10.3 Special Products of Polynomials

What you should learn

GOAL 1 Use special product patterns for the product of a sum and a difference, and for the square of a binomial.

GOAL 2 Use special products as **real-life** models such as a genetic model in **Example 5**.

Why you should learn it

▼ To solve **real-life** problems like finding the percent of second-generation tigers that are white in **Exs. 49 and 50**.

GOAL 1 USING SPECIAL PRODUCT PATTERNS

As you learned in Lesson 10.2, you can always use the FOIL pattern to multiply two binomials. Some pairs of binomials have *special products*. If you learn to recognize these pairs, finding the product of two binomials will sometimes be quicker and easier.

ACTIVITY
Developing Concepts

Investigating Special Product Patterns

1. Use the FOIL pattern to find the products in each group.

1.	**2.**	**3.**
$(x - 2)(x + 2)$	$(x + 3)^2$	$(z - 2)^2$
$(2n + 3)(2n - 3)$	$(3m + 1)^2$	$(6x - 4)^2$
$(4t - 1)(4t + 1)$	$(5s + 2)^2$	$(5p - 7)^2$
$(x + y)(x - y)$	$(x + y)^2$	$(x - y)^2$

2. Describe any patterns that you see in each group.

In this activity, you may have noticed that the product of the sum and difference of two terms has no "middle term." Also, you may have noticed that in the square of a binomial the middle term is twice the product of the terms in the binomial.

SPECIAL PRODUCT PATTERNS

SUM AND DIFFERENCE PATTERN

$(a + b)(a - b) = a^2 - b^2$ **Example:** $(3x - 4)(3x + 4) = 9x^2 - 16$

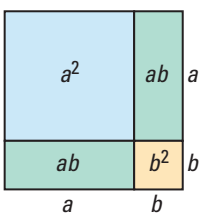
SQUARE OF A BINOMIAL PATTERN

$(a + b)^2 = a^2 + 2ab + b^2$ **Example:** $(x + 4)^2 = x^2 + 8x + 16$

$(a - b)^2 = a^2 - 2ab + b^2$ **Example:** $(2x - 6)^2 = 4x^2 - 24x + 36$

The area model shown at the right gives a geometric representation of the *square of a binomial* pattern $(a + b)^2 = a^2 + 2ab + b^2$.

The area of the large square is $(a + b)^2$, which is equal to the sum of the areas of the two small squares and two rectangles. Note that the two rectangles with area ab produce the middle term $2ab$.



STUDENT HELP

Study Tip

When you use these special product patterns, remember that a and b can be numbers, variables, or sometimes variable expressions.

EXAMPLE 1 Using the Sum and Difference Pattern

Find the product $(5t - 2)(5t + 2)$.

SOLUTION

$$(a - b)(a + b) = a^2 - b^2 \qquad \text{Write pattern.}$$

$$(5t - 2)(5t + 2) = (5t)^2 - 2^2 \qquad \text{Apply pattern.}$$

$$= 25t^2 - 4 \qquad \text{Simplify.}$$

✓**CHECK** You can use the FOIL pattern to check your answer.

$$(5t - 2)(5t + 2) = (5t)(5t) + (5t)(2) + (-2)(5t) + (-2)(2) \qquad \text{Use FOIL.}$$

$$= 25t^2 - 4 \qquad \text{Simplify.}$$

EXAMPLE 2 Squaring a Binomial

Find the product.

a. $(3n + 4)^2$ **b.** $(2x - 7y)^2$

SOLUTION

a. $(a + b)^2 = a^2 + 2ab + b^2$ Write pattern.

$(3n + 4)^2 = (3n)^2 + 2(3n)(4) + 4^2$ Apply pattern.

$= 9n^2 + 24n + 16$ Simplify.

b. $(a - b)^2 = a^2 - 2ab + b^2$ Write pattern.

$(2x - 7y)^2 = (2x)^2 - 2(2x)(7y) + (7y)^2$ Apply pattern.

$= 4x^2 - 28xy + 49y^2$ Simplify.

. . . . . .

The special product patterns can help you use mental math to find products.

EXAMPLE 3 Special Products and Mental Math

Use mental math to find the product.

a. $17 \cdot 23$ **b.** 29^2

SOLUTION

a. $17 \cdot 23 = (20 - 3)(20 + 3)$ Write as product of difference and sum.

$= 400 - 9$ Apply pattern.

$= 391$ Simplify.

b. $29^2 = (30 - 1)^2$ Write as square of binomial.

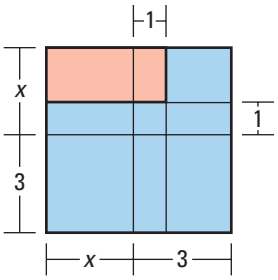
$= 900 - 60 + 1$ Apply pattern.

$= 841$ Simplify.

GOAL 2 APPLYING SPECIAL PRODUCTS

EXAMPLE 4 *Finding an Area*

GEOMETRY CONNECTION Find an expression for the area of the blue region.



SOLUTION

PROBLEM SOLVING STRATEGY

VERBAL MODEL

$$\text{Area of blue region} = \text{Area of entire square} - \text{Area of red region}$$

LABELS

Area of blue region = A (square units)

Area of entire square = $(x + 3)^2$ (square units)

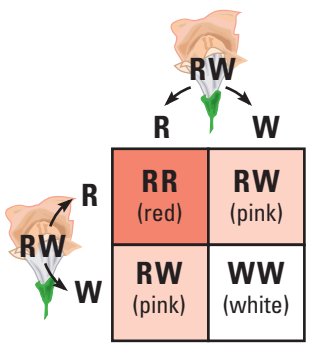
Area of red region = $(x + 1)(x - 1)$ (square units)

ALGEBRAIC MODEL

$$\begin{aligned} A &= (x + 3)^2 - (x + 1)(x - 1) && \text{Write algebraic model.} \\ &= (x^2 + 6x + 9) - (x^2 - 1) && \text{Apply pattern.} \\ &= x^2 + 6x + 9 - x^2 + 1 && \text{Use distributive property.} \\ &= 6x + 10 && \text{Simplify.} \end{aligned}$$

EXAMPLE 5 *Modeling a Punnett Square*

SCIENCE CONNECTION The Punnett square at the right is an area model that shows the possible results of crossing two pink snapdragons, each with one red gene R and one white gene W. Each parent snapdragon passes along only one gene for color to its offspring. Show how the square of a binomial can be used to model the Punnett square.



SOLUTION

Each parent snapdragon has half red genes and half white genes. You can model the genetic makeup of each parent as $0.5R + 0.5W$. The genetic makeup of the offspring can be modeled by the product $(0.5R + 0.5W)^2$.

$$\begin{aligned} (0.5R + 0.5W)^2 &= (0.5R)^2 + 2(0.5R)(0.5W) + (0.5W)^2 \\ &= 0.25R^2 + 0.5RW + 0.25W^2 \end{aligned}$$

Red Pink White

▶ 25% of the offspring will be red, 50% will be pink, and 25% will be white.

FOCUS ON APPLICATIONS

REAL LIFE **PUNNETT SQUARES** are used in genetics to model the mixing of parents' genes in the resulting offspring.

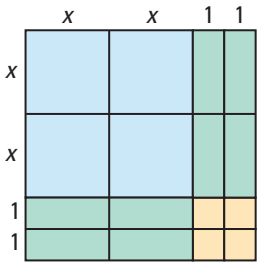
GUIDED PRACTICE

Vocabulary Check ✓

Concept Check ✓

1. What is the sum and difference pattern?

2. Write two expressions for the product shown by the area model at the right.



3. Tell whether the following statement is *true* or *false*. *The product of* $(a + b)$ *and* $(a + b)$ *is* $a^2 + b^2$. Explain.

4. Find the missing term: $(a - b)^2 = a^2 - \boxed{?} + b^2$.

Skill Check ✓

5. Show the product $(x + 2)^2$ using an area model or algebra tiles. Draw a sketch of the model and use it to help you write $(x + 2)^2$ as a trinomial.

Use a special product pattern to find the product.

6. $(x - 6)^2$ **7.** $(w + 11)(w - 11)$ **8.** $(6 + p)^2$

9. $(2y - 3)^2$ **10.** $(3z + 2)^2$ **11.** $(t - 6)(t + 6)$

12. $(x - 3)(x - 3)$ **13.** $(2y + 5)(2y - 5)$ **14.** $(4n + 3)^2$

PRACTICE AND APPLICATIONS

STUDENT HELP

Extra Practice to help you master skills is on p. 806.

SUM AND DIFFERENCE PATTERN **Write the product of the sum and difference.**

15. $(x + 3)(x - 3)$ **16.** $(y - 1)(y + 1)$ **17.** $(2m + 2)(2m - 2)$

18. $(3b - 1)(3b + 1)$ **19.** $(3 + 2x)(3 - 2x)$ **20.** $(6 - 5n)(6 + 5n)$

SQUARE OF A BINOMIAL **Write the square of the binomial as a trinomial.**

21. $(x + 5)^2$ **22.** $(a + 8)^2$ **23.** $(3t + 1)^2$

24. $(2s - 4)^2$ **25.** $(4b - 3)^2$ **26.** $(x - 7)^2$

SPECIAL PRODUCT PATTERNS **Find the product.**

27. $(x + 4)(x - 4)$ **28.** $(x - 3)(x + 3)$ **29.** $(3x + 1)(3x - 1)$

30. $(6x + 5)(6x - 5)$ **31.** $(a + 2b)(a - 2b)$ **32.** $(4n - 8m)(4n + 8m)$

33. $(3y + 8)^2$ **34.** $(9 - 4t)(9 + 4t)$ **35.** $\left(2x + \frac{1}{2}\right)\left(2x - \frac{1}{2}\right)$

36. $(-5 - 4x)^2$ **37.** $(3s + 4t)(3s - 4t)$ **38.** $(-a - 2b)^2$

STUDENT HELP

HOMEWORK HELP

Example 1: Exs. 15–20, 27–42
Example 2: Exs. 21–42
Example 3: Exs. 43–46
Example 4: Exs. 47, 48
Example 5: Exs. 49–52

CHECKING SOLUTIONS **Tell whether the statement is *true* or *false*. If the statement is false, rewrite the right-hand side to make the statement true.**

39. $(9x + 8)(9x - 8) \stackrel{?}{=} 81x^2 - 64$ **40.** $(6y - 7w)^2 \stackrel{?}{=} 36y^2 - 49w^2$

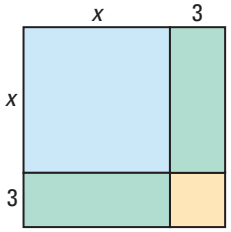
41. $\left(\frac{1}{3}a + 3b\right)^2 \stackrel{?}{=} \frac{1}{9}a^2 + 2ab + 9b^2$ **42.** $\left(\frac{2}{7}n - 3m\right)\left(\frac{2}{7}n - 3m\right) \stackrel{?}{=} \frac{4}{49}n^2 - 9m^2$

MENTAL MATH **Use mental math to find the product.**

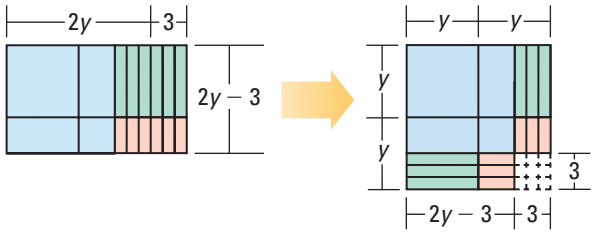
43. $26 \cdot 34$ **44.** $45 \cdot 55$ **45.** 16^2 **46.** 41^2

AREA MODELS **Write two expressions for the area of the figure. Describe the special product pattern that is represented.**

47.



48.



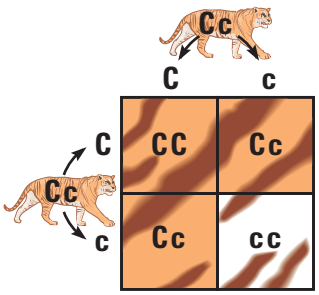
STUDENT HELP

HOMEWORK HELP
Visit our Web site www.mcdougallittell.com for help with Exs. 49–55.

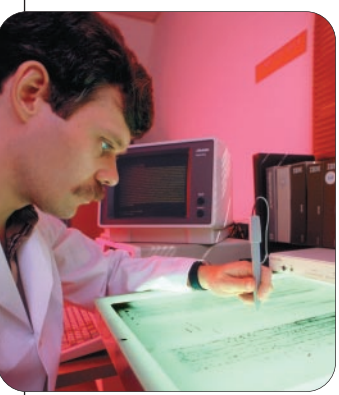
SCIENCE CONNECTION **In tigers, the normal color gene *C* is *dominant* and the white color gene *c* is *recessive*. This means that a tiger whose color genes are *CC* or *Cc* will have normal coloring. A tiger whose color genes are *cc* will be white.**

49. The Punnett square at the right shows the possible results of crossing two tigers that have recessive white genes. Find a model that can be used to represent the Punnett square and write the model as a polynomial.

50. What percent of the offspring will have normal coloring? What percent will be white?



FOCUS ON CAREERS



REAL LIFE **GENETICISTS** do research on the biological and chemical processes that result in the the physical traits of plants and animals.

CAREER LINK
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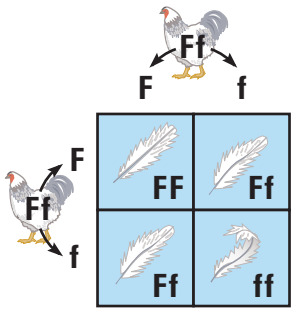
SCIENCE CONNECTION **In Exercises 51 and 52, use the following information.** In chickens, neither the normal-feathered gene *F* nor the frizzle-feathered gene *f* is dominant. So chickens whose feather genes are *FF* will have normal feathers. Chickens with *Ff* will have mildly frizzled feathers. Chickens with *ff* will have extremely frizzled feathers.

51. The Punnett square at the right shows the possible results of crossing two chickens with mildly frizzled feathers. Find a model that can be used to represent the Punnett square and write the model as a polynomial.

52. What percent of the offspring will have normal feathers? What percent will have mildly frizzled feathers? What percent will have extremely frizzled feathers?



INVESTMENT VALUE **In Exercises 53–55, an investment of *P* dollars that gains *r* percent of its value in one year is worth $P(1 + r)$ at the end of that year. An investment that loses *r* percent of its value in one year is worth $P(1 - r)$ at the end of that year.**

53. Write a model for the value of an investment P that loses r percent one year, then gains r percent the following year.

54. According to the model, did the investment increase or decrease in value? By how much?

55. If the investment gains r percent the first year and loses r percent the second year, what is the increase or decrease in the value of the investment?

Test Preparation

QUANTITATIVE COMPARISON **In Exercises 56 and 57, choose the statement that is true about the given numbers.**

Ⓐ The number in column A is greater.

Ⓑ The number in column B is greater.

Ⓒ The two numbers are equal.

Ⓓ The relationship cannot be determined from the given information.

	Column A	Column B
56.	$(3x - 7)(3x + 7)$ when $x = 4$	$(3x - 7)^2$ when $x = 4$
57.	$a^2 + b^2$ when $a = 1$ and $b = -2$	$(a + b)^2$ when $a = 1$ and $b = -2$

58. MULTIPLE CHOICE Which of the following is equal to $(3x + 2)^2 - (x + 5)(x - 5)$?

Ⓐ $8x^2 + 29$ Ⓑ $8x^2 + 12x - 21$

Ⓒ $10x^2 + 6x + 29$ Ⓓ $8x^2 + 12x + 29$

Challenge

WRITING FACTORS **Write a pair of factors that have the given product.**

59. $x^2 + 12x + 36$ **60.** $x^2 - 4$

61. $25x^2 - 30x + 9$ **62.** $49x^2 - 169$

MIXED REVIEW

SIMPLIFYING EXPRESSIONS **Simplify the expression. The simplified expression should have no negative exponents.** (Review 8.3)

63. $\left(\frac{x}{4}\right)^3$ **64.** $\frac{x^3}{x^2}$ **65.** $\left(\frac{4x}{y^3}\right)^3$

66. $x^7 \cdot \frac{1}{x^4}$ **67.** $\frac{3x^2y}{2x} \cdot \frac{6xy^2}{y^3}$ **68.** $\frac{5x^4y}{3xy^2} \cdot \frac{9xy}{x^2y}$

FINDING PARTS OF A GRAPH **Find the coordinates of the vertex and write the equation of the axis of symmetry.** (Review 9.3 for 10.4)

69. $y = 2x^2 + 3x + 6$ **70.** $y = 3x^2 - 9x - 12$ **71.** $y = -x^2 + 4x + 16$

72. $y = -4x^2 - 2x + 5$ **73.** $y = -\frac{1}{2}x^2 + 6x - 4$ **74.** $y = \frac{1}{6}x^2 - \frac{1}{3}x + 2$

CHECKING GRAPHICALLY **Solve the equation algebraically. Check the solutions graphically.** (Review 9.4)

75. $x^2 - 10 = 6$ **76.** $x^2 + 12 = 48$ **77.** $\frac{1}{5}x^2 = 5$

78. $3x^2 = 192$ **79.** $\frac{2}{3}x^2 = 6$ **80.** $2x^2 - 66 = 96$

81. GEOMETRY CONNECTION Is it possible for a rectangle with a perimeter of 52 centimeters to have an area of 148.75 square centimeters? Explain. (Review 9.6)

QUIZ 1

Self-Test for Lessons 10.1–10.3

Use a vertical format or a horizontal format to add or subtract. (Lesson 10.1)

1. $(2x^2 + 7x + 1) + (x^2 - 2x + 8)$
2. $(-4x^3 - 5x^2 + 2x) - (2x^3 + 9x^2 + 2)$
3. $(7t^2 - 3t + 5) - (4t^2 + 10t - 9)$
4. $(5x^3 - x^2 + 3x + 3) + (x^3 - 4x^2 + x)$

Find the product. (Lesson 10.2)

5. $(x + 8)(x - 1)$
6. $(4n + 7)(4n - 7)$
7. $(-2x^2 + x - 4)(x - 2)$
8. $(9m - 4)(3m + 1)$
9. $\left(\frac{1}{2}x - 3\right)\left(\frac{1}{2}x + 5\right)$
10. $-x^2(12x^3 - 11x^2 + 3)$

Use special products to find the product. (Lesson 10.3)

11. $(x - 6)(x + 6)$
12. $(4x + 3)(4x - 3)$
13. $(5 + 3b)(5 - 3b)$
14. $(2x - 7y)(2x + 7y)$
15. $(3x + 6)^2$
16. $(-6 - 8x)^2$

17. **NEW PATIO** You are making a square concrete patio. You want a brick border that is 8 inches wide around the outer edge of the patio, and the total length of a side is $2x$ inches. Draw a diagram and write a polynomial for the area of the floor of the patio not including the brick border. (Lesson 10.3)

MATH & History — Plant Genetics

APPLICATION LINK
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THEN

IN 1866, Gregor Mendel identified the pattern in which pea plants passed their characteristics on to the next generation of plants. In Lesson 10.3, you used special products to model genetic patterns, as shown in Punnett squares.

Mendel identified the pattern for pea plants by breeding many plants under identical circumstances. This enabled him to calculate the experimental probability that a given generation of pea plants would have a given characteristic.

Use the table. Find the experimental probability. (see p. 115)

1. In trial 1, what is the probability that a pea plant is tall?
2. In trial 2, what is the probability that a pea plant is short?

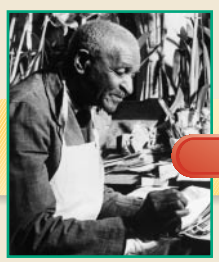
	Short plants	Tall plants
Trial 1	39	121
Trial 2	81	235

NOW

TODAY, knowledge of genetics helps researchers grow disease-resistant crops, more nutritious vegetables, taller trees, and more varieties of flowers.

Machinery dramatically increases farm production.

1860–1890



1896

George Washington Carver heads Agriculture Dept. at Tuskegee Institute.

Barbara McClintock receives Nobel Prize for her work on the genetics of corn.

1983