CHAPTER 3
Extension

Inductive and Deductive Reasoning

What you should learn

GOAL Identify and use inductive and deductive reasoning.

Why you should learn it

To use logical reasoning in mathematics and in everyday life as in **Examples 3 and 4**.

GOAL USING INDUCTIVE AND DEDUCTIVE REASONING

Reasoning is used in mathematics, science, and everyday life. When you reach a conclusion, called a **generalization**, based on several observations, you are using **inductive reasoning**. Such a generalization is not always true. If a counterexample is found, then the generalization is proved to be false.

EXAMPLE 1 *Inductive Reasoning in Everyday Life*

You go to the school library every day for a week to do research for a paper. You notice that the computer you plan to use is not in use at 3:00 each day.

a. Using inductive reasoning, what would you conclude about the computer?

b. What counterexample would show that your generalization is false?

SOLUTION

a. From your observations, you conclude that the computer is available every weekday at 3:00.

b. A counterexample would be to find the computer in use one day at 3:00.

STUDENT HELP

Look Back
For help with counterexamples, see p. 66.

You use inductive reasoning when you make generalizations about number patterns. You assume the next number in the sequence will follow the pattern.

EXAMPLE 2 *Inductive Reasoning and Sequences*

Using inductive reasoning, name the next three numbers of the sequence 1, 5, 9, 13, 17, 21, ...

SOLUTION

The first step is to look for a pattern in the list of numbers. In this case, each number in the list is 4 more than the previous number ($1 + 4 = \mathbf{5}$, $5 + 4 = \mathbf{9}$, $9 + 4 = \mathbf{13}$, $13 + 4 = \mathbf{17}$). The next three numbers would be $21 + 4 = \mathbf{25}$, $25 + 4 = \mathbf{29}$, and $29 + 4 = \mathbf{33}$.

When you use facts, definitions, rules, or properties to reach a conclusion, you are using **deductive reasoning**. A conclusion reached in this way is proved to be true. Deductive reasoning often uses **if-then statements**. The *if* part is called the **hypothesis** and the *then* part is called the **conclusion**. When deductive reasoning has been used to *prove* an if-then statement then the fact that the hypothesis is true implies that the conclusion is true.

EXAMPLE 3 *Use of If-Then Statements*

During the last week of a class, your teacher tells you that if you receive an A on the final exam, you will have earned an A average in the course.

a. Identify the hypothesis and the conclusion of the if-then statement.

b. Supose you receive an A on the final exam. What will your final grade be?

SOLUTION

a. The hypothesis is "you receive an A on the final exam." The conclusion is "you will have earned an A average in the course."

b. You can conclude that you have earned an A in the course.

EXAMPLE 4 *Deductive Reasoning in Mathematics*

To show that the statement "If a, b, and c are real numbers, then $(a + b) + c = (c + b) + a$" is true by deductive reasoning, you can write each step and justify it using the properties of addition.

$(a + b) + c = c + (a + b)$	Commutative property
$= c + (b + a)$	Commutative property
$= (c + b) + a$	Associative property

STUDENT HELP

Look Back
For help with the properties of addition, see p. 73.

EXERCISES

In Exercises 1–4, identify the reasoning as inductive or deductive. Explain.

1. The tenth number in the list 1, 4, 9, 16, 25, . . . is 100.

2. You know that in your neighborhood, if it is Sunday, then no mail is delivered. It is Sunday, so you conclude that the mail will not be delivered.

3. If the last digit of a number is 2, then the number is divisible by 2. You conclude that 98,765,432 is divisible by 2.

4. You notice that for several values of x the value of x^2 is greater than x. You conclude that the square of a number is greater than the number itself.

5. Find a counterexample to show that the conclusion in Exercise 4 is false.

6. Give an example of inductive reasoning and an example of deductive reasoning. Do not use any of the examples already given.

7. Name the next three numbers of the sequence: 0, 3, 6, 9, 12, 15, 18, . . .

In Exercises 8 and 9, identify the hypothesis and the conclusion of the if-then statement.

8. If you add two odd numbers, then the answer will be an even number.

9. If you are in Minnesota in January, then you will be cold.

10. Use deductive reasoning and the properties of addition to show that if z is a real number, then $(z + 2) + (-2) = z$.