.6 - 1.3x$ 41. $-0.625y - 0.184 = 2.506y$
 42. $1.67 + 2.43x = 3.29(x - 5)$ 43. $4.5n - 0.375 = 0.75n + 2.0$

44. **COCOA CONSUMPTION** The 267.9 million people in the United States consumed 639.4 million kilograms of the cocoa produced in the 1996–1997 growing year. Which choice better represents the amount of cocoa consumed per person that year? Explain. **Source:** International Cocoa Organization
- A. 2.38671146 kilograms B. about 2.4 kilograms

CRITICAL THINKING Describe a situation where you would round as indicated.

45. to nearest whole 46. to nearest tenth 47. to nearest hundredth

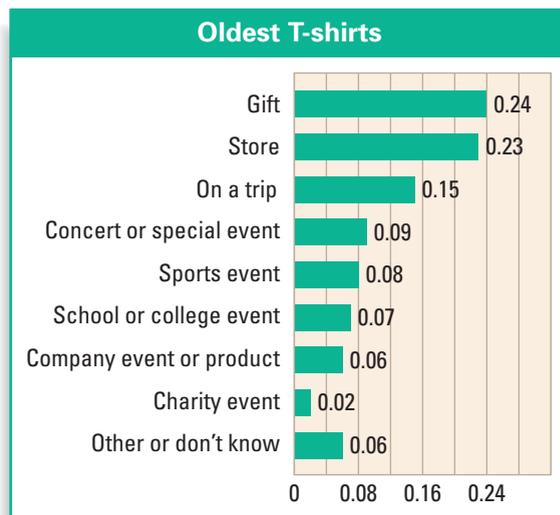
48. **COMPARING ROUNDING METHODS** At the 1998 winter Olympics, Marianne Timmer of the Netherlands won the women's 1000-meter speed skating race with a time of 76.51 seconds. Use a calculator to find Timmer's rates in parts (a)–(d) following each method below. Do the two methods give the same final result after rounding to the nearest tenth in part (d)?

Method 1 Round any decimal answers to the nearest tenth before going on.
Method 2 Use the full calculator display until you round for the final answer.

- a. speed in meters per second b. speed in meters per minute
 c. speed in meters per hour d. speed in kilometers per hour
49. **BUYING DINNER** You have \$8.39 to spend for dinner. You want to leave a 15% tip. What is your price limit for the dinner with tax included?

INTERPRETING DATA In Exercises 50–52, use the data from a survey about T-shirts.

The survey found that 93 out of 100, or 0.93, of the adults responding own at least one T-shirt. The bar graph shows where those adults who own T-shirts got their oldest T-shirt.



Source: Opinion Research

50. Suppose the survey takers counted 186 people who own at least one T-shirt. Choose the equation you could use to find the number of people in the survey. About how many people were surveyed?
- A. $x + 0.93 = 186$ B. $0.93 \div x = 186$ C. $0.93x = 186$

51. **PROBABILITY CONNECTION** What is the probability that an adult chosen at random from a group of adults who own at least one T-shirt received his or her oldest T-shirt as a gift?

52. Suppose a group of adults who own T-shirts includes 42 people who got their oldest T-shirt from a charity event. Estimate the total size of the group.

STUDENT HELP

INTERNET **HOMEWORK HELP**
 Visit our Web site www.mcdougallittell.com for help with Exs. 51 and 52.

FOCUS ON APPLICATIONS



BOTTLE-NOSED WHALES usually stay under the water from 15–20 minutes at a time, but they can stay under for just over an hour. Observers have even reported a two-hour dive!

Test Preparation

★ Challenge

EXTRA CHALLENGE

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53. WHALES A bottle-nosed whale can dive 440 feet per minute. Suppose a bottle-nosed whale is 500 feet deep and dives at this rate. Write and solve an equation to find how long it will take to reach a depth of 2975 feet. Round to the nearest whole minute.

INTERNET SERVICE In Exercises 54 and 55, your Internet service provider charges \$4.95 per month for the first 3 hours of service, plus \$2.50 for each additional hour. Your total charges last month were \$21.83. Let x represent the number of hours you used the service.

54. Which equation models the situation?

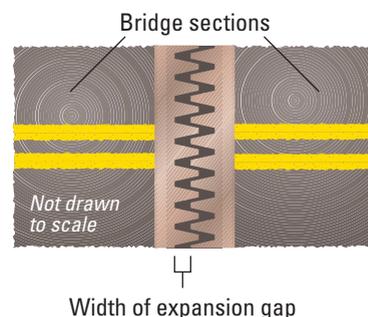
A. $4.95 + 2.5x - 3 = 21.83$

B. $4.95 + 2.5(x - 3) = 21.83$

55. Solve the equation you chose in Exercise 54 and round the result to the nearest hundredth. What does the result represent?

SCIENCE CONNECTION In Exercises 56–58, use the following information.

Bridge sections expand as the temperature goes up, so a small *expansion gap* is left between sections when a bridge is built. As the sections expand, the width of the gap gets smaller.



Suppose that for some bridge the expansion gap is 16.8 millimeters wide at 10°C and decreases by 0.37 millimeter for every 1°C rise in temperature.

56. If the temperature is 18°C , by how many degrees did the temperature rise? By how much would the width of the gap decrease? What would the new width of the gap be? Round to the nearest tenth of a millimeter.

57. The temperature rises to $t^{\circ}\text{C}$. Write expressions for the temperature rise, the decrease in the width of the gap, and the new width of the gap.

58. Use the expressions from Exercise 57. Write and solve an equation to find the temperature at which the gap decreases to 9.4 millimeters.

59. MULTI-STEP PROBLEM You are running water into a laundry sink to get a mixture that is one-half hot water and one-half cold water. The hot water flows more slowly, at a rate of 7.8 liters per minute, so you turn it on first. Two minutes later, you also turn on the cold water which flows at a rate of 12.3 liters per minute. You want to know how long to wait before turning the two faucets off.

a. Let t represent the number of minutes the hot water is on. Write a variable expression for the amount of time the cold water is on.

b. Write an equation that models the situation. Then solve the equation.

c. *Writing* Interpret your solution. If the solution is a decimal, decide what form of the number is most appropriate. Explain your choice.

60. ROUND-OFF ERROR For some decimal equations, you get almost the same solution whether you round early in the solution process or only at the end. For other equations, such as the one in the Activity on page 168, rounding early has a big effect on the solution. Write a decimal equation for which rounding early produces a significant difference in the answer.

MIXED REVIEW

- 61. INPUT-OUTPUT TABLES** Make an input-output table for the function $A = 8 + 2.5t$ when $t = 2, 3, 4, 5,$ and 6 . Describe the domain and the range of the function whose values are shown in the table. (Review 1.7 for 3.7)

OPPOSITES Find the opposite of the number. (Review 2.1)

62. 8 63. -3 64. -4.9 65. 7.9

PROBABILITY Find the probability of the event. (Review 2.8)

66. Choosing the letter N from a bag that contains all 26 letters of the alphabet
67. Choosing a blue marble from a bag that contains 16 blue marbles and 14 white marbles
68. Rolling an even number using a six-sided number cube

 **ACCOUNT ACTIVITY** In Exercises 69–71, use the table. It shows all of the activity in a checking account during June. Deposits are positive and withdrawals are negative. (Review 2.2)

Day	Activity
June 6	$-\$225.00$
June 10	$+\$310.25$
June 17	$+\$152.33$
June 25	$-\$72.45$
June 30	$-\$400.00$

69. How did the amount of money in the account change from the beginning of June through June 10?
70. Find the total amount withdrawn in June.
71. What was the total change in the account balance over the course of the month?

QUIZ 2

Self-Test for Lessons 3.4–3.6

Solve the equation if possible. (Lesson 3.4)

1. $3x + 1 = 5x$ 2. $8 - 2y = 21 - 6y$ 3. $3n = (6 - n)(-3)$
4. $\frac{1}{2}(14 + 8a) = 9a$ 5. $7 - 6d = 3(5 - 2d)$ 6. $-7(b + 1) = 5(b - 2)$

Solve the equation. Round the result to the nearest hundredth. (Lesson 3.6)

7. $15y - 8 = 4y - 3$ 8. $2(2n + 11) = 31 - 3n$
9. $7.6x + 3.7 = 2.8 - 1.6x$ 10. $4.9(3.7x + 1.4) = 34.7x$

11.  **PET SHOW** The city is sponsoring a pet show. The number of cats, birds, and hamsters equals the number of dogs and turtles. There are 3 birds, 3 turtles, and 5 hamsters. There are twice as many dogs as cats. Let n be the number of cats. Draw a diagram to decide which equation models the situation. (Lesson 3.5)

- A. $2n + n = 3 + 3 + 5$ B. $n + 3 = 2n + 3 + 5$
C. $n + 3 + 5 = 2n + 3$ D. $2n + 5 = 3 + 3 + n$

12. Use the results of Exercise 11 to find the number of dogs. (Lesson 3.5)