9.4 Solving Quadratic Equations by Graphing

What you should learn

GOAL 1 Solve a quadratic equation graphically.

GOAL 2 Use quadratic models in **real-life** settings, such as the consumption of Swiss cheese in the United States in **Ex. 52**.

Why you should learn it

▼ To make predictions with **real-life** situations, such as the length of a shot put in **Example 4**.

REAL LIFE

GOAL 1 SOLVING QUADRATIC EQUATIONS GRAPHICALLY

In Lesson 4.7 you learned to use the x-intercept of a linear graph to estimate the solution of a linear equation. A similar procedure applies to quadratic equations.

SOLVING QUADRATIC EQUATIONS USING GRAPHS

The solution of a quadratic equation in one variable x can be solved or checked graphically with the following steps.

STEP 1 Write the equation in the form $ax^2 + bx + c = 0$.

STEP 2 Write the related function $y = ax^2 + bx + c$.

STEP 3 Sketch the graph of the function $y = ax^2 + bx + c$. The solutions, or **roots,** of $ax^2 + bx + c = 0$ are the x-intercepts.

EXAMPLE 1 *Representing a Solution Using a Graph*

Solve $\frac{1}{2}x^2 = 8$ algebraically. Represent your solutions as the x-intercepts of a graph.

SOLUTION

$\frac{1}{2}x^2 = 8$ — **Write original equation.**

$x^2 = 16$ — **Multiply each side by 2.**

$x = \pm 4$ — **Find the square root of each side.**

Represent these solutions using a graph.

1. Write the equation in the form $ax^2 + bx + c = 0$.

   $\frac{1}{2}x^2 = 8$ — **Rewrite original equation.**

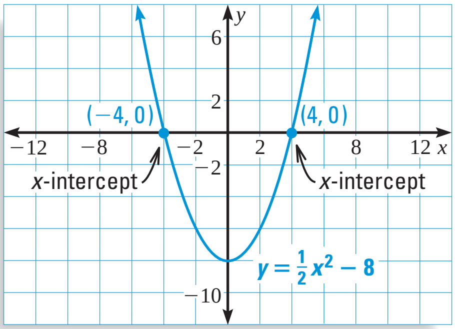
   $\frac{1}{2}x^2 - 8 = 0$ — **Subtract 8 from each side.**

2. Write the related function $y = ax^2 + bx + c$.

   $y = \frac{1}{2}x^2 - 8$

3. Sketch the graph of $y = \frac{1}{2}x^2 - 8$.

   The x-intercepts are ± 4, which are the algebraic solutions.



STUDENT HELP

INTERNET HOMEWORK HELP Visit our Web site www.mcdougallittell.com for extra examples.

EXAMPLE 2 *Solving an Equation Graphically*

Solve $x^2 - x = 2$ graphically. Check your solution algebraically.

SOLUTION

1 Write the equation in the form $ax^2 + bx + c = 0$.

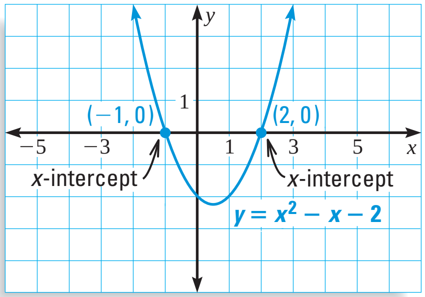
$x^2 - x = 2$ **Write original equation.**

$x^2 - x - 2 = 0$ **Subtract 2 from each side.**

2 Write the related function $y = ax^2 + bx + c$.

$y = x^2 - x - 2$

3 Sketch the graph of the function $y = x^2 - x - 2$.



From the graph, the x-intercepts appear to be $x = -1$ and $x = 2$.

✓CHECK You can check this by substitution.

Check $x = -1$:

$x^2 - x = 2$

$(-1)^2 - (-1) \stackrel{?}{=} 2$

$1 + 1 = 2$

Check $x = 2$:

$x^2 - x = 2$

$2^2 - 2 \stackrel{?}{=} 2$

$4 - 2 = 2$

EXAMPLE 3 *Using a Graphing Calculator*

STUDENT HELP

Look Back For help with using a graphing calculator to graph equations, see p. 248.

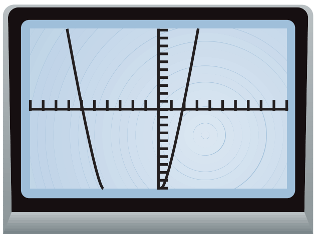
Use a graphing calculator to approximate the solution of $x^2 + 4x - 12 = 0$. Check your solution algebraically.

SOLUTION

$x^2 + 4x - 12 = 0$ **Write original equation.**

$y = x^2 + 4x - 12$ **Write related function.**

Use a graphing calculator to graph the function.



From the graph, you can see that the x-intercepts appear to be $x = 2$ and $x = -6$.

✓CHECK You can check this by substitution.

Check $x = 2$:

$x^2 + 4x - 12 = 0$

$2^2 + 4(2) - 12 \stackrel{?}{=} 0$

$4 + 8 - 12 = 0$

Check $x = -6$:

$x^2 + 4x - 12 = 0$

$(-6)^2 + 4(-6) - 12 \stackrel{?}{=} 0$

$36 + (-24) - 12 = 0$

GOAL 2 USING QUADRATIC MODELS IN REAL LIFE

REAL LIFE
Shot Put

EXAMPLE 4 *Comparing Two Quadratic Models*

A shot put champion performs an experiment for your math class. Assume that both times he releases the shot with the same initial speed but at different angles. The path of each put can be modeled by one of the equations below, where x represents the horizontal distance (in feet) and y represents the height (in feet).

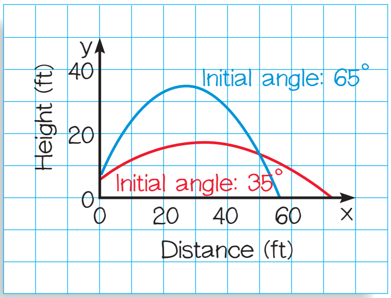
Initial angle of 35°: $y = -0.010735x^2 + 0.700208x + 6$

Initial angle of 65°: $y = -0.040330x^2 + 2.144507x + 6$

Which angle results in a farther throw?

SOLUTION

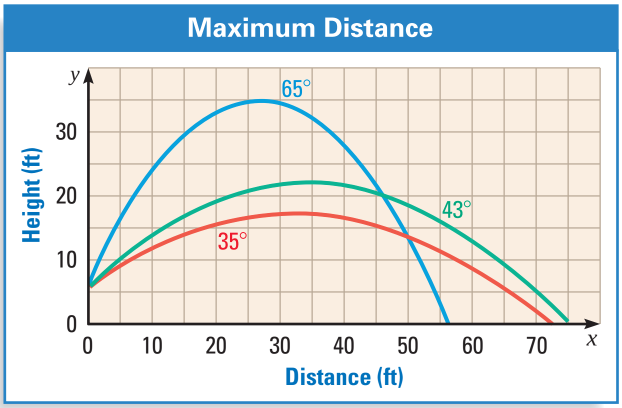
Begin by graphing both models in the same coordinate plane. Use only positive x-values because x represents the distance of the throw.



You can see that the 35° angle produced a greater distance.

. . . .

PARABOLIC MOTION It can be shown that an angle of 45° produces the maximum distance if an object is propelled from ground level. An angle smaller than 45° is better when a shot is released above the ground. If a shot is released from 6 feet above the ground, a 43° angle produces a maximum distance of 74.25 feet.



GUIDED PRACTICE

Vocabulary Check ✓

1. Define the roots of a quadratic equation.

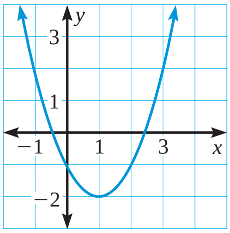
Concept Check ✓

2. Write the steps for solving a quadratic equation using a graph.

Match the quadratic equation with the graph of its related function.

A. $x^2 = 3$ **B.** $x^2 + x = 4$ **C.** $-x^2 = -2x - 1$

3.



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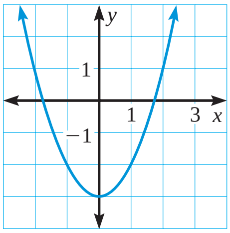

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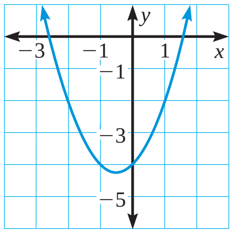


Skill Check ✓

Solve the equation algebraically. Check the solution graphically.

6. $3x^2 = 12$ **7.** $4x^2 = 16$ **8.** $5x^2 = 125$

9. $3x^2 = 27$ **10.** $8x^2 = 32$ **11.** $-2x^2 = -18$

Solve the equation graphically. Check the solutions algebraically.

12. $3x^2 = 48$ **13.** $x^2 - 4 = 5$ **14.** $-x^2 + 7x - 10 = 0$

15. $2x^2 + 6x = -4$ **16.** $\frac{1}{3}x^2 + x - 6 = 0$ **17.** $x - x^2 = -20$

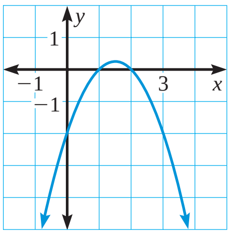
PRACTICE AND APPLICATIONS

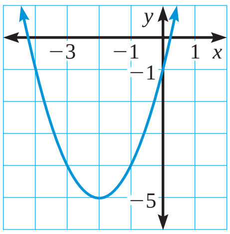
STUDENT HELP

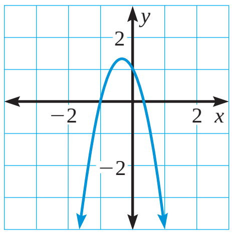
Extra Practice
to help you master skills is on p. 805.

ESTIMATING ROOTS **For each quadratic equation, use the graph to estimate the roots of the equation.**

18. $-x^2 + 3x - 2 = 0$ **19.** $x^2 + 4x - 1 = 0$ **20.** $-3x^2 - 2x + 1 = 0$







STUDENT HELP

HOMEWORK HELP
Example 1: Exs. 21–32
Example 2: Exs. 18–20, 33–44
Example 3: Exs. 45–50
Example 4: Exs. 51–56

CHECKING GRAPHICALLY **Solve the equation algebraically. Check the solutions graphically.**

21. $2x^2 = 32$ **22.** $4x^2 = 16$ **23.** $4x^2 = 100$

24. $\frac{1}{3}x^2 = 3$ **25.** $\frac{1}{4}x^2 = 36$ **26.** $\frac{1}{2}x^2 = 18$

27. $x^2 - 11 = 14$ **28.** $x^2 - 13 = 36$ **29.** $x^2 - 4 = 12$

30. $x^2 - 53 = 11$ **31.** $x^2 + 37 = 118$ **32.** $2x^2 - 89 = 9$

GRAPHICAL REPRESENTATION **Represent the solution graphically. Check the solution algebraically.**

33. $x^2 - x = 6$ **34.** $x^2 + 2x = 3$ **35.** $-x^2 + x = -2$

36. $-x^2 + 3x = -4$ **37.** $2x^2 + 4x = 6$ **38.** $3x^2 + 3x = 6$

39. $x^2 - 4x - 5 = 0$ **40.** $x^2 - x = 12$ **41.** $x^2 + 4x = 21$

42. $8x^2 - 4 = 4x$ **43.** $-7x^2 - 21x = 14$ **44.** $-2x^2 - 4x = -30$

APPROXIMATING SOLUTIONS **Use a graphing calculator to approximate the solution of the equation.**

45. $x^2 - 3x - 4 = 0$ **46.** $x^2 + 6x - 7 = 0$

47. $5x^2 + 5x - 1 = 0$ **48.** $-8x^2 - 24x + 32 = 0$

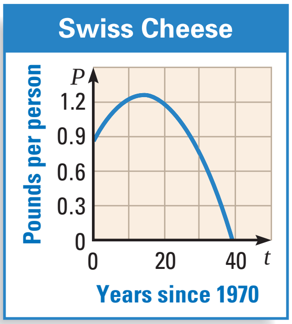
49. $\frac{1}{2}x^2 + 2x - 16 = 0$ **50.** $\frac{5}{4}x^2 + 15x + 40 = 0$

51. **SHOT PUT** A shot put champion releases the shot at a 55° angle. The path of the shot is modeled by the equation below where x represents the horizontal distance (in feet) and y represents the height (in feet).

$$y = -0.021895x^2 + 1.428148x + 6$$

How far does the shot travel? Assume the initial speed is the same as in Example 4. Compare the results with the results in Example 4.

52. **SWISS CHEESE** The consumption of Swiss cheese in the United States from 1970 to 1996 can be modeled by $P = -0.002t^2 + 0.056t + 0.889$, where P is the number of pounds per person and t is the number of years since 1970. According to the graph of the model, in what year would the consumption of Swiss cheese drop to 0? Is this a realistic prediction?



Source: U.S. Department of Agriculture

FOCUS ON APPLICATIONS

REAL LIFE **RECREATIONAL VEHICLES** Motorized homes, travel and camping trailers, and truck campers are all considered recreational vehicles.

RV SALES **In Exercises 53–56, use the following information.** The number of recreational vehicles (RVs) sold in the United States from 1985 to 1991 can be modeled by $N = -9.5t^2 + 48.9t + 343.5$, where N represents the number of vehicles sold (in thousands) and t represents the number of years since 1985.

INTERNET **DATA UPDATE** of Recreation Vehicle Industry Association data at www.mcdougallittell.com

53. Sketch a graph of the model for positive values of x and y.

54. Use the graph to estimate a positive root of the equation

$$0 = -9.5t^2 + 48.9t + 343.5.$$

55. According to the model, in what year will the number of RVs sold in the United States drop to 0?

56. Do you think this prediction is realistic? What factors might explain a decrease or an increase in the number of sales of recreational vehicles?

Test Preparation

57. **MULTIPLE CHOICE** What are the x-intercepts of $y = x^2 - 2x - 3$?

Ⓐ 1 and -3 Ⓑ -2 and -3 Ⓒ 6 and -1 Ⓓ 3 and -1

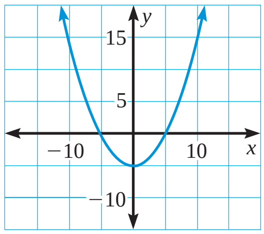
58. **MULTIPLE CHOICE** Choose the equation whose roots are shown in the graph below.

Ⓐ $5x^2 - 1 = 0$

Ⓑ $\frac{1}{5}x^2 - 5 = 0$

Ⓒ $x^2 - 5 = 0$

Ⓓ $\frac{1}{5}x^2 - 1 = 0$



★ Challenge

SOLVING EQUATIONS WITH A GRAPHING CALCULATOR **The solution of a quadratic equation can be found by graphing each side separately and locating the points of intersection. You may wish to consult page 532 for help in approximating solutions.**

59. $3x^2 + 2x + 5 = 6x^2$

60. $-5x^2 + 4x = 2x^2 - 8$

61. $-2x^2 + 5x = 8x^2 - 2$

62. $-x^2 - 2 = 4x^2 + 6x - 3$

63. $0.75x^2 + 2.67x = 6.22x^2 - 4.1$

64. $-4.87x^2 + 1.44x = 5.22x^2 + 6x$

EXTRA CHALLENGE

www.mcdougallittell.com

MIXED REVIEW

SOLVING LINEAR SYSTEMS **Solve the system of linear equations if possible. Does the system have exactly *one solution, no solution,* or *infinitely many solutions*?** (Review 7.5)

65. $-2x + 8y = 11$
$x + 6y = 2$

66. $4x + 5y = 37$
$-5x + 3y = 0$

67. $-2x + 2y = 4$
$x - y = -2$

68. $8x + 4y = -4$
$4x - y = -20$

69. $10x - 6y = -5$
$3y = 5x + 2$

70. $4y = -5x - 3$
$15x + 12y = 9$

EVALUATING EXPRESSIONS **Evaluate the expression to the nearest hundredth.** (Review 9.1 for 9.5)

71. $\frac{5 \pm 3\sqrt{6}}{2}$

72. $\frac{2 \pm 6\sqrt{3}}{3}$

73. $\frac{-3 \pm 2\sqrt{5}}{-1}$

74. $\frac{-2 \pm 4\sqrt{2}}{-2}$

SIMPLIFYING RADICAL EXPRESSIONS **Simplify the radical expression.** (Review 9.2)

75. $\sqrt{40}$

76. $\sqrt{24}$

77. $\sqrt{60}$

78. $\sqrt{200}$

79. $\frac{1}{2}\sqrt{80}$

80. $\frac{1}{3}\sqrt{27}$

81. $\frac{1}{8}\sqrt{32}$

82. $\frac{2}{3}\sqrt{300}$

83. **LUNCH TIME** At lunch, you order 2 pasta dishes and 1 type of salad. Your friend orders 1 pasta dish and 2 types of salads. The restaurant charges the same price for each pasta dish and the same price for each salad. Your bill is \$13.85 and your friend's bill is \$9.85. How much did each pasta dish and each salad cost? (Review 7.4)