9.7 Graphing Quadratic Inequalities

What you should learn

GOAL 1 Sketch the graph of a quadratic inequality.

GOAL 2 Use quadratic inequalities as **real-life** models, as in bridge building in **Example 4**.

Why you should learn it

▼ To model a **real-life** situation such as the height of the two towers on the Golden Gate Bridge in **Ex. 32**.

GOAL 1 GRAPHING A QUADRATIC INEQUALITY

In this lesson you will study the following types of **quadratic inequalities**.

$y < ax^2 + bx + c$ $\quad$ $y \le ax^2 + bx + c$

$y > ax^2 + bx + c$ $\quad$ $y \ge ax^2 + bx + c$

The **graph** of a quadratic inequality consists of the graph of all ordered pairs (x, y) that are solutions of the inequality.

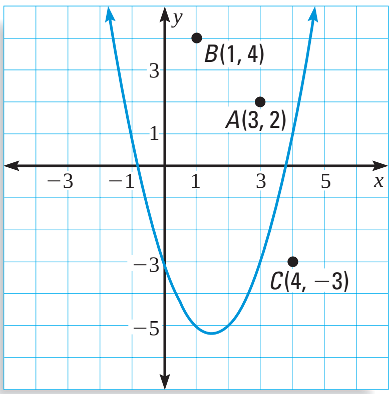
EXAMPLE 1 *Checking Points*

Sketch the graph of $y = x^2 - 3x - 3$. Plot and label the points $A\ (3, 2)$, $B\ (1, 4)$, and $C\ (4, -3)$. Tell whether each point lies inside or outside the parabola.

SOLUTION

1. Sketch the graph of $y = x^2 - 3x - 3$.
2. Plot and label the points $A(3, 2)$, $B(1, 4)$, and $C(4, -3)$.

   A and B lie inside the parabola while C lies outside.



Checking points helps you graph quadratic inequalities by telling you which region to shade. You can use the following guidelines to graph any quadratic inequality.

SKETCHING THE GRAPH OF A QUADRATIC INEQUALITY

STEP 1 Sketch the graph of the equation $y = ax^2 + bx + c$ that corresponds to the inequality. Sketch a dashed parabola for inequalities with $<$ or $>$ to show that the points on the parabola are *not* solutions. Sketch a solid parabola for inequalities with $\le$ or $\ge$ to show that the points on the parabola are solutions.

STEP 2 The parabola you drew separates the coordinate plane into two regions. Test a point that is not on the parabola to find whether it is a solution of the inequality.

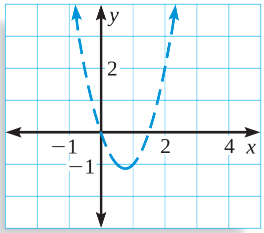
STEP 3 If the test point is a solution, shade its region. If not, shade the other region.

EXAMPLE 2 *Graphing a Quadratic Inequality*

Sketch the graph of $y < 2x^2 - 3x$.

SOLUTION

STEP 1 Sketch the equation $y = 2x^2 - 3x$ that corresponds to the inequality $y < 2x^2 - 3x$. Use a dashed line since the inequality is $<$. The parabola opens up.

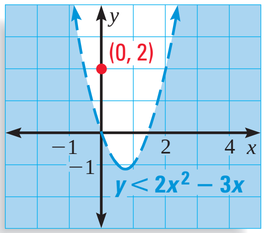


STEP 2 Test a point that is not on the parabola, say (0, 2).

$y < 2x^2 - 3x$	**Write original inequality.**
$2 \overset{?}{<} 2(0)^2 - 3(0)$	**Substitute 0 for *x* and 2 for *y*.**
$2 \not< 0$	**2 is not less than 0.**

Because 2 is not less than 0, the ordered pair (0, 2) is not a solution.

STEP 3 The point (0, 2) is not a solution and it is inside the parabola, so the graph of $y < 2x^2 - 3x$ is all the points outside, but not on, the parabola.



STUDENT HELP

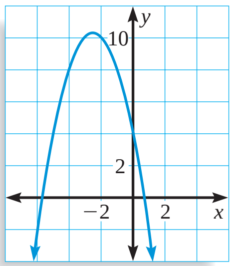
Look Back
For help with checking ordered pairs as solutions, see p. 360.

EXAMPLE 3 *Graphing a Quadratic Inequality*

Sketch the graph of $y \le -x^2 - 5x + 4$.

SOLUTION

STEP 1 Sketch the equation $y = -x^2 - 5x + 4$ that corresponds to the inequality $y \le -x^2 - 5x + 4$. Use a solid line since the inequality is $\le$. The parabola opens down.

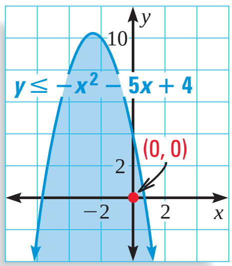


STEP 2 Test a point that is not on the parabola, say (0, 0).

$y \le -x^2 - 5x + 4$	**Write original inequality.**
$0 \overset{?}{\le} -(0)^2 - 5(0) + 4$	**Substitute 0 for *x* and *y*.**
$0 \le 4$	**True.**

Because 0 *is* less than or equal to 4, the ordered pair (0, 0) is a solution.

STEP 3 The point (0, 0) is a solution and it is inside the parabola, so the graph of $y \le -x^2 - 5x + 4$ is all the points inside or on the parabola $y = -x^2 - 5x + 4$.



GOAL 2 USING QUADRATIC INEQUALITIES IN REAL LIFE

EXAMPLE 4 *Using a Quadratic Inequality Model*

FOCUS ON APPLICATIONS

BRIDGE BUILDING The Akashi Kaikyo Bridge is part of a network of bridges built to connect all of Japan's four main islands. It connects the cities of Kobe and Awaji.

APPLICATION LINK
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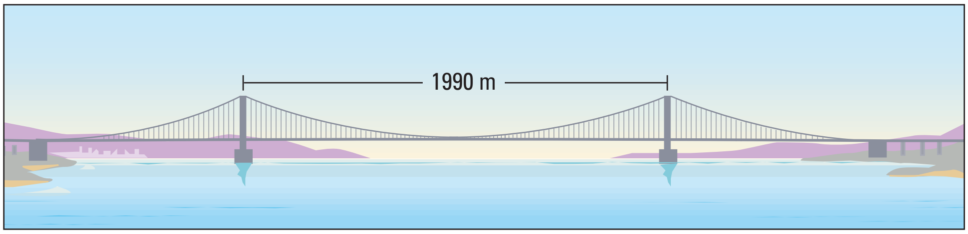
BRIDGE BUILDING The Akashi Kaikyo Bridge in Japan is the longest suspension bridge in the world. The shape of the main cable between the two main towers can be modeled by

$$y = 0.000188x^2 + 15$$

where x is the distance (in meters) from the center of the bridge and y is the height (in meters) above the road.



a. How high are the towers above the road?

b. Write a system of inequalities that describes the region under the main cable, above the roadbed, and between the main towers. Assume that the roadbed is modeled by $y = 0$. Graph the system.

SOLUTION

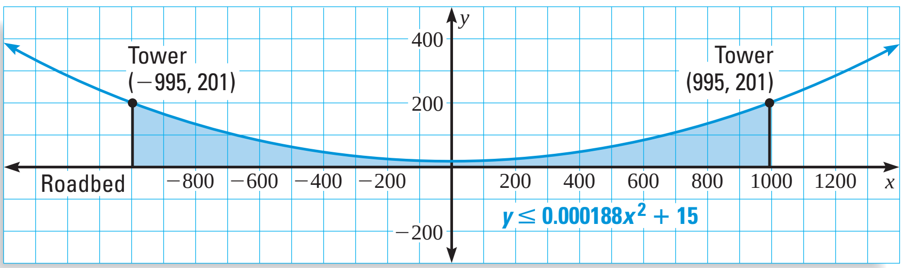
a. The diagram shows that the distance between the two towers is 1990 meters. Because the vertex of the parabola is at the center of the bridge, the x-coordinates of the tops of the towers are ± 995. Substitute either value.

$y = 0.000188x^2 + 15$	Write quadratic equation.
$= 0.000188(995)^2 + 15$	Substitute 995 for x.
≈ 201.12	Approximate with a calculator.

▶ The towers are about 201 meters above the road.

b. The parabola $y = 0.000188x^2 + 15$ opens up. To include the region below the parabola and the main cable, use the inequality $\leq$.

Region below the cable:	$y \leq 0.00188x^2 + 15$
Region between the towers:	$-995 \leq x \leq 995$
Region above the roadbed:	$y \geq 0$



GUIDED PRACTICE

Vocabulary Check ✓

1. Give an example of each of the types of quadratic inequalities.

Concept Check ✓

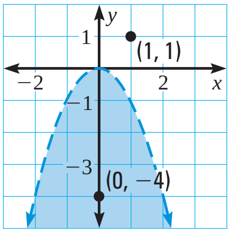
2. Describe the steps used to sketch the graph of a quadratic inequality.

3. Is (0, 3) inside or outside the graph of $y = x^2 + 3x + 2$?

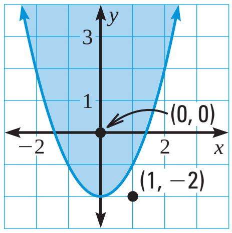
Skill Check ✓

Decide whether each labeled ordered pair is a solution of the inequality.

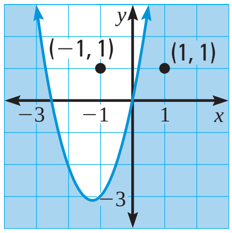
4. $y < -x^2$



5. $y \geq x^2 - 2$



6. $y \leq 2x^2 + 5x$



Sketch the graph of the inequality.

7. $y \leq x^2$

8. $y > -x^2 + 3$

9. $y < x^2 + 2x$

10. $y \geq x^2 - 2x$

11. $y > -2x^2 + 5x$

12. $y < 4x^2 - 2x + 1$

PRACTICE AND APPLICATIONS

STUDENT HELP

Extra Practice to help you master skills is on p. 805.

SOLUTIONS **Decide whether the ordered pair is a solution of the inequality.**

13. $y \geq 2x^2 - x$, (2, 6)

14. $y < x^2 + 9x$, (−3, 10)

15. $y > 4x^2 - 7x$, (2, −10)

16. $y \geq x^2 - 13x$, (−1, 14)

MATCHING INEQUALITIES **In Exercises 17–22, match the graph with its inequality.**

A. $y \geq -2x^2 - 2x + 1$

B. $y > -2x^2 + 4x + 3$

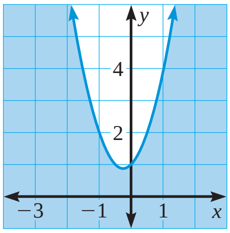
C. $y \leq 2x^2 + x + 1$

D. $y \geq 4x^2 - 6x + 5$

E. $y < 2x^2 - x - 3$

F. $y > -4x^2 + 6x - 5$

17.



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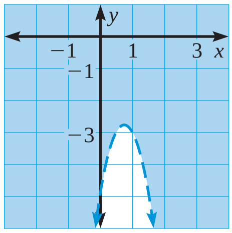

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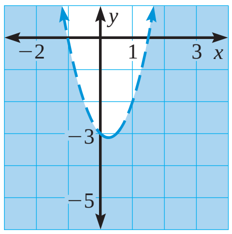

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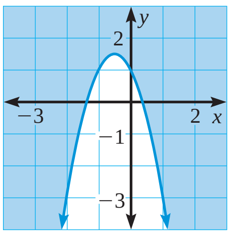

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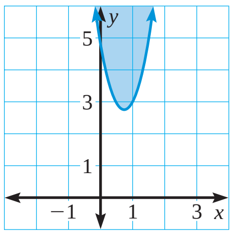

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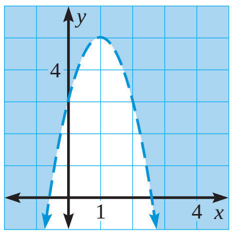


STUDENT HELP

HOMEWORK HELP

Example 1: Exs. 13–22
Example 2: Exs. 17–31
Example 3: Exs. 17–31
Example 4: Exs. 32, 33

SKETCHING GRAPHS **Sketch the graph of the inequality.**

23. $y < -x^2 + x$ **24.** $y \geq x^2 - 3$ **25.** $y \geq x^2 - 5x$

26. $y < -x^2 - 3x - 1$ **27.** $y < x^2 + x + 8$ **28.** $y \leq -x^2 + 3x + 2$

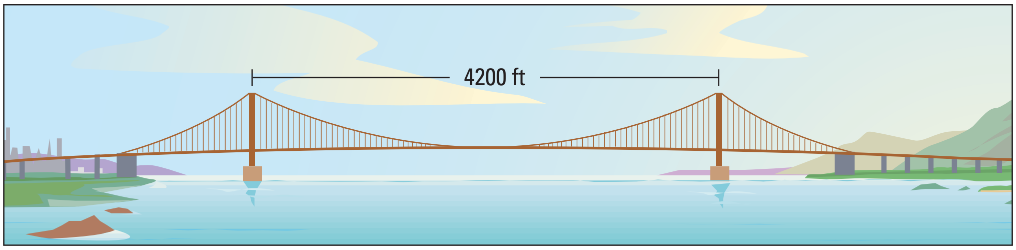
29. $y > -4x^2 - 8x - 4$ **30.** $y \geq -4x^2 - 3x + 8$ **31.** $y \geq 2x^2 + 5x + 3$

STUDENT HELP

INTERNET **HOMEWORK HELP** Visit our Web site www.mcdougallittell.com for help with Exs. 32–33.

GOLDEN GATE BRIDGE **Use the following information for Exercises 32 and 33.**

The Golden Gate Bridge connects northern California to San Francisco. The vertical cables that are suspended from the main cable lie in the region given by $0 \leq y \leq 0.000112x^2 + 5$ where x is horizontal distance from the middle of the bridge (in feet) and y is vertical distance (in feet) above the road.

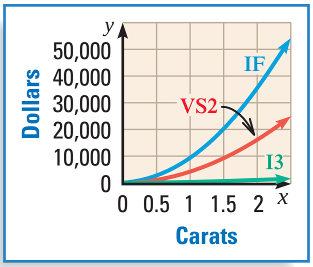


32. How high are the towers above the road?

33. Sketch a graph of the region between the towers and under the main cable.

DIAMONDS **In Exercises 34–38, use the equations below which represent the sample price *y* (in hundreds of dollars) of diamonds of *x* carats with different grades of flawlessness.**

Flawlessness	Equation
IF	$y = 82.12x^2 + 119.13x - 48.15$
VS2	$y = 38.88x^2 + 26.62x - 5.07$
I3	$y = 1.87x^2 + 14.83x - 2.3$



34. Which of the three grades of diamond is the least flawed? How do you know?

35. Which of the three grades of diamond is the most flawed? How do you know?

36. A two-carat diamond is selling for the wholesale price of \$55,000. The diamond is described as having the highest quality. Describe the region in the graph that contains the point (2, 55,000). Is this the highest quality for a two-carat diamond? Explain.

37. The wholesale price of a one-carat diamond is \$12,000. Assume that the price fairly represents the flawlessness of the diamond. Which region contains the point (1, 12,000)? What grade of flawlessness is the diamond?

A. $y > 1.87x^2 + 14.83x - 2.3$ **B.** $y < 1.87x^2 + 14.83x - 2.3$

38. You see a $\frac{1}{2}$-carat diamond on display for a wholesale price of \$4000. Next to it is a two-carat diamond for a wholesale price of \$12,000. Is the more expensive one necessarily less flawed?

FOCUS ON APPLICATIONS

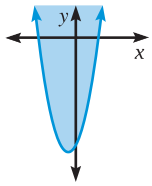
REAL LIFE **DIAMONDS** A carat is a unit of weight for precious stones. The price of a diamond is determined by its weight, cut, clarity, and degree of flawlessness.

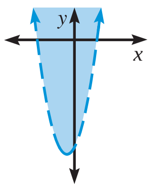
Test Preparation

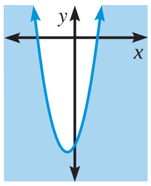
39. MULTIPLE CHOICE Which ordered pair is *not* a solution of the inequality $y \geq 2x^2 - 7x - 10$?

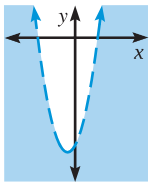
Ⓐ $(0, -4)$ Ⓑ $(-1, -1)$ Ⓒ $(4, -13)$ Ⓓ $(5, 15)$

40. MULTIPLE CHOICE Identify the graph of $y \leq 3x^2 + 2x - 5$.

Ⓐ 

Ⓑ 

Ⓒ 

Ⓓ 

★ Challenge

MILK CONSUMPTION In Exercises 41–43, use the following information. The per capita consumption (pounds per person) of whole milk W and reduced fat milk R from 1980 to 1995 can be modeled by the equations below where t represents years since 1980. ▶ Source: U.S. Department of Agriculture

Whole milk: $W = 0.09t^2 - 5.29t + 114.69$

Reduced fat: $R = -0.23t^2 + 2.40t + 71.39$

41. Sketch a graph of each equation on the same coordinate plane.

42. Shade the region where the per capita consumption of reduced fat milk was greater than that of whole milk. What inequalities represent this region?

43. Find the points where the parabolas intersect. What does each of these points represent?

EXTRA CHALLENGE
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MIXED REVIEW

GRAPHING EQUATIONS Graph the equation. (Review 4.3, 9.3 for 9.8)

44. $y = x + 4$ **45.** $3x + 6y = -18$ **46.** $2x - 4y = 24$

47. $y = x^2 + x + 2$ **48.** $y = -x^2 + 4x + 1$ **49.** $y = 3x^2 - 2x + 6$

DIRECT VARIATION The variables *x* and *y* vary directly. Use the given values to write an equation that relates *x* and *y*. (Review 4.5)

50. $x = 6, y = 42$ **51.** $x = 54, y = -9$ **52.** $x = 7, y = 5$

53. $x = -13, y = -52$ **54.** $x = \frac{3}{4}, y = 3$ **55.** $x = 4.6, y = 1.2$

GRAPHING EXPONENTIAL EQUATIONS Sketch the graph of the exponential equation. (Review 8.5, 8.6)

56. $y = 2^x$ **57.** $y = 0.5^x$ **58.** $y = \frac{1}{2}(2)^x$

59. $y = 0.9^x$ **60.** $y = 4(1.5)^x$ **61.** $y = 5(0.5)^x$

SOLVING QUADRATIC EQUATIONS Use the quadratic formula to solve the equation. (Review 9.5)

62. $x^2 - 2x - 3 = 0$ **63.** $2x^2 - 6x + 4 = 0$ **64.** $2x^2 - 2x - 12 = 0$

65. $-\frac{2}{3}x^2 - 3x + 1 = 0$ **66.** $-7x^2 - 2.5x + 3 = 0$ **67.** $2x^2 + 4x - 3 = 0$