EXPLORING DATA AND STATISTICS

2.8

Probability and Odds

What you should learn

GOAL 1 Find the probability of an event.

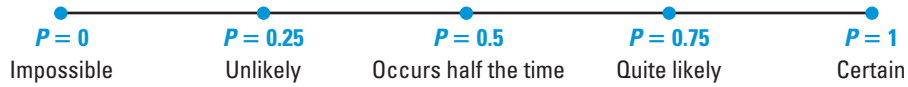
GOAL 2 Find the odds of an event.

Why you should learn it

▼ To represent **real-life** situations, like the probability that a baby sea turtle reaches the ocean in **Ex. 22**.

GOAL 1 FINDING THE PROBABILITY OF AN EVENT

The **probability of an event** is a measure of the likelihood that the event will occur. It is a number between 0 and 1, inclusive.



When you do a probability experiment, the different possible results are called **outcomes**. When an experiment has N *equally likely* outcomes, each of them occurs with probability $\frac{1}{N}$. For example, in the roll of a six-sided number cube, the possible outcomes are 1, 2, 3, 4, 5, and 6, and the probability associated with each outcome is $\frac{1}{6}$.

An **event** consists of a collection of outcomes. In the roll of a six-sided number cube, an "even roll" consists of the outcomes 2, 4, and 6. The **theoretical probability** of an even roll is $\frac{3}{6} = \frac{1}{2}$. The outcomes for an event you wish to have happen are called **favorable outcomes**, and the theoretical probability of this event is calculated by the following rule.

$$\text{Theoretical probability } P = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}}$$

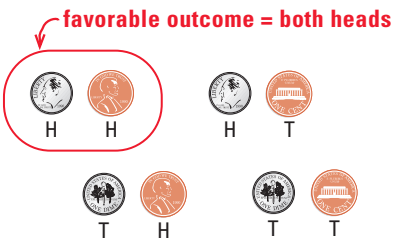
EXAMPLE 1 *Finding the Probability of an Event*

a. You toss two coins. What is the probability P that both are heads?

b. An algebra class has 17 boys and 16 girls. One student is chosen at random from the class. What is the probability P that the student is a girl?

SOLUTION

a. There are four possible outcomes that are equally likely.



$$P = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}} = \frac{1}{4} = 0.25$$

b. Because the student is chosen at random, it is equally likely that any of the 33 students will be chosen. Let "number of girls" be the favorable outcome. Let "number of students" be the total number of outcomes.

$$P = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}} = \frac{\text{Number of girls}}{\text{Number of students}} = \frac{16}{33} \approx 0.485$$

Another type of probability is **experimental probability**. This type of probability is based on repetitions of an actual experiment and is calculated by the following rule.

$$\text{Experimental probability } P = \frac{\text{Number of favorable outcomes observed}}{\text{Total number of trials}}$$

ACTIVITY
Developing Concepts

Investigating Experimental Probability

Partner Activity Toss three coins 20 times and record the number of heads for each of the 20 tosses.

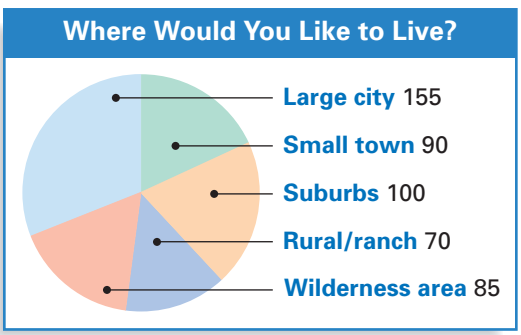
1. Use your results to find the experimental probability of getting three heads when three coins are tossed.
2. Combine your results with those of all the other pairs in your class. Then use the combined results to find the experimental probability of getting three heads when three coins are tossed.
3. Find the theoretical probability of getting three heads when three coins are tossed. How does it compare with your results from **Step 2**?

A survey is a type of experiment. Probabilities based on the outcomes of a survey can be used to make predictions.

EXAMPLE 2 *Using a Survey to Find a Probability*

SURVEYS Use the circle graph at the right showing the responses of 500 teens to a survey asking "Where would you like to live?"

If you were to ask a randomly chosen teen this question, what is the experimental probability that the teen would say "large city?"



▶ Source: *Youth Views*

FOCUS ON CAREERS

REAL LIFE MARKET RESEARCHERS use surveys to predict the needs and interests of their potential customers. Then they can make their products better.

INTERNET CAREER LINK
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SOLUTION

Let "choosing large city" be the favorable outcome. Let "number surveyed" be the total number of trials.

$$\text{Experimental probability } P = \frac{\text{Number choosing large city}}{\text{Number surveyed}}$$

$$= \frac{155}{500}$$

$$= 0.31$$

GOAL 2 FINDING THE ODDS OF AN EVENT

STUDENT HELP

Study Tip

Odds are always read as the ratio of one quantity *to* another. For example, $\frac{4}{3}$ is read as "four *to* three," not as "four thirds."

THE ODDS OF AN EVENT

When all outcomes are equally likely, the **odds** that an event will occur are given by the formula below.

$$\text{Odds} = \frac{\text{Number of favorable outcomes}}{\text{Number of unfavorable outcomes}}$$

EXAMPLE 3 *Finding the Odds of an Event*

You randomly choose an integer from 0 through 9. What are the odds that the integer is 4 or more?

SOLUTION There are 6 favorable outcomes: 4, 5, 6, 7, 8, and 9. There are 4 unfavorable outcomes: 0, 1, 2, and 3.

$$\text{Odds} = \frac{\text{Number of favorable outcomes}}{\text{Number of unfavorable outcomes}} = \frac{6}{4} = \frac{3}{2}$$

▶ The odds that the integer is 4 or more are 3 to 2.

. . . . . .

If you know the probability that an event will occur, then you can find the odds.

$$\text{Odds} = \frac{\text{Probability event will occur}}{\text{Probability event will not occur}} = \frac{\text{Probability event will occur}}{1 - (\text{Probability event will occur})}$$

EXAMPLE 4 *Finding Odds from Probability*

The probability that a randomly chosen household has a cat is 0.27. What are the odds that a household has a cat? ▶Source: American Veterinary Medical Association

SOLUTION

$$\text{Odds} = \frac{\text{Probability event will occur}}{1 - (\text{Probability event will occur})}$$

$$= \frac{0.27}{1 - 0.27}$$ **Substitute for probabilities.**

$$= \frac{0.27}{0.73}$$ **Simplify denominator.**

$$= \frac{27}{73}$$ **Multiply numerator and denominator by 100.**

▶ The odds are 27 to 73 that a randomly chosen household has a cat.

GUIDED PRACTICE

Vocabulary Check ✓

1. Using the results of a student lunch survey, you determine that the probability that a randomly chosen student likes green beans is 0.38. Is this probability theoretical or experimental?

Concept Check ✓

2. The probability that an event will occur is 0.4. Is it more likely that the event will occur, or is it more likely that the event will *not* occur?

3. The odds that an event will occur are 3 to 4. Is it more likely that the event will occur, or is it more likely that the event will *not* occur?

Skill Check ✓

Tell whether the event is best described as *impossible, unlikely, likely to occur half the time, quite likely,* or *certain.* Explain your reasoning.

4. The probability of rain is 80%, or 0.8.

5. The odds in favor of winning a race are $\frac{1}{3}$.

6. The odds of being chosen for a committee are 1 to 1.

TEST PLANNING **Suppose it is equally likely that a teacher will chose any day from Monday, Tuesday, Wednesday, Thursday, and Friday to have the next test.**

7. What is the probability that the next test will be on a Friday?

8. What are the odds that the next test will be on a day starting with the letter T?

PRACTICE AND APPLICATIONS

STUDENT HELP

Extra Practice to help you master skills is on p. 798.

PROBABILITY **Find the probability of randomly choosing a red marble from the given bag of red and white marbles.**

9. Number of red marbles: 16
   Total number of marbles: 64

10. Number of red marbles: 8
    Total number of marbles: 40

11. Number of white marbles: 7
    Total number of marbles: 20

12. Number of white marbles: 24
    Total number of marbles: 32

ODDS **Find the odds of randomly choosing the indicated letter from a bag that contains the letters in the name of the given state.**

13. S; MISSISSIPPI

14. N; PENNSYLVANIA

15. A; NEBRASKA

16. G; VIRGINIA

STUDENT HELP

HOMEWORK HELP

Example 1: Exs. 9–12
Example 2: Exs. 21–24, 26, 28, 29
Example 3: Exs. 13–16, 25, 27, 30
Example 4: Exs. 17, 18, 20

PROBABILITY TO ODDS **Given the probability, find the odds.**

17. The probability of randomly choosing a club from a deck of cards is 0.25.

18. The probability of tossing a total of 7 using two number cubes is $\frac{1}{6}$.

COIN TOSS **You toss two coins.**

19. What is the theoretical probability that only one is tails?

20. Use the theoretical probability to find the odds that only one is tails.

21. NUMBER CUBES You toss a six-sided number cube 20 times. For twelve of the tosses the number tossed was 3 or more.

a. What is the experimental probability that a number tossed is 3 or more?

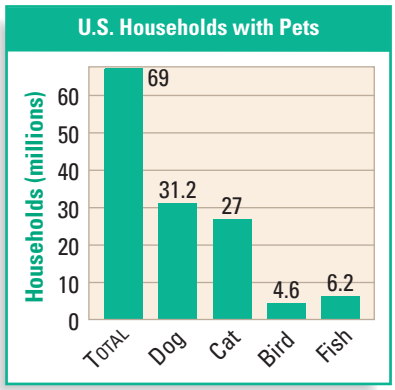
b. What are the odds that a number tossed is 3 or more?

22. BABY SEA TURTLES A sea turtle buries 90 eggs in the sand. From the 50 eggs that hatch, 37 turtles do not make it to the ocean. What is the probability that an egg chosen at random hatched and the baby turtle made it to the ocean?

STUDENT HELP

INTERNET HOMEWORK HELP
Visit our Web site www.mcdougallittell.com for help with Exs. 23–27.

PETS In Exercises 23–25, use the graph.



▶ Source: American Veterinary Medical Assoc.

23. What is the probability that a pet-owning household chosen at random owns a dog?

24. What is the probability that a pet-owning household chosen at random does *not* own a fish?

25. There are approximately 98.8 million households in the United States. If a household is chosen at random, what are the odds that the household owns a pet?

MOVING In Exercises 26 and 27, use the table, which shows the percent of citizens from various age groups who changed homes within the United States from 1995 to 1996.

26. What is the probability that a citizen from the 15–19 age group changed homes?

27. What are the odds that a citizen from the 25–29 age group moved to a home in a different state?

Percent of U.S. Citizens of Given Ages who Moved

Age group	Total	Same county	Different county, same state	Different state
15 to 19	15	10	3	2
20 to 24	33	21	6	5
25 to 29	32	20	7	5
30 to 44	16	10	3	3

▶ Source: U.S. Bureau of the Census

FOCUS ON APPLICATIONS

REAL LIFE **FAULT LINES**
A fault line, like the one shown above, may appear in the earth's surface in a region that is prone to earthquakes.

EARTHQUAKES In Exercises 28–30, use the table, which shows the number of earthquakes of magnitude 4.0 or greater in the western United States since 1900. The magnitude of an earthquake indicates its severity.

28. What is the probability that the magnitude of an earthquake is from 6.0 to 6.9?

29. What is the probability that the magnitude of an earthquake is *not* from 4.0 to 4.9?

30. What are the odds that the magnitude of an earthquake is from 7.0 to 7.9?

Magnitude	Number of earthquakes
8 and higher	1
7.0–7.9	18
6.0–6.9	129
5.0–5.9	611
4.0–4.9	3171

▶ Source: U.S. Geological Survey

31. **WARDROBE** Your cousin spills spaghetti sauce on her shirt and asks to borrow a clean shirt from you for the rest of the day. You decide to let her choose from a selection of 4 sweatshirts, 1 hockey shirt, 8 T-shirts, and 3 tank tops. If it is equally likely that your cousin will choose any shirt, what are the odds that she will choose a sweatshirt?

32. *Writing* Suppose you randomly choose a marble from a bag holding 11 green, 4 blue, and 5 yellow marbles. Use probability *and* odds to express how likely it is that you choose a yellow marble. Compare the two ways of expressing how likely it is to choose a yellow marble. If you find one (probability or odds) easier to understand or more useful than the other, explain why.

Test Preparation

MULTIPLE CHOICE **Use the table, which shows the number of students in each grade for three high schools and the number that get to each school by riding the bus.**

3rd District High School Student Bus Riders						
	Marshall		Jordan		King	
Grade	Total	Bus riders	Total	Bus riders	Total	Bus riders
9th	247	198	326	195	265	151
10th	232	176	311	205	273	142
11th	194	141	304	196	264	139
12th	211	142	285	173	231	106

33. If you are in the tenth grade, what is the probability that you ride a bus to get to school?

Ⓐ $\frac{141}{194} \approx 0.73$ Ⓑ $\frac{196}{304} \approx 0.64$ Ⓒ $\frac{523}{816} \approx 0.64$ Ⓓ $\frac{139}{264} \approx 0.53$

34. If you go to Marshall High School, what are the odds that you do *not* ride the bus to get to school?

Ⓐ 657 to 227 Ⓑ 657 to 884 Ⓒ 227 to 657 Ⓓ 227 to 884

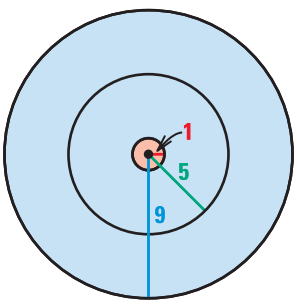
Challenge

BULL'S-EYE **In Exercises 35–38, use the drawing of the dart board below, and the following information.**

Assume that when a dart is thrown and hits the board, the dart is equally likely to hit any point on the board. The probability that a dart lands within the red bull's-eye circle is given by the following equation.

$$P = \frac{\text{Area of bull's-eye}}{\text{Area of board}}$$

(*Hint*: See the Table of Formulas on p. 813.)



35. What is the area of the bull's-eye?

36. What is the probability that a dart lands within the middle circle?

37. What is the probability that a dart lands between the middle and outer circles?

38. What are the odds that a dart lands on the bull's-eye?

EXTRA CHALLENGE
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MIXED REVIEW

MENTAL MATH **Write a question that can be used to solve the equation. Then use mental math to solve the equation.** (Review 1.4)

39. $x + 17 = 25$ **40.** $a - 5 = 19$ **41.** $2b - 1 = 10$

42. $11t = 110$ **43.** $\frac{y}{15} = 8$ **44.** $3x + 15 = 24.6$

TRANSLATING VERBAL SENTENCES **Translate the sentence into an equation or an inequality.** (Review 1.5, 2.1)

45. 17 less than a number z is 9.

46. 8 more than a number r is less than 17.

47. -3 is the sum of a number y and -6.

48. -9 is equal to a number y decreased by 21.

EVALUATING EXPRESSIONS **Evaluate the expression.** (Review 2.3 for 3.1)

49. $-8 + 4 - 9$ **50.** $12 - (-8) - 5$ **51.** $20 + (-17) - 8$

52. $-17 + 25 - 34$ **53.** $-6.3 + 4.1 - 9.5$ **54.** $2 - 11 + 5 - (-16)$

55. $-29.4 - (-8) + 4$ **56.** $\frac{1}{2} + \left(-\frac{4}{5}\right) - \frac{2}{3}$ **57.** $-1\frac{3}{8} + 4\frac{3}{4} - 7\frac{1}{2}$

QUIZ 3

Self-Test for Lessons 2.7 and 2.8

Find the quotient. (Lesson 2.7)

1. $-28 \div \frac{4}{7}$ **2.** $\frac{36}{\frac{2}{3}}$ **3.** $\frac{32}{\frac{1}{4}}$ **4.** $48 \div (-12)$

5. $75 \div (-15)$ **6.** $-144 \div 9$ **7.** $-120 \div \frac{3}{8}$ **8.** $-\frac{13}{27} \div \left(-1\frac{4}{9}\right)$

In Exercises 9–16, simplify the division expression. (Lesson 2.7)

9. $42x \div (-6)$ **10.** $-56 \div (-8x)$ **11.** $9x \div \frac{1}{2}$ **12.** $20 \div \frac{4}{x}$

13. $25x \div \frac{5}{7}$ **14.** $15t \div \left(-\frac{3}{4}\right)$ **15.** $66y \div \left(-\frac{6}{5}\right)$ **16.** $-\frac{2x}{18} \div (-4)$

17. What is the probability that you randomly choose a purple marble from a bag containing 11 red, 6 green, and 8 purple marbles? (Lesson 2.8)

Tell whether the event is best described as *impossible, unlikely, likely to occur half the time, quite likely,* or *certain.* Explain your reasoning. (Lesson 2.8)

18. The probability of snow is 20%, or 0.2.

19. The probability of taking the test is 1.

20. The odds in favor of winning a car are $\frac{1}{30{,}000}$.

21. The odds of being chosen for the softball team are 15 to 6.