EXPLORING DATA AND STATISTICS

11.3

# Direct and Inverse Variation

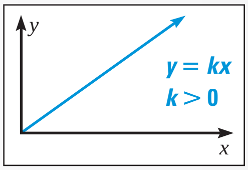
## GOAL 1 USING DIRECT AND INVERSE VARIATION

In Lesson 4.5 you studied *direct variation*. Now you will review variation problems and learn about a different type of variation, called **inverse variation**.

### MODELS FOR DIRECT AND INVERSE VARIATION

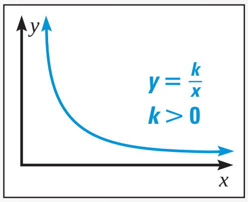
**DIRECT VARIATION** The variables $x$ and $y$ *vary directly* if for a constant $k$

$$\frac{y}{x} = k, \text{ or } y = kx,\ k \neq 0.$$




**INVERSE VARIATION** The variables $x$ and $y$ *vary inversely* if for a constant $k$

$$xy = k, \text{ or } y = \frac{k}{x},\ k \neq 0.$$




The number $k$ is the **constant of variation.**

**What you should learn**

GOAL 1 Use direct and inverse variation.

GOAL 2 Use direct and inverse variation to model **real-life** situations, such as the snowshoe problem in **Exs. 43 and 44**.

**Why you should learn it**

▼ To solve **real-life** situations such as relating the banking angle of a bicycle to its turning radius in **Example 3**.

REAL LIFE

### EXAMPLE 1 *Using Direct and Inverse Variation*

When $x$ is 2, $y$ is 4. Find an equation that relates $x$ and $y$ in each case.

**a.** $x$ and $y$ vary directly **b.** $x$ and $y$ vary inversely

**SOLUTION**

**a.** $\frac{y}{x} = k$ **Write direct variation model.**

$\frac{4}{2} = k$ **Substitute 2 for $x$ and 4 for $y$.**

$2 = k$ **Simplify.**

▶ The direct variation that relates $x$ and $y$ is $\frac{y}{x} = 2$, or $y = 2x$.

**b.** $xy = k$ **Write inverse variation model.**

$(2)(4) = k$ **Substitute 2 for $x$ and 4 for $y$.**

$8 = k$ **Simplify.**

▶ The inverse variation that relates $x$ and $y$ is $xy = 8$, or $y = \frac{8}{x}$.

## EXAMPLE 2 *Comparing Direct and Inverse Variation*

Compare the direct variation model and the inverse variation model you found in Example 1 using $x = 1, 2, 3$, and 4.

**a.** numerically **b.** graphically

### SOLUTION

**a.** Use the models $y = 2x$ and $y = \frac{8}{x}$ to make a table.

**Direct Variation:** Because $k$ is positive, $y$ increases as $x$ increases. As $x$ increases by 1, $y$ increases by 2.

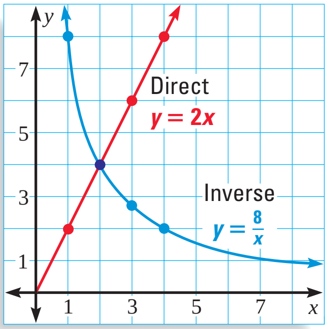
**Inverse Variation:** Because $k$ is positive, $y$ decreases as $x$ increases. As $x$ doubles (from 1 to 2), $y$ is halved (from 8 to 4).

| *x*-value | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| Direct, $y = 2x$ | 2 | 4 | 6 | 8 |
| Inverse, $y = \frac{8}{x}$ | 8 | 4 | $\frac{8}{3}$ | 2 |

**b.** Use the table of values. Plot the points and then connect the points with a smooth curve.

**Direct Variation:** The graph for this model is a line passing through the origin.

**Inverse Variation:** The graph for this model is a curve that approaches the $x$-axis as $x$ increases and approaches the $y$-axis as $x$ gets close to 0.



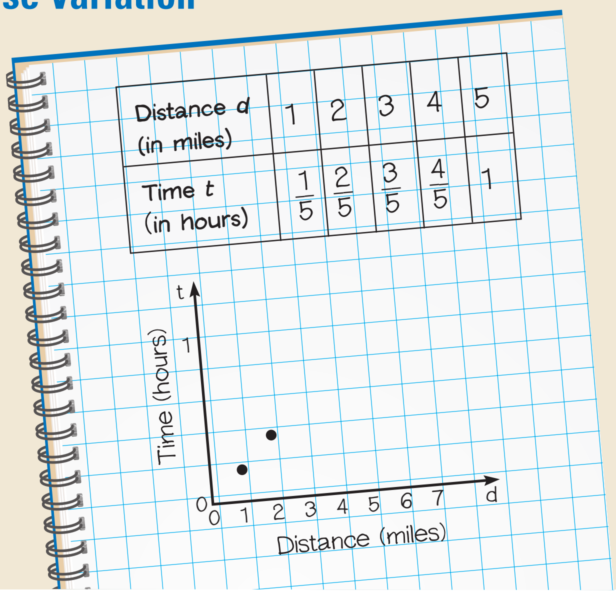

Direct and inverse variation models represent functions because for each value of $x$ there is exactly one value of $y$. For inverse variation, the domain excludes 0.

**ACTIVITY** Developing Concepts

## Investigating Direct and Inverse Variation

**Make a table and draw a graph for each situation. Then decide whether the quantities vary *directly*, *inversely*, or *neither*.**

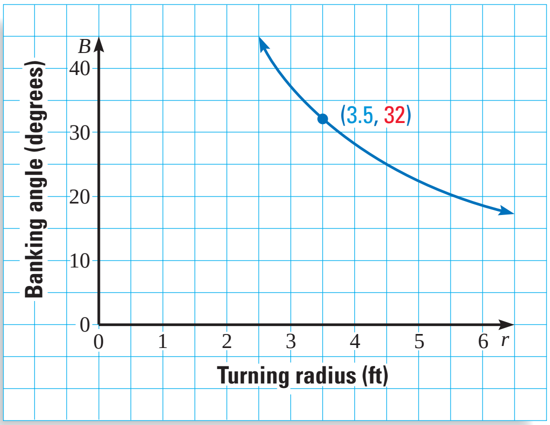
1. You are walking at a rate of 5 miles per hour. How are your distance $d$ and time $t$ related?
2. You take a 5-mile walk. How are your rate $r$ and time $t$ related?
3. A ball is dropped from a height of 32 feet. How are the height $h$ and time $t$ related?
4. A rectangle has an area of 12 square inches. How are the length $l$ and width $w$ related?

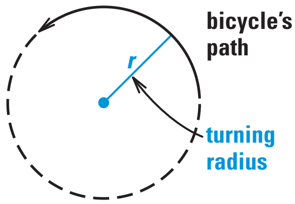


## GOAL 2 USING DIRECT AND INVERSE VARIATION IN REAL LIFE

### EXAMPLE 3 *Writing and Using a Model*

**BICYCLING** The graph below shows a model for the relationship between the banking angle and the turning radius for a bicycle traveling at a particular speed. For the values shown, the banking angle $B$ and the turning radius $r$ can be approximated by an inverse variation.




**a.** Find an inverse variation model that approximately relates $B$ and $r$.

**b.** Use the model to approximate the angle for a radius of 5 feet.

**c.** Use the graph to describe how the banking angle changes as the turning radius gets smaller.

**SOLUTION**

**a.** From the graph, you can see that $B = 32°$ when $r = 3.5$ feet.

$B = \frac{k}{r}$ Write inverse variation model.

$32 = \frac{k}{3.5}$ Substitute 32 for $B$ and 3.5 for $r$.

$112 = k$ Solve for $k$.

▶ The model is $B = \frac{112}{r}$, where $B$ is in degrees and $r$ is in feet.

**b.** Substitute 5 for $r$ in the model found in part (a).

$$B = \frac{112}{5} = 22.4$$

▶ When the turning radius is 5 feet, the banking angle is about 22°.

**c.** As the turning radius gets smaller, the banking angle becomes greater. The bicyclist leans at greater angles. Notice that the increase in the banking angle becomes more rapid when the turning radius is small. For instance, the increase in the banking angle is about 10° for a 1-foot decrease from 4 feet to 3 feet. The increase in the banking angle is only about 4° for a 1-foot decrease from 6 feet to 5 feet.

**FOCUS ON APPLICATIONS**


REAL LIFE **BICYCLE BANKING** A bicyclist tips the bicycle when making a turn. The angle $B$ of the bicycle from the vertical direction is called the *banking angle*.

## GUIDED PRACTICE

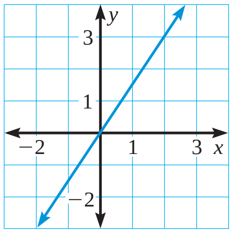
**Vocabulary Check ✓**

**1.** What does it mean for two quantities to vary directly? to vary inversely?

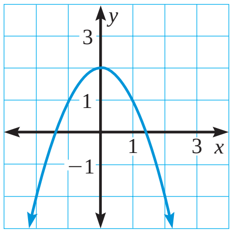
**Concept Check ✓**

**Does the graph show *direct variation, inverse variation*, or *neither*? Explain.**

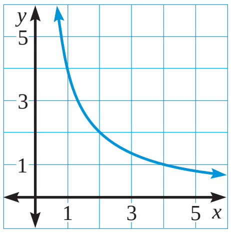
**2.**



**3.**



**4.**



**5.** One student says that the equation $y = -2x$ is an example of direct variation. Another student says it is inverse variation. Which is correct? Explain.

**Skill Check ✓**

**Does the equation model *direct variation, inverse variation,* or *neither*?**

**6.** $x = \frac{4}{y}$ **7.** $y = 7x - 2$ **8.** $a = 12b$ **9.** $ab = 9$

**When $x = 4$, $y = 6$. For the given type of variation, find an equation that relates $x$ and $y$. Then find the value of $y$ when $x = 8$.**

**10.** $x$ and $y$ vary directly **11.** $x$ and $y$ vary inversely

## PRACTICE AND APPLICATIONS

**STUDENT HELP**

**Extra Practice** to help you master skills is on p. 807.

**DIRECT VARIATION EQUATIONS** **The variables $x$ and $y$ vary directly. Use the given values to write an equation that relates $x$ and $y$.**

**12.** $x = 3, y = 9$ **13.** $x = 2, y = 8$ **14.** $x = 18, y = 6$

**15.** $x = 8, y = 24$ **16.** $x = 36, y = 12$ **17.** $x = 27, y = 3$

**18.** $x = 24, y = 16$ **19.** $x = 45, y = 81$ **20.** $x = 54, y = 27$

**INVERSE VARIATION EQUATIONS** **The variables $x$ and $y$ vary inversely. Use the given values to write an equation that relates $x$ and $y$.**

**21.** $x = 2, y = 5$ **22.** $x = 3, y = 7$ **23.** $x = 16, y = 1$

**24.** $x = 11, y = 2$ **25.** $x = \frac{1}{2}, y = 8$ **26.** $x = \frac{13}{5}, y = 5$

**27.** $x = 12, y = \frac{3}{4}$ **28.** $x = 5, y = \frac{1}{3}$ **29.** $x = 30, y = 7.5$

**30.** $x = 1.5, y = 50$ **31.** $x = 45, y = \frac{3}{5}$ **32.** $x = 10.5, y = 7$

**STUDENT HELP**

**HOMEWORK HELP**
**Example 1:** Exs. 12–32
**Example 2:** Exs. 33–38
**Example 3:** Exs. 43–46

**DIRECT OR INVERSE VARIATION** **Make a table of values for $x = 1, 2, 3$, and 4. Use the table to sketch a graph. Decide whether $x$ and $y$ vary *directly* or *inversely*.**

**33.** $y = \frac{4}{x}$ **34.** $y = \frac{3}{2x}$ **35.** $y = 3x$ **36.** $y = \frac{6}{x}$

**FOCUS ON APPLICATIONS**

**REAL LIFE** **SNOWSHOES** Snowshoes are made of lightweight materials that distribute a person's weight over a large area. This makes it possible to walk over deep snow without sinking.

**VARIATION MODELS FROM DATA** **Decide if the data in the table show *direct* or *inverse* variation. Write an equation that relates the variables.**

**37.**

| *x* | 1 | 3 | 5 | 10 | 0.5 |
|---|---|---|---|---|---|
| *y* | 5 | 15 | 25 | 50 | 2.5 |

**38.**

| *x* | 1 | 3 | 4 | 10 | 0.5 |
|---|---|---|---|---|---|
| *y* | 30 | 10 | 7.5 | 3 | 60 |

**VARIATION MODELS IN CONTEXT** **State whether the variables have *direct variation*, *inverse variation*, or *neither*.**

**39.** **BASE AND HEIGHT** The area $B$ of the base and the height $h$ of a prism with a volume of 10 cubic units are related by the equation $Bh = 10$.

**40.** **MASS AND VOLUME** The mass $m$ and the volume $V$ of a substance are related by the equation $2V = m$, where 2 is the density of the substance.

**41.** **DINNER AND BREAKFAST** Alicia cut a pizza into 8 pieces. The number of pieces $d$ that Alicia ate for dinner and the number of pieces $b$ that she can eat for breakfast are related by the equation $b = 8 - d$.

**42.** **HOURS AND PAY RATE** The number of hours $h$ that you must work to earn \$480 and your hourly rate of pay $p$ are related by the equation $ph = 480$.

**SNOWSHOES** **In Exercises 43 and 44, use the following information.** When a person walks, the pressure $P$ on each boot sole varies inversely with the area $A$ of the sole. Denise is walking through deep snow, wearing boots that have a sole area of 29 square inches each. The boot-sole pressure is 4 pounds per square inch when she stands on one foot.

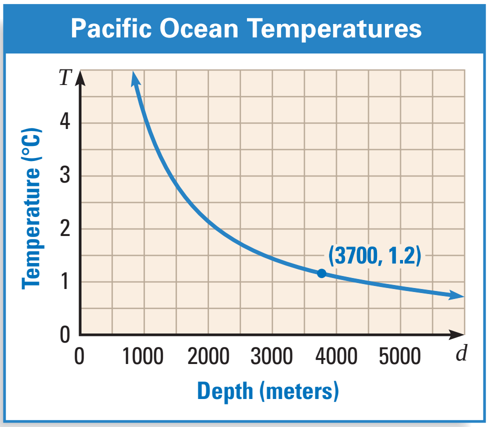
**43.** The constant of variation is Denise's weight in pounds. What is her weight?

**44.** If Denise wears snowshoes, each with an area 11 times that of her boot soles, what is the snowshoe pressure when she stands on one foot?

**OCEAN TEMPERATURES** **The graph for Exercises 45 and 46 shows water temperatures for part of the Pacific Ocean. Assume temperature varies inversely with depth at depths greater than 900 meters.**

**45.** Find a model that relates the temperature $T$ and the depth $d$. Round $k$ to the nearest hundred.

**46.** Find the temperature at a depth of 5000 meters. Round to the nearest tenth of a degree.




**FUEL MILEAGE** **In Exercises 47–49, you are taking a trip on the highway in a car that gets a gas mileage of about 26 miles per gallon for highway driving. You start with a full tank of 12 gallons of gasoline.**

**47.** Find your rate of gas consumption (the gallons of gas used to drive 1 mile).

**48.** Use Exercise 47 to write an equation relating the number of gallons of gas in your tank $g$ and the number of miles $m$ that you have driven on your trip.

**49.** Do the variables $g$ and $m$ vary *directly*, *inversely*, or *neither*? Explain.

**STUDENT HELP**

INTERNET **HOMEWORK HELP** Visit our Web site www.mcdougallittell.com for help with Exs. 47–49.

**Test Preparation**

**50. MULTI-STEP PROBLEM** Use $x$ to represent one dimension of a rectangle, and use $y$ to represent the other dimension.

**a.** Make a table of possible values of $x$ and $y$ if the area of the rectangle is 12 square inches. Then use your table to sketch a graph.

**b.** Do $x$ and $y$ vary *directly*, *inversely*, or *neither*? Explain your reasoning.

**c.** Make a table of possible values of $x$ and $y$ if the area of the rectangle is 24 square inches. Then use your table to sketch a graph in the same coordinate plane you used for your graph in part (a).

**d. CRITICAL THINKING** How is the area of the first rectangle related to the area of the second rectangle? For a given value of $x$, how is the value of $y$ for the first rectangle related to the value of $y$ for the second rectangle? For a given value of $y$, how are the values of $x$ related?

★ **Challenge**

**CONSTANT RATIOS In Exercises 51 and 52, use the following information.**

In a direct variation, the ratio $\frac{y}{x}$ is constant. If $(x_1, y_1)$ and $(x_2, y_2)$ are solutions of the equation $\frac{y}{x} = k$, then $\frac{y_1}{x_1} = k$ and $\frac{y_2}{x_2} = k$. Use the proportion $\frac{y_1}{x_1} = \frac{y_2}{x_2}$ to find the missing value.

**51.** Find $x_2$ when $x_1 = 2$, $y_1 = 3$, and $y_2 = 6$.

**52.** Find $y_2$ when $x_1 = -4$, $y_1 = 8$, and $x_2 = -1$.

**CONSTANT PRODUCTS In an inverse variation, the product $xy$ is constant. If $(x_1, y_1)$ and $(x_2, y_2)$ are solutions of $xy = k$, then $x_1y_1 = x_2y_2$. Use this equation to find the missing value.**

EXTRA CHALLENGE
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**53.** Find $y_2$ when $x_1 = 4$, $y_1 = 5$, and $x_2 = 8$.

**54.** Find $x_2$ when $x_1 = 9$, $y_1 = -3$, and $y_2 = 12$.

# MIXED REVIEW

**SIMPLIFYING FRACTIONS Simplify the fraction.** (Skills Review, p. 781–783)

**55.** $\frac{36}{48}$ **56.** $\frac{27}{108}$ **57.** $\frac{96}{180}$ **58.** $\frac{-15}{125}$

**PROBABILITY Find the probability.** (Review 2.8 for 11.4)

**59.** You roll a die. What is the probability that you will roll a four?

**60.** You roll a die. What is the probability that you will roll an odd number?

**SIMPLIFYING FRACTIONS Simplify the fraction.** (Review 8.3 for 11.4)

**61.** $\frac{y^4 \cdot y^7}{y^5}$ **62.** $\frac{5xy}{5x^2}$ **63.** $\frac{-3xy^3}{3x^3y}$ **64.** $\frac{56x^2y^5}{64x^2y}$

**CLASSIFYING POLYNOMIALS Identify the leading coefficient, and classify the polynomial by degree and by number of terms.** (Review 10.1)

**65.** $-5x - 4$ **66.** $8x^4 + 625$ **67.** $x - x^3$ **68.** $16 - 4x + 3x^2$

# QUIZ 1

*Self-Test for Lessons 11.1–11.3*

**Solve the proportion.** (Lesson 11.1)

**1.** $\frac{x}{10} = \frac{4}{5}$ **2.** $\frac{3}{x} = \frac{7}{9}$ **3.** $\frac{x}{4x - 8} = \frac{2}{x}$ **4.** $\frac{6x + 4}{5} = \frac{2}{x}$

**The variables *x* and *y* vary inversely. Use the given values to write an equation that relates *x* and *y*.** (Lesson 11.3)

**5.** $x = 3, y = 4$ **6.** $x = 12, y = 1$ **7.** $x = 24, y = \frac{3}{4}$

**CAR WASH In Exercises 8 and 9, use the survey results.** (Lesson 11.2)

**8.** In the survey, 413 people said they hand-wash their cars at home. How many people were surveyed? Round to the nearest whole number.

**9.** Use the result of Exercise 8 to find the number of people in the survey who wash their cars at an automatic car wash.

| Method | Percent of people |
|---|---|
| Hand-wash at home | 45.7 |
| Automatic car wash | 22.6 |
| Manually wash it myself at a car wash | 16.3 |
| Others manually wash it at a car wash | 14.8 |
| Other | 0.7 |

▶ Source: Maritz Marketing Research

## MATH & History — History of Polling

APPLICATION LINK www.mcdougallittell.com

**THEN** **IN THE 1930S,** opinion polls were conducted by interviewers knocking on people's doors. Later telephone polling became popular and a sampling method using random digit-dialing was developed.

**NOW** **DURING THE 1990S,** television stations used computers to evaluate exit poll data and present real-time election results.

**After the polls closed in one of the 1998 governor's races, exit poll data reported that the top three candidates had 37%, 34%, and 28% of the vote.**

1. A total of 2,075,280 votes had been counted so far. If the exit polls accurately described the vote count, about how many votes did each candidate have?
2. Use your results from Exercise 1. The final vote count included 16,486 more votes. If these votes were all for one candidate instead of being distributed as exit polls predicted, would the outcome of the election have been changed?


**1948**
Opinion polls incorrectly predict Dewey's win.

**1970s on**

Telephone polling becomes common


**Now**
Internet sites enable voters to learn early results