

3.4

Solving Equations with Variables on Both Sides

What you should learn

GOAL 1 Collect variables on one side of an equation.

GOAL 2 Use equations to solve **real-life** problems such as renting video games in **Example 5**.

Why you should learn it

▼ To solve **real-life** problems, such as deciding whether to join a rock-climbing gym in **Exercise 46**.



GOAL 1 COLLECTING VARIABLES ON ONE SIDE

Some equations have variables on both sides. To solve such equations, you may first collect the variable terms on the side with the *greater* variable coefficient.

EXAMPLE 1 Collect Variables on Left Side

Solve $7x + 19 = -2x + 55$.

SOLUTION Look at the coefficients of the x -terms. Since 7 is greater than -2 , collect the x -terms on the left side.

$$7x + 19 = -2x + 55$$

Write original equation.

$$7x + 19 + 2x = -2x + 55 + 2x$$

Add $2x$ to each side.

$$9x + 19 = 55$$

Simplify.

$$9x + 19 - 19 = 55 - 19$$

Subtract 19 from each side.

$$9x = 36$$

Simplify.

$$\frac{9x}{9} = \frac{36}{9}$$

Divide each side by 9.

$$x = 4$$

Simplify.

✓ **CHECK** $7x + 19 = -2x + 55$

Write original equation.

$$7(4) + 19 \stackrel{?}{=} -2(4) + 55$$

Substitute 4 for each x .

$$47 = 47$$

Solution is correct.

EXAMPLE 2 Collect Variables on Right Side

Solve $80 - 9y = 6y$.

SOLUTION Look at the coefficients of the y -terms. Think of $80 - 9y$ as $80 + (-9y)$. Since 6 is greater than -9 , collect the y -terms on the right side.

$$80 - 9y = 6y$$

Write original equation.

$$80 - 9y + 9y = 6y + 9y$$

Add $9y$ to each side.

$$80 = 15y$$

Simplify.

$$\frac{80}{15} = \frac{15y}{15}$$

Divide each side by 15.

$$\frac{16}{3} = y$$

Simplify.

STUDENT HELP

Look Back

For help with terms of an expression, see page 80.

Linear equations do not always have one solution. An **identity** is an equation that is true for all values of the variable. Some linear equations have no solution.

STUDENT HELP



HOMEWORK HELP

Visit our Web site www.mcdougallittell.com for extra examples.

EXAMPLE 3 *Many Solutions or No Solution*

Solve the equation.

a. $3(x + 2) = 3x + 6$

b. $x + 2 = x + 4$

SOLUTION

a. $3(x + 2) = 3x + 6$ **Write original equation.**

$3x + 6 = 3x + 6$ **Use distributive property.**

$6 = 6$ **Subtract $3x$ from each side.**

▶ All values of x are solutions, because $6 = 6$ is always true. The original equation is an *identity*.

b. $x + 2 = x + 4$ **Write original equation.**

$2 \neq 4$ **Subtract x from each side.**

▶ The original equation has no solution, because $2 \neq 4$ for any value of x .

EXAMPLE 4 *Solving More Complicated Equations*

Solve the equation.

a. $4(1 - x) + 3x = -2(x + 1)$

b. $\frac{1}{4}(12x + 16) = 10 - 3(x - 2)$

SOLUTION Simplify the equation before you decide whether to collect the variable terms on the right side or the left side of the equation.

a. $4(1 - x) + 3x = -2(x + 1)$ **Write original equation.**

$4 - 4x + 3x = -2x - 2$ **Use distributive property.**

$4 - x = -2x - 2$ **Add like terms.**

$4 + x = -2$ **Add $2x$ to each side.**

$x = -6$ **Subtract 4 from each side.**

b. $\frac{1}{4}(12x + 16) = 10 - 3(x - 2)$ **Write original equation.**

$3x + 4 = 10 - 3x + 6$ **Use distributive property.**

$3x + 4 = 16 - 3x$ **Simplify.**

$6x + 4 = 16$ **Add $3x$ to each side.**

$6x = 12$ **Subtract 4 from each side.**

$x = 2$ **Divide each side by 6.**

STUDENT HELP

Study Tip

You can multiply by the reciprocal first in part (b) of Example 4, but distributing the $\frac{1}{4}$ is easier here. Before you solve, look to see which method seems easier.

GOAL 2 SOLVING REAL-LIFE PROBLEMS



Membership Fees

EXAMPLE 5 Using a Verbal Model

A video store charges \$8 to rent a video game for five days. You must be a member to rent from the store, but the membership is free. A video game club in town charges only \$3 to rent a game for five days, but membership in the club is \$50 per year. Which rental plan is more economical?

SOLUTION

Find the number of rentals for which the two plans would cost the same.



VERBAL MODEL	$\underbrace{\text{Store rental fee} \cdot \text{Number rented}}_{\text{Video store}} = \underbrace{\text{Club rental fee} \cdot \text{Number rented}}_{\text{Video club}} + \text{Club membership fee}$	
LABELS	Store rental fee = 8	(dollars per game)
	Number rented = x	(games)
	Club rental fee = 3	(dollars per game)
	Club membership fee = 50	(dollars)
ALGEBRAIC MODEL	$8 \cdot x = 3 \cdot x + 50$ <p style="text-align: right;">Write algebraic model.</p> $5x = 50$ <p style="text-align: right;">Subtract $3x$ from each side.</p> $x = 10$ <p style="text-align: right;">Divide each side by 5.</p>	

If you rent 10 video games in a year, the cost would be the same at either the store or the club. If you rent more than 10, the club is more economical. If you rent fewer than 10, the store is more economical.

UNIT ANALYSIS Check that *dollars* are the units of the solution.

$$\frac{\text{dollars}}{\text{game}} \cdot \cancel{\text{games}} = \frac{\text{dollars}}{\text{game}} \cdot \cancel{\text{games}} + \text{dollars}$$

✓ **CHECK** A table can help you check the result of Example 5.

Number rented	2	4	6	8	10	12	14	16
Cost at store	\$16	\$32	\$48	\$64	\$80	\$96	\$112	\$128
Cost at club	\$56	\$62	\$68	\$74	\$80	\$86	\$92	\$98

Store is cheaper.

Club is cheaper.

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CAREER LINK

Visit our Web site www.mcdougallittell.com for information about video game designers.

GUIDED PRACTICE

Vocabulary Check ✓

Concept Check ✓

Skill Check ✓

1. Is the equation $-2(4 - x) = 2x - 8$ an identity? Explain why or why not.
2. Decide if the statement is *true* or *false*. *The solution of $x = 2x$ is zero.*
3. Solve $9(9 - x) = 4x - 10$. Explain what you are doing at each step.

Solve the equation if possible. Does the equation have *one solution*, is it an *identity*, or does it have *no solution*?

4. $2x + 3 = 7x$
5. $12 - 2a = -5a - 9$
6. $x - 2x + 3 = 3 - x$
7. $5x + 24 = 5(x - 5)$
8. $\frac{2}{3}(6c + 3) = 6(c - 3)$
9. $6y - (3y - 6) = 5y - 4$
10.  **FUNDRAISING** You are making pies to sell at a fundraiser. It costs \$3 to make each pie, plus a one-time cost of \$20 for a pastry blender and a rolling pin. You plan to sell the pies for \$5 each. Which equation should you use to find the number of pies you need to sell to break even?
 - A. $3x = 20 + 5x$
 - B. $3x + 20 = 5x$
 - C. $3x - 20 = 5x$
 - D. $20 - 5x = 3x$
11. Find the number of pies you need to sell to break even in Exercise 10.

PRACTICE AND APPLICATIONS

STUDENT HELP

Extra Practice
to help you master
skills is on p. 799.

WRITING Solve the equation and describe each step you use.

12. $7 - 4c = 10c$
13. $-8x + 7 = 4x - 5$
14. $x + 2 = 3x - 1$
15. $7(1 - y) = -3(y - 2)$
16. $\frac{1}{5}(10a - 15) = 3 - 2a$
17. $5(y - 2) = -2(12 - 9y) + y$

SOLVING EQUATIONS Solve the equation if possible.

18. $4x + 27 = 3x$
19. $12y + 21 = 9y$
20. $-2m = 16m - 9$
21. $4n = -28n - 3$
22. $12c - 4 = 12c$
23. $-30d + 12 = 18d$
24. $6 - (-5r) = 5r - 3$
25. $6s - 11 = -2s + 5$
26. $12p - 7 = -3p + 8$
27. $-12q + 4 = 8q - 6$
28. $-7 + 4m = 6m - 5$
29. $-7 + 11g = 9 - 5g$
30. $8 - 9t = 21t - 17$
31. $24 - 6r = 6(4 - r)$
32. $3(4 + 4x) = 12x + 12$
33. $-4(x - 3) = -x$
34. $10(-4 + y) = 2y$
35. $8a - 4(-5a - 2) = 12a$
36. $9(b - 4) - 7b = 5(3b - 2)$
37. $-2(6 - 10n) = 10(2n - 6)$
38. $-(8n - 2) = 3 + 10(1 - 3n)$
39. $\frac{1}{2}(12n - 4) = 14 - 10n$
40. $\frac{1}{4}(60 + 16s) = 15 + 4s$
41. $\frac{3}{4}(24 - 8b) = 2(5b + 1)$

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HOMEWORK HELP

Example 1: Exs. 12–41
Example 2: Exs. 12–41
Example 3: Exs. 18–41
Example 4: Exs. 12–41
Example 5: Exs. 45, 46

ERROR ANALYSIS In Exercises 42 and 43, describe the errors.

42. ~~$$\begin{aligned} 7(c - 6) &= (4 - c)(3) \\ 7c - 6 &= 4 - 3c \\ 10c &= 10 \\ c &= 1 \end{aligned}$$~~

43. ~~$$\begin{aligned} 2(4b - 3) &= 8b - 6 \\ 8b - 6 &= 8b - 6 \\ -6 &= -6 \\ b &= -6 \end{aligned}$$~~

44. **CRITICAL THINKING** A student was confused by the results when solving this equation. Explain what the result means.

$$\begin{aligned} 7(y - 2) &= -y + 8y - 14 \\ 7y - 14 &= 7y - 14 \\ 7y &= 7y \\ y &= y \end{aligned}$$

45. **COMPUTER TIME** A local computer center charges nonmembers \$5 per session to use the media center. If you pay a membership fee of \$25, you pay only \$3 per session. Write an equation that can help you decide whether to become a member. Then solve the equation and interpret the solution.
46. **ROCK CLIMBING** A rock-climbing gym charges nonmembers \$16 per day to use the gym and \$8 per day for equipment rental. Members pay a yearly fee of \$450 for unlimited climbing and \$6 per day for equipment rental. Write and solve an equation to find how many times you must use the gym to justify becoming a member.

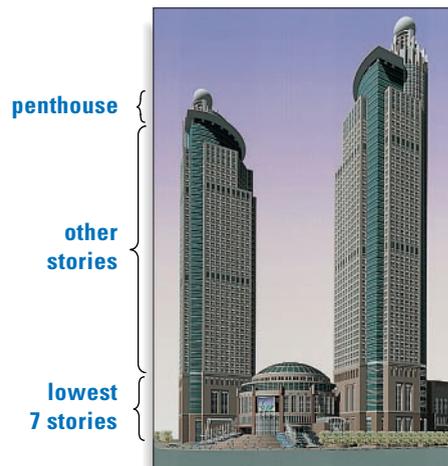
TALL BUILDINGS In Exercises 47–49, use the following information.

In designing a tall building, many factors affect the height of each story. How the space will be used is important. At the Grand Gateway at Xu Hui in Shanghai, the lowest 7 stories have a combined height of about 126 feet. These stories next to a shopping mall are unusually tall. The building's other 43 stories have a more typical height. If the average height of the other stories had been used for the lowest 7 stories, then the building could have fit about $3\frac{1}{2}$ more stories.

► Source: Callison Architecture. (Actual dimensions have been simplified.)

Combined height of lowest 7 stories	+	Number of other stories	·	Average height of other stories	=
Possible number of stories	·	Average height of other stories			

47. Explain why you would use $53\frac{1}{2}$ for the “possible number of stories” in the verbal model.
48. Let h represent the average height of the other stories. Finish assigning labels and write the algebraic model.
49. Solve the equation. Be sure to simplify the answer. About how many feet tall are the stories above the 7th floor?



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REAL LIFE ARCHITECT
Designing a building that is functional, attractive, safe, and economical requires teamwork and many stages of planning. Most architects use computers as they write reports and create plans.

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Test Preparation

50. **MULTIPLE CHOICE** Which equations are equivalent?

- I. $7x - 9 = 7$ II. $-9 = 7 - 5x$ III. $3(2x - 3) = 7 - x$
 (A) I and III (B) II and III (C) All (D) None

51. **MULTIPLE CHOICE** Which equation has more than one solution?

- (A) $18y + 13 = 12y - 25$ (B) $6y - (3y - 6) = -14 + 3y$
 (C) $-\frac{1}{2}(30x - 18) = 9 - 15x$ (D) $\frac{1}{5}(2x - 5) = 3x + 7$

52. **MULTIPLE CHOICE** Solve $\frac{1}{3}(7x + 5) = 3x - 5$.

- (A) 15 (B) -5 (C) 10 (D) $-\frac{5}{2}$

★ Challenge

SUBSCRIPTIONS In Exercises 53 and 54, use the table. It shows monthly expenses and income of a magazine for different numbers of subscribers.

Number of subscribers	50	100	150	200	250	300	350	400
Income	\$75	\$150	\$225	\$300	\$375	\$450	\$525	\$600
Expenses	\$150	\$200	\$250	\$300	\$350	\$400	\$450	\$500

53. **EXPLAINING A PATTERN** Look for patterns in the table. Write an equation that you can use to find how many subscribers the magazine needs for its income to equal its expenses.

54. Explain how to use the table to check your answer in Exercise 53.

EXTRA CHALLENGE

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MIXED REVIEW

EQUIVALENTS Write the percent as a decimal. (Skills Review, pages 784–785)

55. 28% 56. 40% 57. 3% 58. 19.5%

PERCENT OF A NUMBER Find the number. (Skills Review, page 786)

59. 45% of 84 60. 7% of 28.5 61. 76% of 540
 62. 16.3% of 132 63. 8% of \$928.50 64. 5.5% of \$74

UNIT ANALYSIS Find the resulting unit of measure. (Review 1.1 for 3.5)

65. (dollars per hour) • (hours) 66. (years) • (people per year)
 67. (miles) ÷ (miles per hour) 68. (meters) • (kilometers per meter)

GEOMETRY The two triangles are similar. Find the length of the side marked x . (Review 3.2)

