6.3 Solving Compound Inequalities

What you should learn

GOAL 1 Write, solve, and graph compound inequalities.

GOAL 2 Model a **real-life** situation with a compound inequality, such as the distances in **Example 6**.

Why you should learn it

▼ To describe **real-life** situations, such as elevations on Mount Rainier in **Example 2**.

GOAL 1 SOLVING COMPOUND INEQUALITIES

In Lesson 6.1 you studied four types of simple inequalities. In this lesson you will study *compound inequalities*. A **compound inequality** consists of two inequalities connected by *and* or *or*.

EXAMPLE 1 *Writing Compound Inequalities*

Write an inequality that represents the set of numbers and graph the inequality.

a. All real numbers that are greater than zero *and* less than or equal to 4.

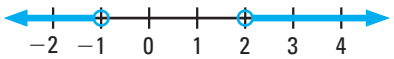
b. All real numbers that are less than -1 *or* greater than 2.

SOLUTION

a. $0 < x \leq 4$

This inequality is also written as $0 < x$ *and* $x \leq 4$.

b. $x < -1$ *or* $x > 2$



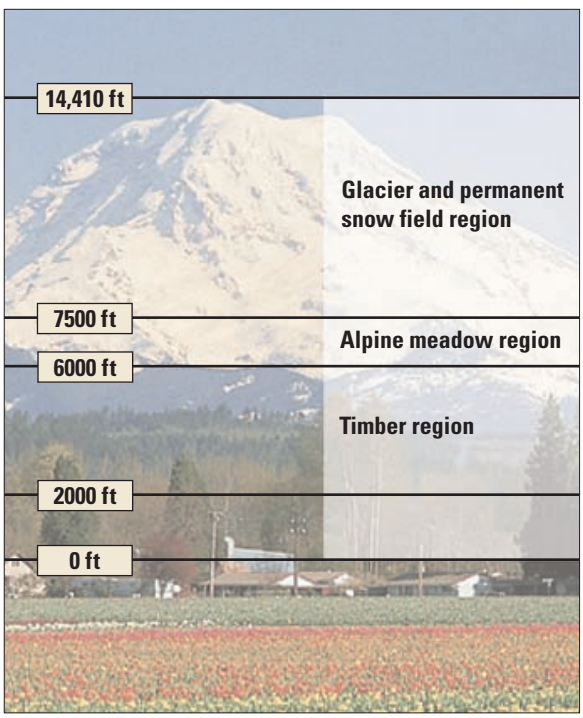
EXAMPLE 2 *Compound Inequalities in Real Life*

Write an inequality that describes the elevations of the regions of Mount Rainier.

a. Timber region below 6000 ft

b. Alpine meadow region below 7500 ft

c. Glacier and permanent snow field region



SOLUTION Let y represent the approximate elevation (in feet).

a. Timber region: $2000 \leq y < 6000$

b. Alpine meadow region: $6000 \leq y < 7500$

c. Glacier and permanent snow field region: $7500 \leq y \leq 14{,}410$

STUDENT HELP

INTERNET HOMEWORK HELP
Visit our Web site www.mcdougallittell.com for extra examples.

EXAMPLE 3 Solving a Compound Inequality with And

Solve $-2 \leq 3x - 8 \leq 10$. Graph the solution.

SOLUTION

Isolate the variable x between the two inequality symbols.

$-2 \leq 3x - 8 \leq 10$ **Write original inequality.**

$6 \leq 3x \leq 18$ **Add 8 to each expression.**

$2 \leq x \leq 6$ **Divide each expression by 3.**

▶ The solution is all real numbers that are greater than or equal to 2 *and* less than or equal to 6.

EXAMPLE 4 Solving a Compound Inequality with Or

Solve $3x + 1 < 4$ *or* $2x - 5 > 7$. Graph the solution.

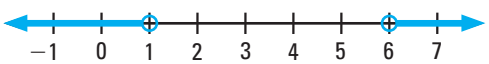
SOLUTION A solution of this inequality is a solution of either of its simple parts. You can solve each part separately.

$3x + 1 < 4$ *or* $2x - 5 > 7$

$3x < 3$ *or* $2x > 12$

$x < 1$ *or* $x > 6$

▶ The solution is all real numbers that are less than 1 *or* greater than 6.



STUDENT HELP

Study Tip
When you multiply or divide by a negative number to solve a compound inequality, remember that you have to reverse *both* inequality symbols.

EXAMPLE 5 Reversing Both Inequality Symbols

Solve $-2 < -2 - x < 1$. Graph the solution.

SOLUTION

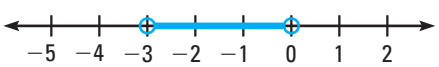
Isolate the variable x between the two inequality signs.

$-2 < -2 - x < 1$ **Write original inequality.**

$0 < -x < 3$ **Add 2 to each expression.**

$0 > x > -3$ **Multiply each expression by −1 and *reverse* both inequality symbols.**

▶ To match the order of numbers on a number line, this compound inequality is usually written as $-3 < x < 0$. The solution is all real numbers that are greater than -3 *and* less than 0.



GOAL 2 MODELING REAL-LIFE PROBLEMS

REAL LIFE *Distances*

EXAMPLE 6 *Modeling with a Compound Inequality*

You have a friend Bill who lives three miles from school and another friend Mary who lives two miles from the same school. You wish to estimate the distance d that separates their homes.

a. What is the smallest value d might have?

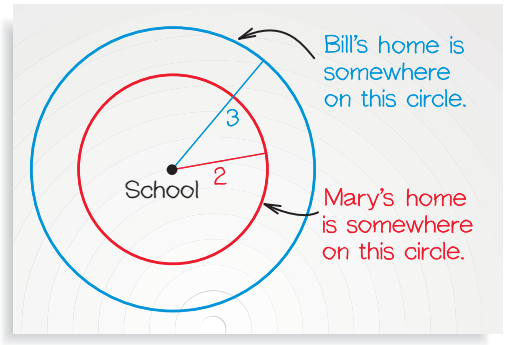
b. What is the largest value d might have?

c. Write an inequality that describes all the possible values that d might have.

PROBLEM SOLVING STRATEGY

SOLUTION

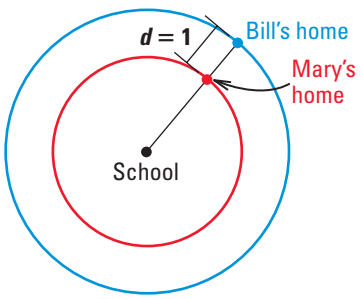
DRAW A DIAGRAM A good way to begin this problem is to draw a diagram with the school at the center of a circle.



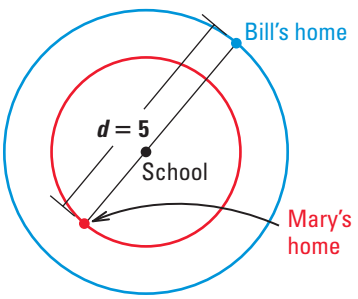
Bill's home is somewhere on the circle with radius 3 miles and center at the school.

Mary's home is somewhere on the circle with radius 2 miles and center at the school.

a. If both homes are on the same line going toward school, the distance is 1 mile.



b. If both homes are on the same line but in opposite directions from school, the distance is 5 miles.



c. The values of d can be described by the inequality $1 \leq d \leq 5$.

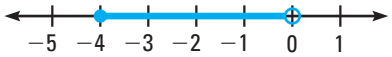
GUIDED PRACTICE

Vocabulary Check ✓

Concept Check ✓

1. Write a compound inequality and explain why it is a compound inequality.

2. Write a compound inequality that describes the graph at the right.



Skill Check ✓

WRITING INEQUALITIES **Write an inequality that represents the statement and graph the inequality.**

3. x is less than 5 and greater than 2.

4. x is greater than 3 or less than −1.

Solve the inequality.

5. $7 < 4 + x < 8$

6. $-1 < 2x + 3 \leq 13$

7. $6 < 4x - 2 \leq 14$

8. $4x - 1 > 7$ or $5x - 1 < -6$

9. $2x + 3 < -1$ or $3x - 5 > -2$

10. $4 \leq -8 - x < 7$

11. In Example 6, write an inequality that describes the situation if you live two miles from school and your friend lives one mile from the same school.

PRACTICE AND APPLICATIONS

STUDENT HELP

Extra Practice to help you master skills is on p. 802.

WRITING INEQUALITIES **Write an inequality that represents the statement and graph the inequality.**

12. x is less than 8 and greater than 3.

13. x is greater than 7 or less than 5.

14. x is less than 5 and is at least 0.

15. x is less than 4 and is at least −9.

16. x is less than −2 and is at least −4.

17. x is greater than −6 and less than −1.

SOLVING INEQUALITIES **Solve the inequality. Write a sentence that describes the solution.**

18. $6 < x - 6 \leq 8$

19. $-5 < x - 3 < 6$

20. $-4 < 2 + x < 1$

21. $8 \leq 2x + 6 \leq 18$

22. $6 + 2x > 20$ or $8 + x \leq 0$

23. $-3x - 7 \geq 8$ or $-2x - 11 \leq -31$

24. $-4 \leq -3x - 13 \leq 26$

25. $-13 \leq 5 - 2x < 9$

GRAPHING INEQUALITIES **Solve the inequality and graph the solution.**

26. $3 \leq 2x < 7$

27. $-25 < 5x < -20$

28. $-5 < 3x + 4 < 19$

29. $-4 \leq 9x - 1 < 5$

30. $2x + 7 < 3$ or $5x + 5 \geq 10$

31. $3x + 8 > 17$ or $2x + 5 \leq 7$

STUDENT HELP

HOMEWORK HELP

Example 1: Exs. 12–17
Example 2: Exs. 36–38
Example 3: Exs. 18–35
Example 4: Exs. 18–35
Example 5: Exs. 18–35
Example 6: Exs. 41–44

CHECKING SOLUTIONS **Solve the inequality and graph the solution. Then check graphically whether the given x-value is a solution by graphing the x-value on the same number line.**

32. $-4 < 4x - 8 < 12; x = 1$

33. $-1 < 7x - 15 \leq 20; x = 5$

34. $-2x \geq 6$ or $2x + 1 > 5; x = 0$

35. $7 \leq -2x + 21 < 31; x = 0$

FOCUS ON APPLICATIONS

REAL LIFE **FINE ART** The painting above is *Still Life with Apples* by Paul Cézanne. Another of Cézanne's paintings, *Still Life with Curtain, Pitcher and Bowl of Fruit,* sold at auction for $60.5 million in 1999.

36. PRICES OF FINE ART In 1958, the painting *Still Life with Apples* by Paul Cézanne sold at auction for $252,000. The painting was auctioned again in 1993 and sold for $28.6 million. Write a compound inequality that represents the different values that the painting probably was worth between 1958 and 1993.

37. TELEVISION ADVERTISING In 1967 a 60-second TV commercial during the first Super Bowl cost $85,000. In 1998 advertisers paid $2.6 million for 60 seconds of commercial time (two 30-second spots). Write a compound inequality that represents the different prices that 60 seconds of commercial time during the Super Bowl probably cost between 1967 and 1998.

38. ANTELOPES The table gives the weights of some adult antelopes. The eland is the largest antelope in Africa. The royal antelope is the smallest of all. Write a compound inequality that represents the different weights of these adult antelopes.

Antelope	Weight (lb)
Eland	2000
Kudu	700
Nyala	280
Springbok	95
Royal	7

39. LOGICAL REASONING Explain why the inequality $3 < x < 1$ has no solution.

40. LOGICAL REASONING Explain why the inequality $x < 2$ *or* $x > 1$ has every real number as a solution.

SCIENCE CONNECTION **Use the diagram of distances in our solar system.**

41. Which compound inequality best describes the distance d (in miles) between the Sun and any of the nine planets?

A. $10^7 < d < 10^{10}$

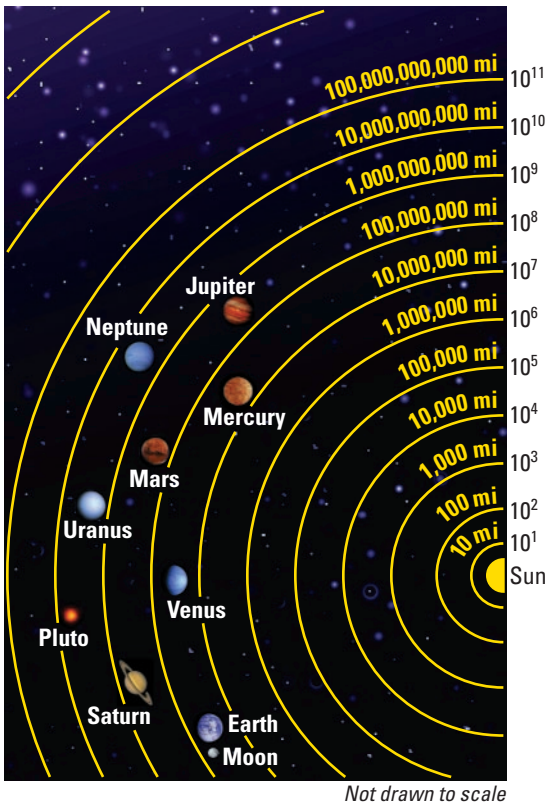
B. $10^6 < d < 10^{11}$

C. $10^8 < d < 10^{11}$

42. Write an inequality to describe an estimate of the distance d (in miles) between the Sun and Mercury.

43. Write an inequality to describe an estimate of the distance d (in miles) between the Sun and Saturn.

44. The Moon's distance d (in miles) from Earth varies from about 220,000 miles to about 250,000 miles. Write an inequality to represent this fact.



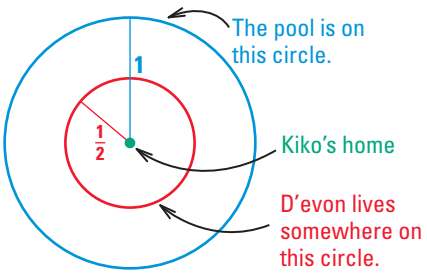
Not drawn to scale

Test Preparation

45. MULTI-STEP PROBLEM You have a friend Kiko who lives one mile from a pool. Another friend D'evon lives 0.5 mile from Kiko. Kiko walks from her home to the pool, where she meets D'evon. They both walk from the pool to D'evon's home and then to Kiko's home.

a. Write an inequality that describes the values, d, of the possible distances that D'evon could live from the pool.

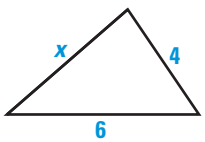
b. Write an inequality that describes the values, D, of the possible distances that Kiko could have walked. (Assume that each part of the walk followed a straight path.)



Challenge

GEOMETRY CONNECTION **Write a compound inequality that must be satisfied by the length of the side labeled *x*. Use the fact that the sum of the lengths of any two sides of a triangle is greater than the length of the third side.**

46.



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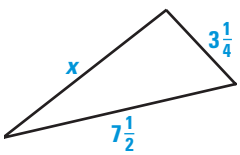

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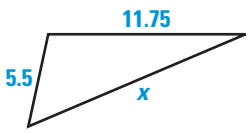

EXTRA CHALLENGE
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MIXED REVIEW

EVALUATING EXPRESSIONS **Evaluate the expression.** (Review 1.1)

49. $x + 5$ when $x = 2$

50. $6.5a$ when $a = 4$

51. $m - 20$ when $m = 30$

52. $\frac{x}{15}$ when $x = 30$

53. $5x$ when $x = 3.3$

54. $4.2p$ when $p = 4.1$

SOLVING EQUATIONS **Solve the equation.** (Review 3.1, 3.2 for 6.4)

55. $x + 17 = 9$

56. $8 = x + 2\frac{1}{2}$

57. $x - 4 = 12$

58. $x - (-9) = 15$

59. $\frac{1}{2}x = -6$

60. $-3x = -27$

61. $4x = -28$

62. $-\frac{3}{4}x = 21$

63. $x + 3.2 = 11$

64. $x - 4 = 16.7$

65. $\frac{x}{-6} = -\frac{1}{2}$

66. $\frac{5}{6}x = -25$

67. ICE SKATING An ice skating rink charges \$4.75 for admission and skate rental. If you bring your own skates, the admission is \$3.25. You can buy a pair of ice skates for \$45. How many times must you go ice skating to justify buying your own skates? (Review 3.4)

HIKING **You are hiking a six-mile trail at a constant rate in Topanga Canyon. You begin at 10 A.M. At noon you are two miles from the end of the trail.** (Review 5.5)

68. Write a linear equation that gives the distance d (in miles) from the end of the trail in terms of time t. Let t represent the number of hours since 10 A.M.

69. Find the distance you are from the end of the trail at 11 A.M.

QUIZ 1

Self-Test for Lessons 6.1–6.3

In Exercises 1–13, solve the inequality and graph the solution. (Lessons 6.2 and 6.3)

1. $x + 2 < 7$

2. $-3 + x \leq -11$

3. $3.4x \leq 13.6$

4. $5 \leq -\frac{x}{2}$

5. $-4x - 2 \geq 14$

6. $-5 < x - 8 < 4$

7. $-x - 4 > 3x - 12$

8. $x + 3 \leq 2(x - 7)$

9. $-10 \leq -4x - 18 \leq 30$

10. $-3 < x + 6$ or $-\frac{x}{3} > 4$

11. $2 - x < -3$ or $2x + 14 < 12$

12. $2x - 6 < -8$ or $10 - 5x < -19$

13. $6x - 2 > -7$ or $-3x - 1 > 11$

14. **AMUSEMENT PARK** A person must be at least 52 inches tall to ride the *Power Tower* ride at Cedar Point in Ohio. Write an inequality that describes the required heights. (Lesson 6.1)

15. **TEMPERATURES** The lowest temperature ever recorded was $-128.6°F$ at the Soviet station Vostok in Antarctica. The highest temperature ever recorded was $136°F$ at Azizia, Libya. Write a compound inequality whose solution includes all of the other temperatures T ever recorded. (Lesson 6.3)

▶ Source: National Climatic Data Center

MATH & History — History of Communication

APPLICATION LINK
www.mcdougallittell.com

THEN **IN 1860** you could send a message from St. Joseph, Missouri, to Sacramento, California, in 10 to 11 days via Pony Express. By 1884 you could send a message in at least $4\frac{1}{2}$ days by transcontinental railroad.

NOW **BY 1972** it took less than a second to send a message across the country via e-mail.

Write an inequality to represent the time it took to send the message.

1. Via Pony Express. Let t represent the time (in days).
2. Via transcontinental railroad. Let t represent the time (in hours).
3. Via e-mail. Let t represent the time (in seconds).