CHAPTER 11

Chapter Summary

WHAT did you learn?

WHY did you learn it?

WHAT did you learn?	WHY did you learn it?
Solve proportions. (11.1)	Estimate the number of clay figures in an archaeological dig. (p. 645)
Use equations to solve percent problems. (11.2)	Compare responses to a survey. (p. 654)
Use direct and inverse variation. (11.3)	Relate the banking angle of a bicycle to its turning radius. (p. 658)
Simplify rational expressions. (11.4)	Use rational expressions to model real-life situations. (p. 668)
Find geometric probability. (11.4)	Determine the probability of a meteor strike. (p. 668)
Multiply and divide rational expressions. (11.5)	Analyze service industry sales. (p. 674)
Add and subtract rational expressions. (11.6)	Calculate the time it takes to make a trip. (p. 678)
Divide polynomials. (11.7)	Provide alternate forms of rational expression models. (p. 688)
Solve rational equations. (11.8)	Study changes in batting averages. (p. 691)
Graph rational functions. (11.8)	Provide a visual representation of a fundraising problem. (p. 695)

How does Chapter 11 fit into the BIGGER PICTURE of algebra?

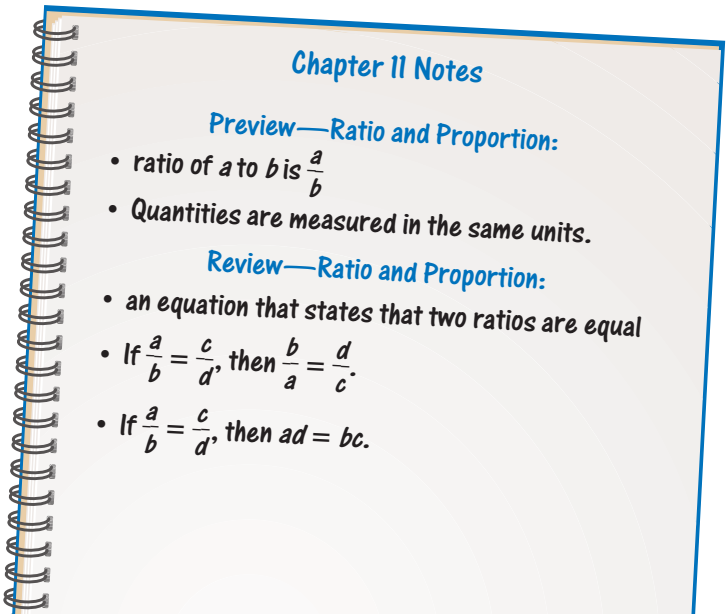
In this chapter you studied rational expressions—fractions whose numerators and denominators are polynomials. Rational expressions occur frequently in real life as proportions, percents, probabilities, and direct and inverse variations. Understanding these will enable you to model and solve a variety of real-life problems.

Techniques used in these problems include simplifying, multiplying, dividing, adding, and subtracting rational expressions. Real-life problems can also be modeled with rational equations and represented by graphing rational functions.

STUDY STRATEGY

How did you use your notes to see what you learned?

The notes you made, using the **Study Strategy** on page 642, may include the ideas shown.



CHAPTER 11

Chapter Review

VOCABULARY

- proportion, p. 643
- extremes of a proportion, p. 643
- means of a proportion, p. 643
- solving a proportion, p. 643
- extraneous solution, p. 644
- base number of a percent equation, p. 649
- inverse variation, p. 656
- constant of variation, p. 656
- rational number, p. 664
- rational expression, p. 664
- simplified rational expression, p. 664
- geometric probability, p. 666
- least common denominator, p. 677
- polynomial long division, p. 685
- rational equation, p. 690
- rational function, p. 692
- hyperbola, p. 692
- center of a hyperbola, p. 692
- asymptote, p. 692

Examples on pp. 643–645

11.1 Ratio and Proportion

EXAMPLE Solve the proportion $\frac{12}{7} = \frac{5}{x}$.

$\frac{12}{7} = \frac{5}{x}$ Write original proportion.

$12 \cdot x = 7 \cdot 5$ Use cross product property.

$x = \frac{35}{12}$ Divide each side by 12.

Solve the proportion. Check for extraneous solutions.

1. $\frac{x}{2} = \frac{4}{7}$
2. $\frac{7}{10} = \frac{9 + x}{x}$
3. $\frac{x^2 - 16}{x + 4} = \frac{x - 4}{3}$
4. $\frac{5}{x + 6} = \frac{x - 6}{x}$

11.2 Percents

Examples on pp. 649–652

EXAMPLES

$20 is 40% of what amount of money?

a is p percent of b

$20 = 0.4b$

$\frac{20}{0.4} = b$

$50 = b$

▶ $20 is 40% of $50.

$75 is what percent of $60?

a is p percent of b

$75 = p(60)$

$\frac{75}{60} = p$

$1.25 = p$, or $p = 125\%$

▶ $75 is 125% of $60.

Solve the percent problem.

5. How much is 80% of $95?
6. 24 inches is 250% of what length?
7. $90 is 75% of what amount of money?
8. 35 feet is what percent of 175 feet?

11.3 Direct and Inverse Variation

Examples on pp. 656–658

EXAMPLES When x is 5, y is 25. Write an equation that relates x and y in each case.

***x* and *y* vary directly:**

$$\frac{y}{x} = k$$

$$\frac{25}{5} = k$$

$$5 = k$$

An equation that relates x and y is $\frac{y}{x} = 5$, or $y = 5x$.

***x* and *y* vary inversely:**

$$xy = k$$

$$(5)(25) = k$$

$$125 = k$$

An equation that relates x and y is $xy = 125$, or $y = \frac{125}{x}$.

When *x* is 17, *y* is 51. Find an equation that relates *x* and *y* in each case.

9. x and y vary directly

10. x and y vary inversely

11.4 Simplifying Rational Expressions

Examples on pp. 664–666

EXAMPLE To simplify a rational expression, look for common factors.

$$\frac{2x^2 + 3x - 2}{2x^2 + 5x + 2} = \frac{(2x - 1)(x + 2)}{(2x + 1)(x + 2)}$$ Factor numerator and denominator. Divide out common factor $(x + 2)$.

$$= \frac{2x - 1}{2x + 1}$$ Simplified form

Simplify the expression.

11. $\frac{3x}{9x^2 + 3}$

12. $\frac{6x^2}{12x^4 + 18x^2}$

13. $\frac{7x^3 - 21x}{-14x^2}$

14. $\frac{5x^2 + 21x + 4}{25x + 100}$

11.5 Multiplying and Dividing Rational Expressions

Examples on pp. 670–672

EXAMPLE To divide rational expressions, multiply by the reciprocal.

$$\frac{6x^2 + x - 1}{2x + 1} \div (9x - 3) = \frac{6x^2 + x - 1}{2x + 1} \cdot \frac{1}{9x - 3}$$ Multiply by reciprocal.

$$= \frac{(2x + 1)(3x - 1)}{(2x + 1) \cdot 3(3x - 1)}$$ Multiply numerators and denominators. Factor and divide out common factors.

$$= \frac{1}{3}$$ Simplified form

Simplify the expression.

15. $\frac{12x^2}{5x^3} \cdot \frac{25x^4}{3x}$

16. $\frac{9x^3}{x^3 - x^2} \div \frac{x - 8}{x^2 - 9x + 8}$

17. $\frac{x^2 + 3x + 2}{x^2 + 7x + 12} \div \frac{x^2 + 5x + 4}{x^2 + 5x + 6}$

11.6 Adding and Subtracting Rational Expressions

Examples on pp. 676–678

EXAMPLE Simplify $\frac{x}{x-5} - \frac{2}{x+2}$. The LCD is $(x-5)(x+2)$.

$$\frac{x(x+2)}{(x-5)(x+2)} - \frac{2(x-5)}{(x-5)(x+2)}$$ Rewrite fractions using LCD.

$$= \frac{x^2+2x}{(x-5)(x+2)} - \frac{2x-10}{(x-5)(x+2)}$$ Simplify numerators.

$$= \frac{(x^2+2x)-(2x-10)}{(x-5)(x+2)} = \frac{x^2+10}{(x-5)(x+2)}$$ Subtract fractions and simplify.

Simplify the expression.

18. $\frac{6x}{x+4} - \frac{5x-4}{x+4}$ **19.** $\frac{2x+1}{8x} - \frac{x}{12x}$ **20.** $\frac{x+3}{3x-1} + \frac{4}{x-3}$ **21.** $\frac{-5x-10}{x^2-4} + \frac{4x}{x-2}$

11.7 Dividing Polynomials

Examples on pp. 684–686

EXAMPLES There are two cases to look for when you divide polynomials.

CASE 1: Monomial divisor

To divide a polynomial by a monomial, divide each term by the monomial.

CASE 2: Binomial divisor

To divide a polynomial by a binomial, factor out common factors if possible. If not, use long division.

Divide.

22. Divide $3x^2 - x - 1$ by $x - 2$.

23. Divide $6x^2 - 36x + 5$ by $6x$.

24. Divide $4x^2 + 6x - 5$ by $2x - 1$.

25. Divide $5x^2 + 13x - 6$ by $5x - 2$.

11.8 Rational Equations and Functions

Examples on pp. 690–693

EXAMPLE Solve the equation $\frac{2x}{9} - \frac{1}{x} = \frac{1}{3}$. The LCD is $9x$.

$$9x \cdot \frac{2x}{9} - 9x \cdot \frac{1}{x} = 9x \cdot \frac{1}{3}$$ Multiply each side by $9x$.

$$2x^2 - 9 = 3x$$ Simplify.

$$2x^2 - 3x - 9 = 0$$ Write in standard form.

$$(2x+3)(x-3) = 0$$ Factor left side.

When you set each factor equal to 0, you find that the solutions are $-\frac{3}{2}$ and 3.

Solve the equation.

26. $\frac{1}{4} - \frac{6}{x} = \frac{3}{x}$ **27.** $\frac{x+2}{2} = \frac{4}{x}$ **28.** $\frac{6}{x+4} + \frac{3}{4} = \frac{2x+1}{3x+12}$

CHAPTER 11

Chapter Test

Solve the proportion. Check for extraneous solutions.

1. $\frac{6}{x} = \frac{17}{5}$ **2.** $\frac{x}{4} = \frac{x+8}{x}$ **3.** $\frac{x}{-3} = \frac{7}{x-10}$ **4.** $\frac{x^2-64}{x+8} = \frac{x-8}{2}$

Solve the percent problem.

5. What is 34% of 100 liters?

6. What is 86% of $350?

7. 24 yards is 12% of what distance?

8. 36 T-shirts is what percent of 900 T-shirts?

Make a table of values for $x = 1, 2, 3$, and 4. Use the table to sketch a graph. Decide whether x and y vary *directly* or *inversely*.

9. $y = 4x$ **10.** $y = \frac{50}{x}$ **11.** $y = \frac{9}{2}x$ **12.** $y = \frac{15}{2x}$

Simplify the expression.

13. $\frac{56x^6}{4x^4}$ **14.** $\frac{5x^2-15x}{15x^4}$ **15.** $\frac{x^2-x-6}{x^2-4}$

16. $\frac{6x^2}{8x} \cdot \frac{-4x^3}{2x^2}$ **17.** $\frac{x+3}{x^3-x^2-6x} \div \frac{x^2-9}{x^2+x-12}$ **18.** $\frac{x^3+x^2}{x^2-16} \cdot \frac{x+4}{3x^4+x^3-2x^2}$

19. $\frac{3x^2+6x}{4x} \div \frac{15}{8x^2}$ **20.** $\frac{12x-4}{x-1} + \frac{4x}{x-1}$ **21.** $\frac{5}{2x^2} + \frac{4}{3x}$

22. $\frac{8}{5x} - \frac{4}{x^2}$ **23.** $\frac{4}{x+3} + \frac{3x}{x-2}$ **24.** $\frac{5x+1}{x-3} - \frac{2x}{x-1}$

Divide.

25. Divide $4x^3 - 15x^2 - 6x$ by $3x$.

26. Divide $81x^2 - 25$ by $9x - 5$.

27. Divide $2x^2 + 11x + 12$ by $x + 4$.

28. Divide $5x^2 + 4x - 7$ by $x + 2$.

Solve the equation.

29. $\frac{3}{4x-9} = \frac{x}{3}$ **30.** $\frac{5}{9} + \frac{2}{9x} = \frac{3}{x}$ **31.** $\frac{5}{x+3} - \frac{3}{x-2} = \frac{5}{3x-6}$

Sketch a graph of the function.

32. $y = \frac{1}{x-4} + 3$ **33.** $y = 5 - \frac{2}{x}$ **34.** $y = \frac{x-5}{x+2}$

35. **BAGEL SHOP** You invest $30,000 to start a bagel shop. You can produce bagels for $1.20 per dozen. How many dozen must you produce before your average cost per dozen (including your initial investment of $30,000) drops to $1.80?

36. **CARNIVAL GAME** At a carnival game, a dart is thrown at the board shown at the right. Assume it is equally likely to land anywhere on the board. Write a model that gives the probability that the dart will land in the small rectangle. Evaluate the model when $x = 5$.