7.1

Solving Linear Systems by Graphing

What you should learn

GOAL 1 Solve a system of linear equations by graphing.

GOAL 2 Model a **real-life** problem using a linear system, such as predicting the number of visits at Internet sites in **Example 3**.

Why you should learn it

▼ To solve **real-life** problems, such as comparing the number of people who live inland to the number of people living on the coastline in **Exs. 37–39**.

REAL LIFE

GOAL 1 GRAPHING A LINEAR SYSTEM

In this chapter you will study *systems of linear equations* in two variables. Here are two equations that form a **system of linear equations** or simply a **linear system**.

$x + 2y = 5$ **Equation 1**

$2x - 3y = 3$ **Equation 2**

A **solution of a system of linear equations** in two variables is an ordered pair (x, y) that satisfies each equation in the system.

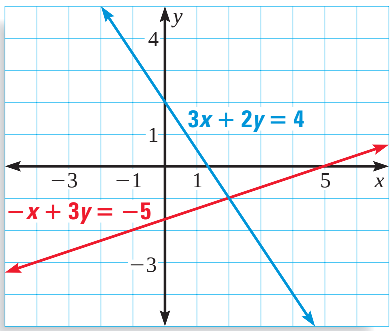
Because the solution of a linear system satisfies each equation in the system, the solution must lie on the graph of both equations. When the solution has integer values, it is possible to find the solution by graphical methods.

EXAMPLE 1 Checking the Intersection Point

Use the graph at the right to solve the system of linear equations. Then check your solution algebraically.

$3x + 2y = 4$ **Equation 1**

$-x + 3y = -5$ **Equation 2**



SOLUTION

The graph gives you a visual model of the solution.

The lines appear to intersect once at $(2, -1)$.

✓ **CHECK** To check $(2, -1)$ as a solution algebraically, substitute 2 for x and -1 for y in each equation.

EQUATION 1	EQUATION 2
$3x + 2y = 4$	$-x + 3y = -5$
$3(2) + 2(-1) \stackrel{?}{=} 4$	$-(2) + 3(-1) \stackrel{?}{=} -5$
$6 - 2 \stackrel{?}{=} 4$	$-2 - 3 \stackrel{?}{=} -5$
$4 = 4$	$-5 = -5$

▶ Because $(2, -1)$ is a solution of each equation, $(2, -1)$ is the solution of the system of linear equations. Because the lines in the graph of this system intersect at only one point, $(2, -1)$ is the only solution of the linear system.

STUDENT HELP

Look Back
For help with checking solutions, see p. 210.

SOLVING A LINEAR SYSTEM USING GRAPH-AND-CHECK

To use the graph-and-check method to solve a system of linear equations in two variables, use the following steps.

STEP 1 Write each equation in a form that is easy to graph.

STEP 2 Graph both equations in the same coordinate plane.

STEP 3 Estimate the coordinates of the point of intersection.

STEP 4 Check the coordinates algebraically by substituting into each equation of the original linear system.

STUDENT HELP

INTERNET

HOMEWORK HELP Visit our Web site www.mcdougallittell.com for extra examples.

EXAMPLE 2 *Using the Graph-and-Check Method*

Solve the linear system graphically. Check the solution algebraically.

$x + y = -2$ **Equation 1**

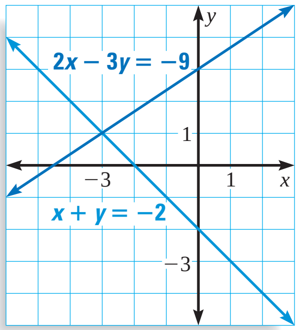
$2x - 3y = -9$ **Equation 2**

SOLUTION

1 Write each equation in a form that is easy to graph, such as slope-intercept form.

$y = -x - 2$ Slope: -1, y-intercept: -2

$y = \frac{2}{3}x + 3$ Slope: $\frac{2}{3}$, y-intercept: 3



2 Graph these equations.

3 The two lines appear to intersect at $(-3, 1)$.

4 To check $(-3, 1)$ as a solution algebraically, substitute -3 for x and 1 for y in each original equation.

EQUATION 1

$$x + y = -2$$
$$-3 + 1 \stackrel{?}{=} -2$$
$$-2 = -2$$

EQUATION 2

$$2x - 3y = -9$$
$$2(-3) - 3(1) \stackrel{?}{=} -9$$
$$-6 - 3 \stackrel{?}{=} -9$$
$$-9 = -9$$

▶ Because $(-3, 1)$ is a solution of each equation in the linear system, it is a solution of the linear system.

In Example 2 you used the slope-intercept form of each equation in the system to graph the lines. However, there are other methods. For instance, in Lesson 4.3 you learned how to make a quick graph of a linear equation using intercepts.

In the Graphing Calculator Activity at the end of this lesson, you will learn how to graph a system of linear equations using a graphing calculator.

GOAL 2 MODELING A REAL-LIFE PROBLEM

EXAMPLE 3 *Writing and Using a Linear System*

INTERNET In the fall, the math club and the science club each created an Internet site. You are the webmaster for both sites. It is now January and you are comparing the number of times each site is visited each day.

Science Club: There are currently 400 daily visits and the visits are increasing at a rate of 25 daily visits per month.

Math Club: There are currently 200 daily visits and the visits are increasing at a rate of 50 daily visits per month.

Predict when the number of visits at the two sites will be the same.

SOLUTION

PROBLEM SOLVING STRATEGY

VERBAL MODEL

Daily visits = Current visits to science site + Monthly increase (sci) · Number of months

Daily visits = Current visits to math site + Monthly increase (math) · Number of months

LABELS

Daily visits = V (daily visits)

Current visits (science) = **400** (daily visits)

Increase (science) = **25** (daily visits per month)

Number of months = t (months)

Current visits (math) = **200** (daily visits)

Increase (math) = **50** (daily visits per month)

ALGEBRAIC MODEL

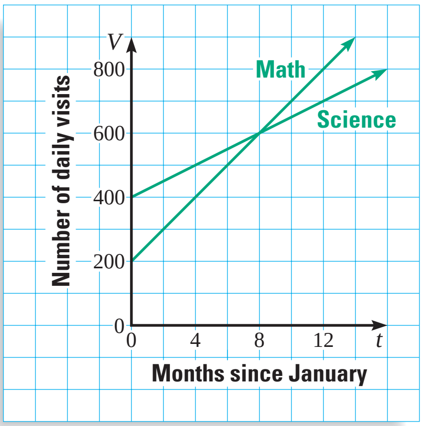
$V = 400 + 25t$ **Equation 1 (science)**

$V = 200 + 50t$ **Equation 2 (math)**

Use the graph-and-check method to solve the system. The point of intersection of the two lines appears to be (8, 600). Check this solution in Equation 1 and in Equation 2.

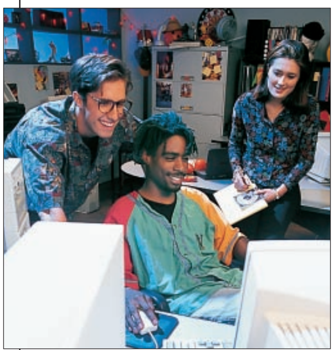
$600 = 400 + 25(8)$

$600 = 200 + 50(8)$



▶ If the monthly increases continue at the same rates, the sites will have the same number of visits by the eighth month after January, which is September.

FOCUS ON CAREERS



WEBMASTER
Webmasters build Web sites for clients. They often update their technical skills to meet the demands for first-class Web sites. Some also help design Web pages and are responsible for updating the content.

CAREER LINK
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GUIDED PRACTICE

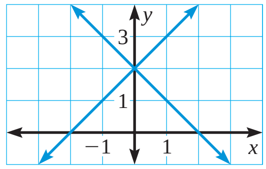
Vocabulary Check ✓

1. Explain what it means to solve a system of linear equations.

Concept Check ✓

2. Explain how to use the graph at the right to solve the system of linear equations.

$y = -x + 2$
$y = x + 2$



Ex. 2

Skill Check ✓

Graph the linear system below. Then decide if the ordered pair is a solution of the system.

$-x + y = -2$
$2x + y = 10$

3. $(-4, -2)$ **4.** $(4, -2)$ **5.** $(-4, 2)$ **6.** $(4, 2)$

7. Confirm your answer for Exercise 3 algebraically.

Use the graph-and-check method to solve the system of linear equations.

8. $y = 2x - 1$
$y = x + 1$

9. $y = -2x + 3$
$y = x - 3$

10. $y = \frac{1}{2}x + 2$
$y = -x + 5$

PRACTICE AND APPLICATIONS

STUDENT HELP

Extra Practice to help you master skills is on p. 803.

CHECKING FOR SOLUTIONS **Decide whether the ordered pair is a solution of the system of linear equations.**

11. $3x - 2y = 11$
$-x + 6y = 7$ $(5, 2)$

12. $6x - 3y = -15$
$2x + y = -3$ $(-2, 1)$

13. $x + 3y = 15$
$4x + y = 6$ $(3, -6)$

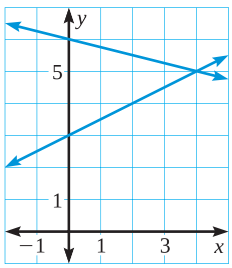
14. $-5x + y = 19$
$x - 7y = 3$ $(-4, -1)$

15. $-15x + 7y = 1$
$3x - y = 1$ $(3, 5)$

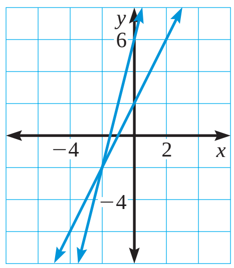
16. $-2x + y = -11$
$-x - 9y = -15$ $(6, 1)$

SOLVING SYSTEMS GRAPHICALLY **Use the graph to solve the linear system. Check your solution algebraically.**

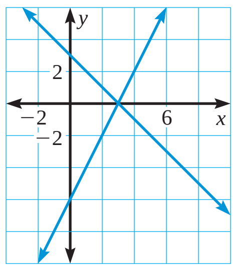
17. $-x + 2y = 6$
$x + 4y = 24$



18. $2x - y = -2$
$4x - y = -6$



19. $x + y = 3$
$-2x + y = -6$



STUDENT HELP

HOMEWORK HELP
Example 1: Exs. 11–19
Example 2: Exs. 20–34
Example 3: Exs. 35, 36

GRAPH AND CHECK **Graph and check to solve the linear system.**

20. $y = -x + 3$
$y = x + 1$

21. $y = -6$
$x = 6$

22. $y = 2x - 4$
$y = -\frac{1}{2}x + 1$

23. $2x - 3y = 9$
$x = -3$

24. $5x + 4y = 16$
$y = -16$

25. $x - y = 1$
$5x - 4y = 0$

26. $3x + 6y = 15$
$-2x + 3y = -3$

27. $7y = -14x + 42$
$7y = 14x + 14$

28. $0.5x + 0.6y = 5.4$
$-x + y = 9$

29. $15x - 10y = -80$
$6x + 8y = -80$

30. $-3x + y = 10$
$7x + y = 20$

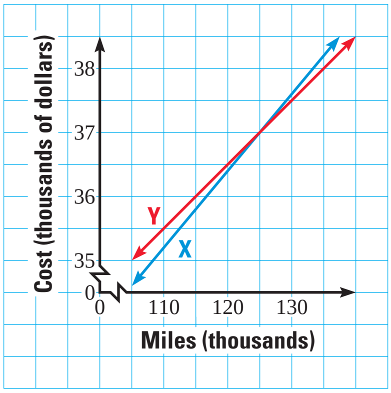
31. $x - 8y = -40$
$-5x + 8y = 8$

32. $\frac{1}{5}x + \frac{3}{5}y = \frac{12}{5}$
$-\frac{1}{5}x + \frac{3}{5}y = \frac{6}{5}$

33. $\frac{3}{4}x - \frac{1}{4}y = -\frac{1}{2}$
$\frac{1}{4}x - \frac{3}{4}y = \frac{3}{2}$

34. $2.8x + 1.4y = 1.4$
$0.7x - 0.7y = 1.4$

35. **COMPARING CARS** Car model X costs \$22,000 to purchase and an average of \$.12 per mile to maintain. Car model Y costs \$24,500 to purchase and an average of \$.10 per mile to maintain. Use the graph to determine how many miles must be driven for the total cost of the two models to be the same.

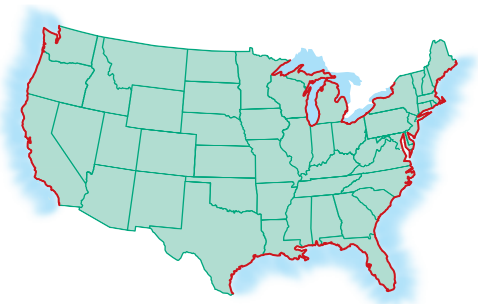


Ex. 35

36. **AEROBICS CLASSES** A fitness club offers two water aerobics classes. There are currently 40 people regularly going to the morning class, and attendance is increasing at a rate of 2 people per month. There are currently 22 people regularly going to the evening class, and attendance is increasing at a rate of 8 people per month. Predict when the number of people in each class will be the same.

COASTAL POPULATION **In Exercises 37–39, use the table below, which gives the percents of people in the contiguous United States living within 50 miles of a coastal shoreline and those living further inland.**

Population in United States	1940	1997
Living within 50 miles of a coastal shoreline	46%	53%
Living farther inland	54%	47%



DATA UPDATE U.S. Bureau of the Census at www.mcdougallittell.com

37. For each location, write a linear model to represent the percent at time t, where t represents the number of years since 1940.

38. Graph the linear equations you wrote.

39. From your graphs, estimate when the percent of people living near the coast equaled the percent living inland.

40. **LAUNDRY** You do 4 loads of laundry each week at a launderette where each load costs \$1.25. You could buy a washing machine that costs \$400. Washing 4 loads at home will cost about \$1 per week for electricity. How many loads of laundry must you do in order for the costs to be equal?

Test Preparation

41. **MULTIPLE CHOICE** Which ordered pair is a solution of the linear system?

$$x + y = 0.5$$
$$x + 2y = 1$$

(A) $(0, -2)$ (B) $(-0.5, 0)$ (C) $(0, 0.5)$ (D) $(0, -0.5)$

42. **MULTIPLE CHOICE** If $y - x = -3$ and $4x + y = 2$, then $x = \underline{\ ?\ }$.

(A) 1 (B) -1 (C) 5 (D) -5

★ Challenge

43. **SOLVE USING A SYSTEM** You know how to solve the equation $\frac{1}{2}x + 2 = \frac{3}{2}x - 12$ algebraically. This equation can also be solved graphically by solving the linear system.

$$y = \frac{1}{2}x + 2$$

$$y = \frac{3}{2}x - 12$$

a. Explain how the linear system is related to the original equation.

b. Solve the system graphically.

c. Check that the x-coordinate from part (b) satisfies the original equation $\frac{1}{2}x + 2 = \frac{3}{2}x - 12$ by substituting the x-coordinate for x.

EXTRA CHALLENGE
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Use the method shown in Exercise 43 to solve the equation graphically.

44. $\frac{4}{5}x = 3x - 11$ **45.** $x + 1 = \frac{3}{2}x + 2$ **46.** $1.2x - 2 = 3.4x - 13$

MIXED REVIEW

SOLVING EQUATIONS **Solve the equation.** (Review 3.3 for 7.2)

47. $3x + 7 = -2$ **48.** $15 - 2a = 7$ **49.** $21 = 7(w - 2)$

50. $2y + 5 = 3y$ **51.** $2(z - 3) = 12$ **52.** $-(3 - x) = -7$

WRITING EQUATIONS **Write an equation of the line that passes through the point and has the given slope. Use slope-intercept form.** (Review 5.2)

53. $(3, 0), m = -4$ **54.** $(-4, 3), m = 1$ **55.** $(1, -5), m = 4$

56. $(-4, -1), m = -2$ **57.** $(2, 3), m = 2$ **58.** $(-1, 5), m = \frac{2}{3}$

Verrazano-Narrows Bridge

59. **SUSPENSION BRIDGES** The Verrazano-Narrows Bridge in New York City is the longest suspension bridge in North America, with a main span of 4260 feet. Write an inequality that describes the length in feet x of every other suspension bridge in North America. Graph the inequality on a number line. (Review 6.1)