CHAPTER 9

Chapter Summary

WHAT did you learn?	Why did you learn it?
Evaluate and approximate square roots. (9.1)	Find solutions using a quadratic model for mineral hardness. (p. 509)
Solve a quadratic equation.	
• by finding square roots (9.1)	Estimate the time for an object to fall. (p. 506)
• by sketching its graph (9.4)	Compare shot-put throws. (p. 528)
• by using the quadratic formula. (9.5)	Model vertical motion problems. (p. 535)
Simplify radicals. (9.2)	Compare the speeds of two sailboats. (p. 513)
Sketch the graph of a quadratic function. (9.3)	Estimate the maximum height of a water spray. (p. 522)
Find the number of solutions of a quadratic equation by using the discriminant. (9.6)	Decide whether an outcome is possible in a camping situation. (p. 543)
Sketch the graph of a quadratic inequality. (9.7)	Sketch the region between the towers and under the main cable of a suspension bridge. (p. 550)
Choose an algebraic model that best fits a collection of data. (9.8)	Recognize the characteristics of different models in real-life settings. (pp. 555 and 556)

How does Chapter 9 fit into the BIGGER PICTURE of algebra?

A quadratic equation contains the square of a variable. Try to remember the "algebra-geometry" connection between equations in one and two variables. For instance, the equation $x^2 - 9 = 0$ has two solutions, or roots, which correspond to the two x-intercepts of the graph of the function $y = x^2 - 9$. A quadratic function has a U-shaped graph called a *parabola*.

This chapter focuses on quadratic models, which have many applications, such as vertical motion and parabolic path problems. The last lesson helps you decide whether to apply a linear, an exponential, or a quadratic model to fit a collection of real-life data.

STUDY STRATEGY

How did explaining ideas help you to understand a topic?

Talking with a classmate about Lesson 9.4 may have led you to think about ideas that you would understand fully in Lesson 9.6.

Talking about x-intercepts following Lesson 9.4

"Let's see what the graph of $y = x^2$ looks like. It has only one x-intercept. When I solve $x^2 = 0$, I get $x = \sqrt{0} = 0$. It has only one solution, too!"

"If a quadratic function has a graph with no x-intercepts, then I don't think the equation will have any solutions either."

CHAPTER 9

Chapter Review

VOCABULARY

- square root, p. 503
- positive square root, p. 503
- negative square root, p. 503
- radicand, p. 503
- perfect square, p. 504
- irrational number, p. 504
- radical expression, p. 504
- quadratic equation in standard form, p. 505
- leading coefficient, p. 505
- simplest form of a radical expression, p. 512
- quadratic function in standard form, p. 518
- parabola, p. 518
- vertex, p. 518
- axis of symmetry, p. 518
- roots, p. 526
- quadratic formula, p. 533
- discriminant, p. 541
- quadratic inequalities, p. 548
- graph of a quadratic inequality, p. 548

9.1 SOLVING QUADRATIC EQUATIONS BY FINDING SQUARE ROOTS

Examples on pp. 503–506

EXAMPLE To find the real solutions of a quadratic equation of the form $ax^2 + c = 0$, isolate x^2 and then find the square root of each side.

$2x^2 - 98 = 0$	**Write original equation.**
$2x^2 = 98$	**Add 98 to each side.**
$x^2 = 49$	**Divide each side by 2.**
$x = \pm\sqrt{49}$	**Find square roots.**
$x = \pm 7$	**49 is a perfect square.**

Solve the equation.

1. $x^2 - 144 = 0$ **2.** $8y^2 = 968$ **3.** $4t^2 + 19 = 19$

4. $16y^2 - 80 = 0$ **5.** $\frac{1}{5}a^2 = 5$ **6.** $\frac{1}{3}x^2 - 7 = -4$

9.2 SIMPLIFYING RADICALS

Examples on pp. 511–513

EXAMPLES Use the product property and the quotient property.

a. $\sqrt{28} = \sqrt{4 \cdot 7}$	**Factor using perfect square factor.**
$= \sqrt{4} \cdot \sqrt{7} = 2\sqrt{7}$	**Use product property and simplify.**
b. $\sqrt{\frac{169}{625}} = \frac{\sqrt{169}}{\sqrt{625}} = \frac{13}{25}$	**Use quotient property and simplify.**

Simplify the expression.

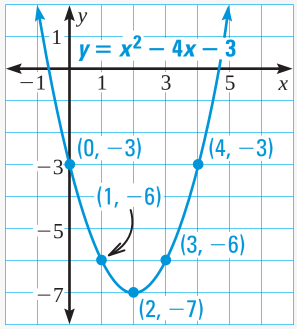
7. $\sqrt{45}$ **8.** $\sqrt{441}$ **9.** $\sqrt{\frac{36}{64}}$ **10.** $\sqrt{\frac{99}{25}}$

9.3 GRAPHING QUADRATIC FUNCTIONS

Examples on pp. 518–520

EXAMPLE To sketch the graph of $y = x^2 - 4x - 3$, first find the x-coordinate of the vertex: $-\frac{b}{2a} = -\frac{-4}{2(1)} = 2$. Make a table of values using x-values to the left and right of $x = 2$. Plot the points and connect them to form a parabola.

x	-1	0	1	2	3	4	5
y	2	-3	-6	-7	-6	-3	2



Sketch the graph of the function. Label the vertex.

11. $y = x^2 - x - 5$ **12.** $y = -x^2 - 3x + 2$ **13.** $y = -4x^2 + 6x + 3$

9.4 SOLVING QUADRATIC EQUATIONS BY GRAPHING

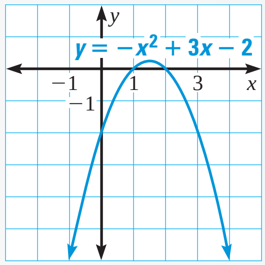
Examples on pp. 526–528

EXAMPLE To solve the quadratic equation $-x^2 + 3x = 2$ by graphing, first rewrite the equation in the form $ax^2 + bx + c = 0$.

$$-x^2 + 3x - 2 = 0.$$

Sketch the graph of the related function $y = -x^2 + 3x - 2$.

▶ From the graph, the x-intercepts appear to be $x = 1$ and $x = 2$. Check these in the original equation.



Solve the equation by graphing. Check the solution algebraically.

14. $x^2 + 2 = 3x$ **15.** $x^2 - 2x = 15$ **16.** $\frac{1}{2}x^2 + 5x = -8$ **17.** $-x^2 - 2x = -24$

9.5 SOLVING QUADRATIC EQUATIONS BY THE QUADRATIC FORMULA

Examples on pp. 533–535

EXAMPLE Solve equations of the form $ax^2 + bx + c = 0$ by substituting the values of a, b, and c into the quadratic formula. Solve $x^2 + 6x - 16 = 0$.

$$x = \frac{-6 \pm \sqrt{6^2 - 4(1)(-16)}}{2(1)} = \frac{-6 \pm \sqrt{36 + 64}}{2} = \frac{-6 \pm \sqrt{100}}{2} = \frac{-6 \pm 10}{2}$$

▶ The equation has two solutions: $x = \frac{-6 + 10}{2} = 2$ and $x = \frac{-6 - 10}{2} = -8$.

Use the quadratic formula to solve the equation.

18. $3x^2 - 4x + 1 = 0$ **19.** $-2x^2 + x + 6 = 0$ **20.** $10x^2 - 11x + 3 = 0$

9.6 APPLICATIONS OF THE DISCRIMINANT

Examples on pp. 541–543

EXAMPLES Use the discriminant, $b^2 - 4ac$, to find the number of solutions of a quadratic equation $ax^2 + bx + c = 0$.

EQUATION	DISCRIMINANT	NUMBER OF SOLUTIONS
$3x^2 + 6x + 2 = 0$	$6^2 - 4(3)(2) = 12$	2
$2x^2 + 8x + 8 = 0$	$8^2 - 4(2)(8) = 0$	1
$x^2 + 7x + 15 = 0$	$7^2 - 4(1)(15) = -11$	0

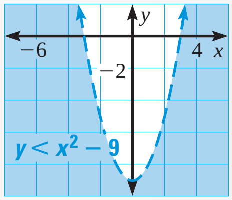
Tell if the equation has *two solutions*, *one solution*, or *no real solution*.

21. $3x^2 - 12x + 12 = 0$ **22.** $2x^2 + 10x + 6 = 0$ **23.** $-x^2 + 3x - 5 = 0$

9.7 GRAPHING QUADRATIC INEQUALITIES

Examples on pp. 548–550

EXAMPLE To sketch the graph of the quadratic inequality $y < x^2 - 9$, first sketch the graph of the parabola $y = x^2 - 9$. Use a dashed parabola for inequalities with $>$ or $<$, and a solid parabola for inequalities with $\geq$ or $\leq$.



Then test a point that is not on the parabola, say (0, 0). If the test point is a solution, shade its region. If not, shade the other region.

Sketch the graph of the inequality.

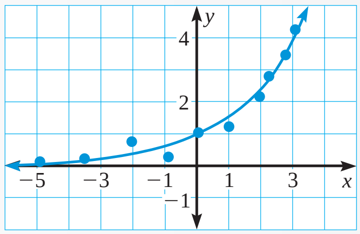
24. $x^2 - 3 \geq y$ **25.** $-x^2 - 2x + 3 \leq y$ **26.** $\frac{1}{2}x^2 + 3x - 4 < y$

9.8 COMPARING LINEAR, EXPONENTIAL, AND QUADRATIC MODELS

Examples on pp. 554–556

EXAMPLE To choose the type of model that best fits a set of data, first make a scatter plot of the data. Then decide whether the points lie on a line, an exponential curve, or a parabola.

The points in the graph appear to lie on an exponential curve. An exponential model could be used to model the set of data.



Make a scatter plot and name the type of model that best fits the data.

27. (−3, 4), (−2, 1), (−1, 0), (0, 1), (1, 4), (2, 9), (3, 16)

28. (−3, −7), (−2, −4), (−1, −1), (0, 2), (1, 5), (2, 8), (3, 11)

29. $\left(-3, \frac{1}{8}\right), \left(-2, \frac{1}{4}\right), \left(-1, \frac{1}{2}\right)$, (0, 1), (1, 2), (2, 4), (3, 8)

CHAPTER 9

Chapter Test

Simplify the expression.

1. $-\sqrt{9}$ **2.** $\sqrt{0.0064}$ **3.** $\pm\sqrt{121}$ **4.** $\sqrt{8^2 - 4(2)(8)}$

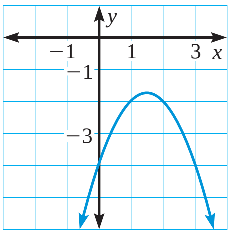
5. $\sqrt{192}$ **6.** $\sqrt{5} \cdot \sqrt{30}$ **7.** $\sqrt{\frac{27}{147}}$ **8.** $\frac{10\sqrt{8}}{\sqrt{16}}$

Solve the equation by finding square roots or using the quadratic formula.

9. $8x^2 = 800$ **10.** $4x^2 + 11 = 12$ **11.** $2x^2 + 5x - 7 = 0$

12. $-2x^2 + 4x + 6 = 0$ **13.** $64x^2 - 5 = 11$ **14.** $10x^2 + 17x - 11 = 0$

Match the equation or inequality with its graph.

A.


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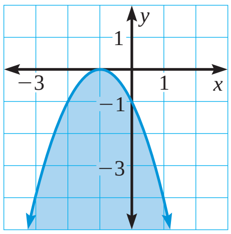

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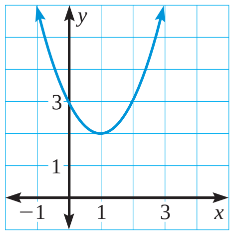

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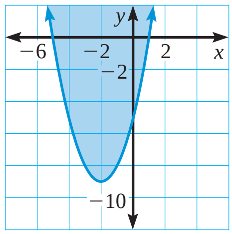


15. $y = x^2 - 2x + 3$ **16.** $y = -x^2 + 3x - 4$ **17.** $y \geq x^2 + 4x - 5$ **18.** $y \leq -x^2 - 2x - 1$

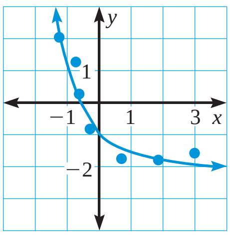
Decide how many solutions the equation has. Check the results by graphing.

19. $5x^2 - 20x - 60 = 0$ **20.** $-3x^2 + 4x - 2 = 0$ **21.** $x^2 - 4x + 4 = 0$

Sketch the graph of the equation or the inequality.

22. $y = \frac{1}{3}x^2 + 2x - 3$ **23.** $y = -x^2 + 5x - 6$ **24.** $y < x^2 + 7x + 6$ **25.** $y \leq -\frac{1}{2}x^2 + 2x + \frac{5}{2}$

Name the type of model suggested by the graph.

26.


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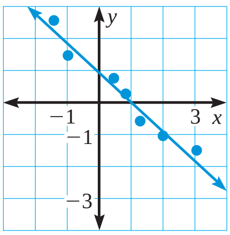

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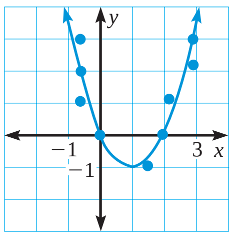

VERTICAL MOTION In Exercises 29–31, you are standing on a bridge over a creek, holding a stone 20 feet above the water.

29. You release the stone. How long will it take the stone to hit the water?

30. You take another stone and toss it straight up with an initial velocity of 30 feet per second. How long will it take the stone to hit the water?

31. If you throw a stone straight up into the air with an initial velocity of 50 feet per second, could the stone reach a height of 60 feet above the water?